

CAM-SE-CSLAM: Consistent finite-volume transport with spectral-element dynamics



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Introduction

CAM-SE (Community Atmosphere Model – Spectral Elements) is based on a continuous Galerkin spectral finite element method in the horizontal directions and a hybrid sigma-pressure floating Lagrangian vertical coordinate.

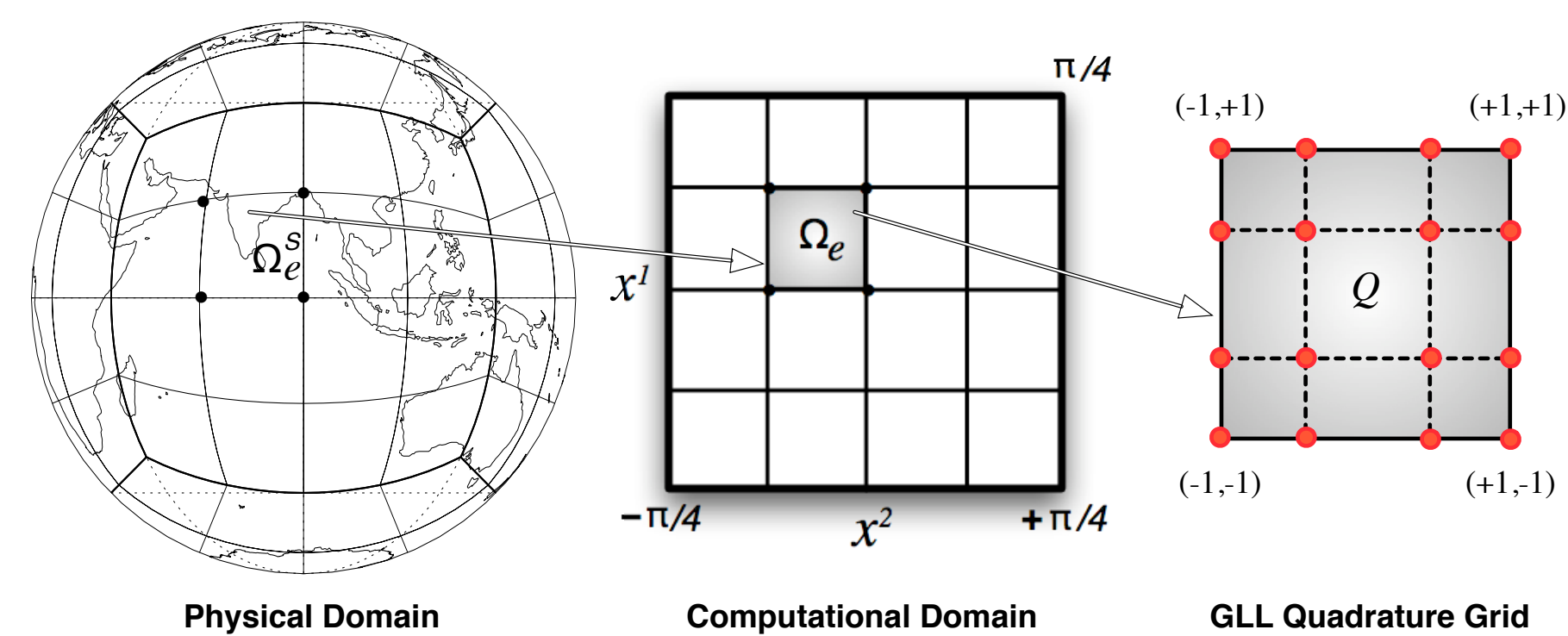


Fig. 9.22 A schematic diagram showing the mapping between each spherical tile (element) Ω_e^c of the physical domain (cubed-sphere) \mathcal{S} onto a planar element Ω on the computational domain (cube). For a DG discretization each element on the cube is further mapped onto a unique reference element Q , which is defined by the Gauss-Lobatto-Legendre (GLL) quadrature points. The horizontal discretization of the HOMME dynamical cores relies on this grid system.

Properties:

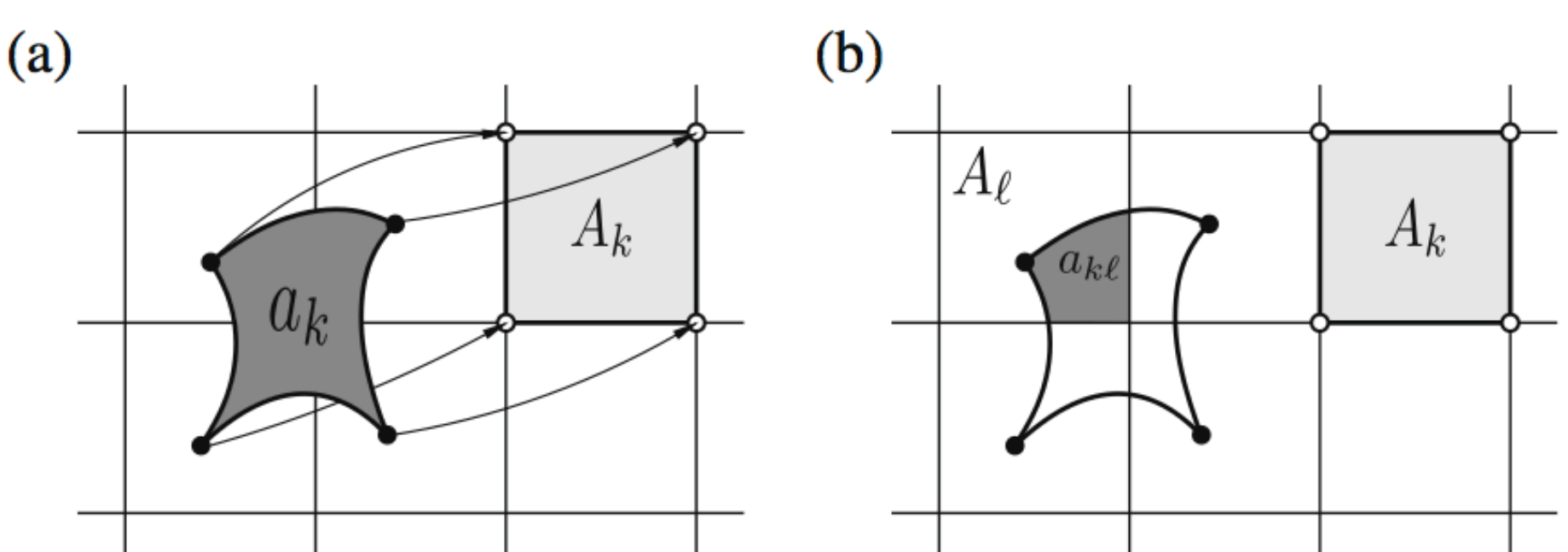
- Discretization is mimetic \Rightarrow mass-conservation and total energy conservation on element
- Conserves axial angular momentum very well (Lauritzen et al., 2014)
- Supports static mesh-refinement and retains formal order of accuracy!
- Highly scalable
- AMIP-climate similar to current model
- Computational throughput for many-tracer applications

CSLAM (Conservative semi-Lagrangian Multi-tracer scheme) is based on a cell-integrated semi-Lagrangian approach that, unlike CAM-SE, allows for long time-steps, is locally and globally conservative, has a linear correlation shape-preserving limiter/filter and is geometrically very flexible meaning that the method can accommodate any spherical grid constructed from great-circle arcs. Since geometric information computed for the transport of one trace species can be re-used for each additional tracer, the scheme (CSLAM) is termed multi-tracer efficient.

Consider the finite-volume Lagrangian form of continuity equation for air (pressure level thickness, Δp), and tracer (mixing ratio, q):

$$\int_{A_k} \psi_k^{n+1} dA = \int_{a_k} \psi_k^n dA, \quad \psi = \Delta p, \Delta p q, \quad (1)$$

where n is time-level index.



Problem: Coupling

The continuity equations for air and tracers are coupled:

$$\int_{A_k} \Delta p_k^{n+1} dA = \int_{a_k} \delta p_k^n dA, \quad (2)$$

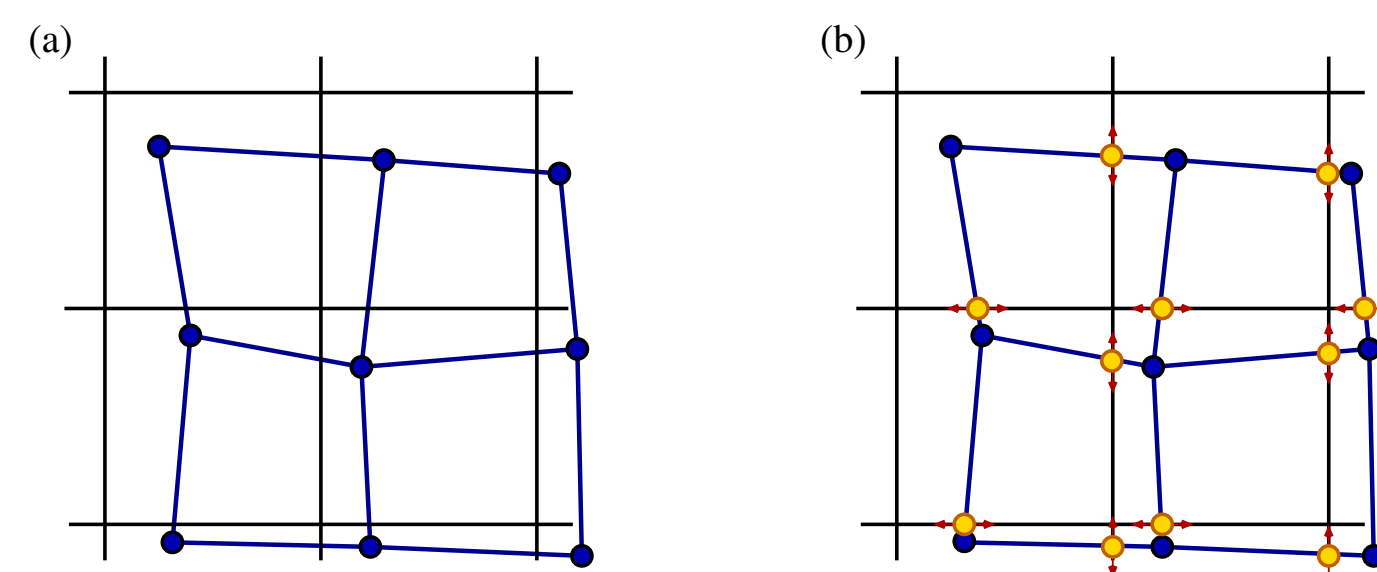
$$\int_{A_k} (\Delta p q)_k^{n+1} dA = \int_{a_k} (\delta p_k q)^n dA. \quad (3)$$

If $q = 1$ then (3) should reduce to (2).

The continuity equation for air mass (2) is solved with CAM-SE and the continuity for tracers (3) is solved with CSLAM. We need the CSLAM solution (3) to reduce to the CAM-SE solution for air mass (2) when $q=1$:

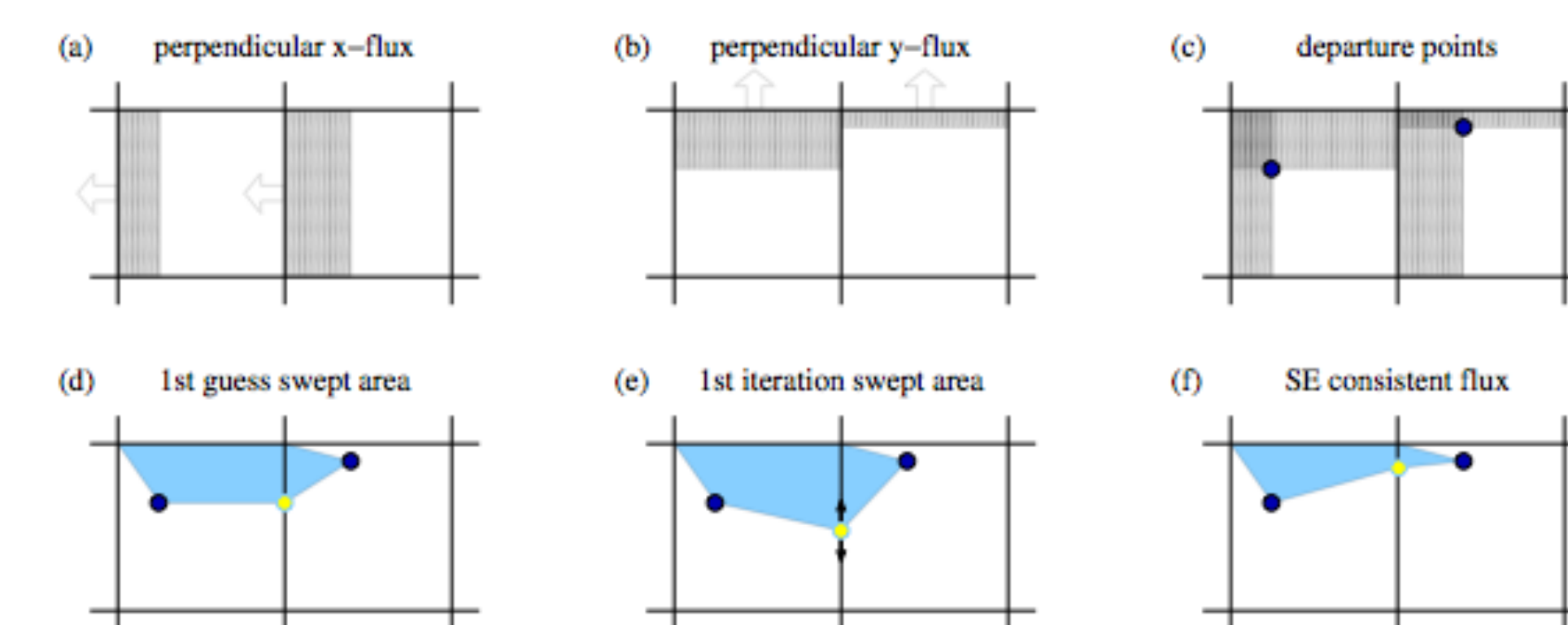
Find upstream area, a_k , so that CSLAM predicted mass field is equal to CAM-SE predicted mass field integrated over a CSLAM control volume A_k :

$$\int_{A_k} \Delta p^{n+1}(\text{CAM-SE}) dA = \int_{a_k} \delta p^n(\text{CSLAM}) dA \quad (4)$$



Solution: flux-form

Consistent SE-CSLAM algorithm: step-by-step example



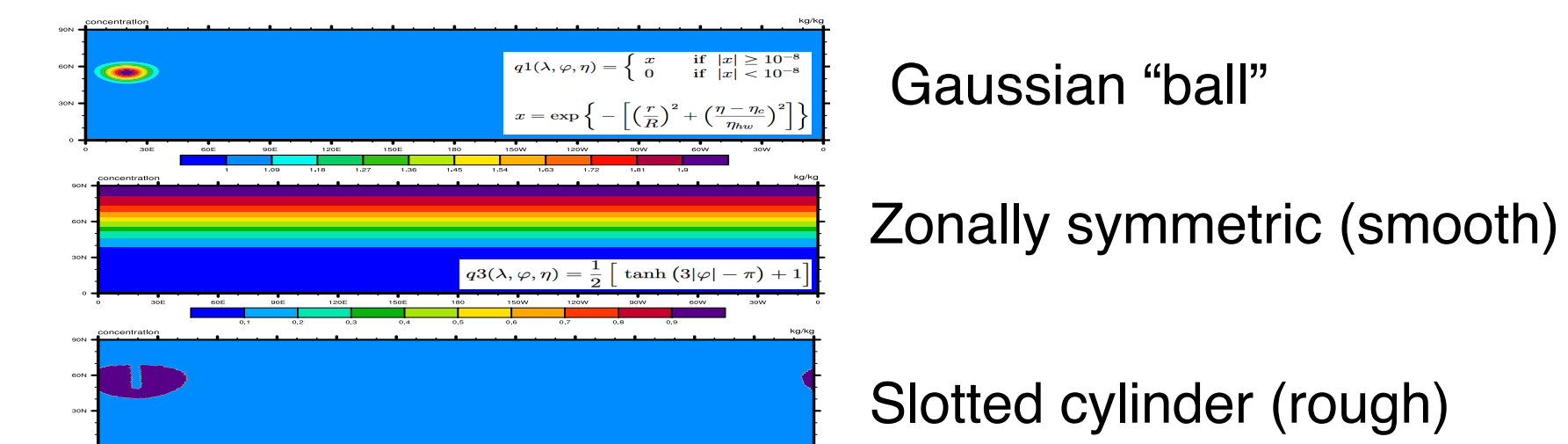
Well-posed? As long as flow deformation $|\frac{\partial u}{\partial x}| \Delta t \leq 1$ (Lipschitz criterion)

- Compute air mass-flux through CSLAM control volume sides using method developed by Taylor et al.
- Find swept fluxes (using Newton iteration) so that CSLAM swept flux matches CAM-SE flux to round-off:

- a-b: Find perpendicular (x and y) CSLAM fluxes that match CAM-SE fluxes
- c: flux areas define departure points
- d: add extra point to swept side and iterate so that 2D CSLAM flux match CAM-SE

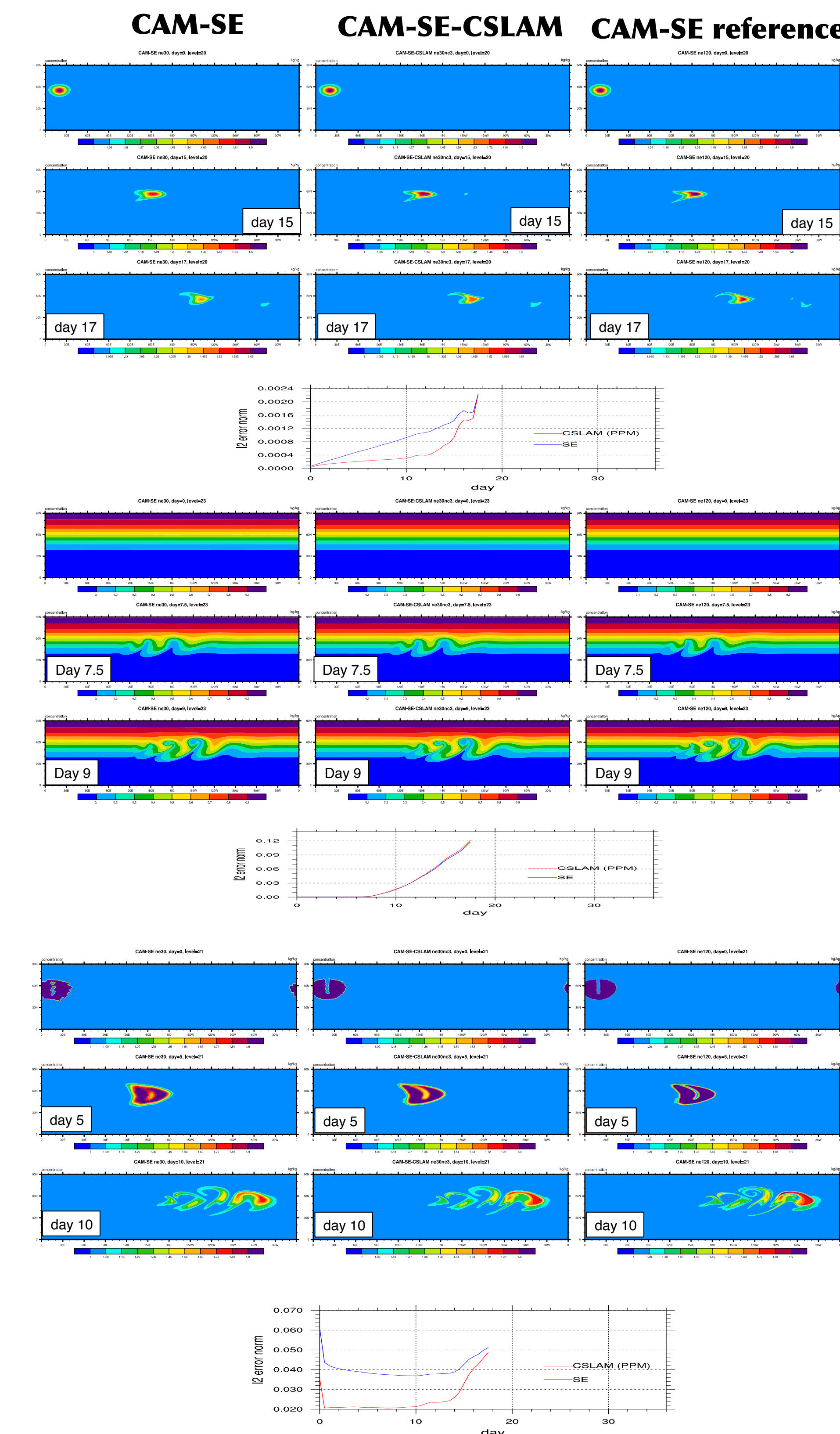
Results

Setup: Transport inert and passive tracers in Jablonowski & Williamson (2006) baroclinic wave



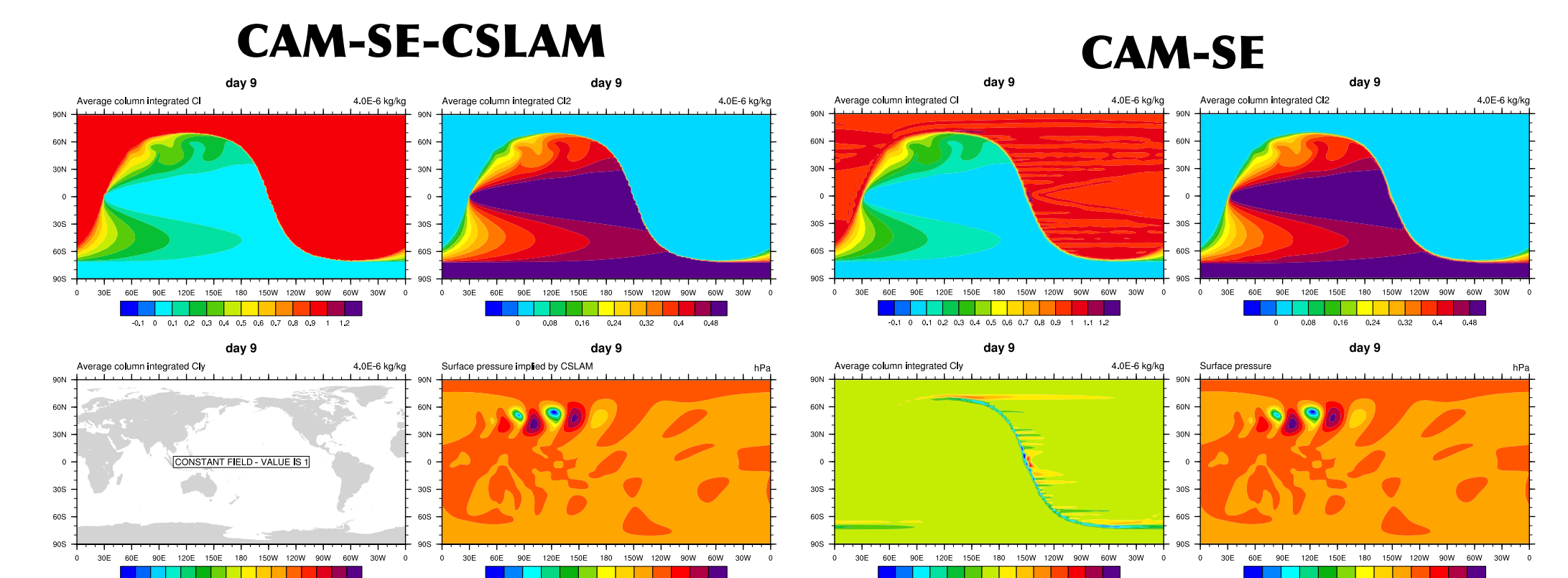
Simulations are performed with 30 vertical levels with 1 degree CAM-SE (left column), CAM-SE-CSLAM (middle column) and reference solution is 0.25 degree CAM-SE.

Surface pressure evolution for CAM-SE and CAM-SE-CSLAM match to round-off every time-step (not shown)



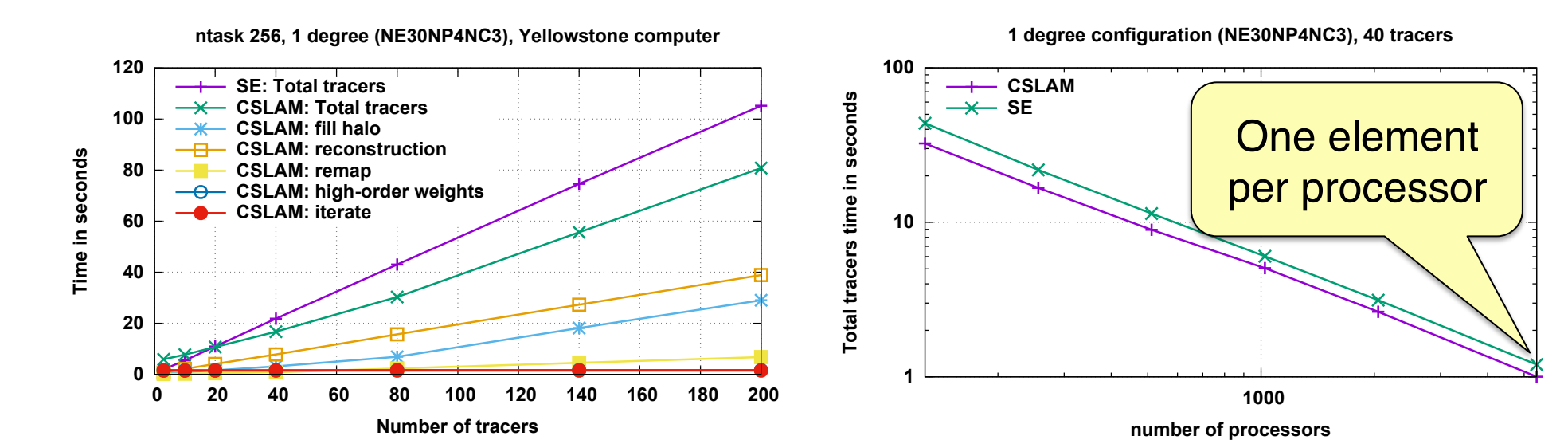
The terminator 'toy'-chemistry test: A simple tool to assess errors in transport schemes (Lauritzen et al., 2015, GMD)

Transport 2 reactive species (C1 and C2) in Jablonowski and Williamson (2006) flow. The sources and sinks for C1 and C2 are given by a simple, but non-linear, "toy" chemistry that mimics photolysis-driven conditions near the solar terminator, where strong gradients in the spatial distribution of the species develop near its edge. Despite the large spatial variations in each species, the weighted sum $C1y=2C1+C2$ should always be preserved



Performance

Throughput data produced on NCAR's Yellowstone supercomputer.



Summary

- CSLAM has been consistently coupled with spectral element (SE) dynamical core, i.e. CSLAM conserves tracer mass, is shape-preserving (monotone), and preserves a constant mixing ratio distribution (free-stream preserving).
- CSLAM takes a 3x longer time-step for tracers than SE
- CSLAM has been shown to be more accurate for tracers with steep gradients compared to SE and equally accurate for smooth tracer distributions
- CSLAM preserves linear correlation even under forced conditions in 'toy' terminator chemistry test whereas SE does not
- CSLAM is faster than SE when transporting more than approximately 18 tracers
- Even though CSLAM needs a halo of 3 cell width, it scales to one element (3x3 CSLAM control volumes) per processor