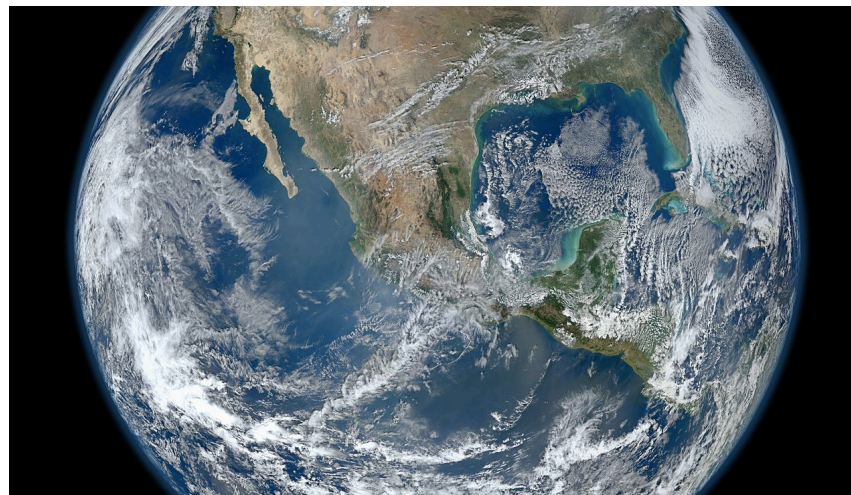


Desired Properties of Transport Schemes for Coupled Atmospheric-Chemistry Models



Peter Hjort Lauritzen

**Atmospheric Modeling and Prediction Section (AMP)
Climate and Global Dynamics Laboratory (CGD)
National Center for Atmospheric Research (NCAR)**



NCAR

Meteorology And Climate Modeling for Air Quality (MAC-MAQ)
Sacramento, September 16-18, 2015



Desired Properties of Transport Schemes for Coupled Atmospheric-Chemistry Models

AMP develops and maintains NCAR's global **Community Atmosphere Model (CAM)** which is a component of NCAR's Community Earth System Model (CESM)

I specialize in the resolved-scale fluid flow solver (a.k.a. the dynamical core) and its coupling to physics (sub-grid-scale parameterizations)

**Atmospheric Modeling and Prediction Section (AMP)
Climate and Global Dynamics Laboratory (CGD)
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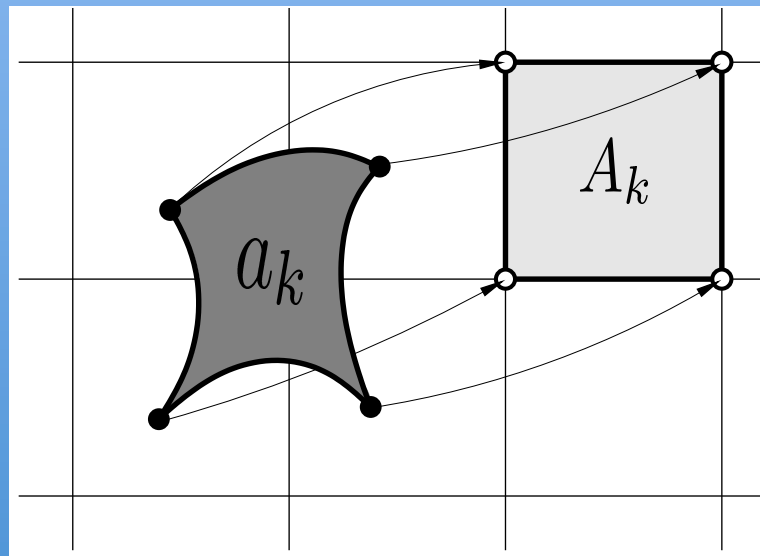
Desired Properties of Transport Schemes for Coupled Models

A1

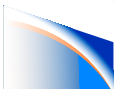
Consider the finite-volume Lagrangian form of continuity equation for air (pressure level thickness, Δp), and tracer (mixing ratio, q):

$$\int_{A_k} \psi_k^{n+1} dA = \int_{a_k} \psi_k^n dA, \quad \psi = \Delta p, \Delta p q, \quad (1)$$

where n time-level.



**No sources/
sinks**



Overall, tracer transport needs to be:

A. Efficient (on massively parallel computers):

Most atmospheric models solve at least a handful of continuity equations and, in most cases, many more:

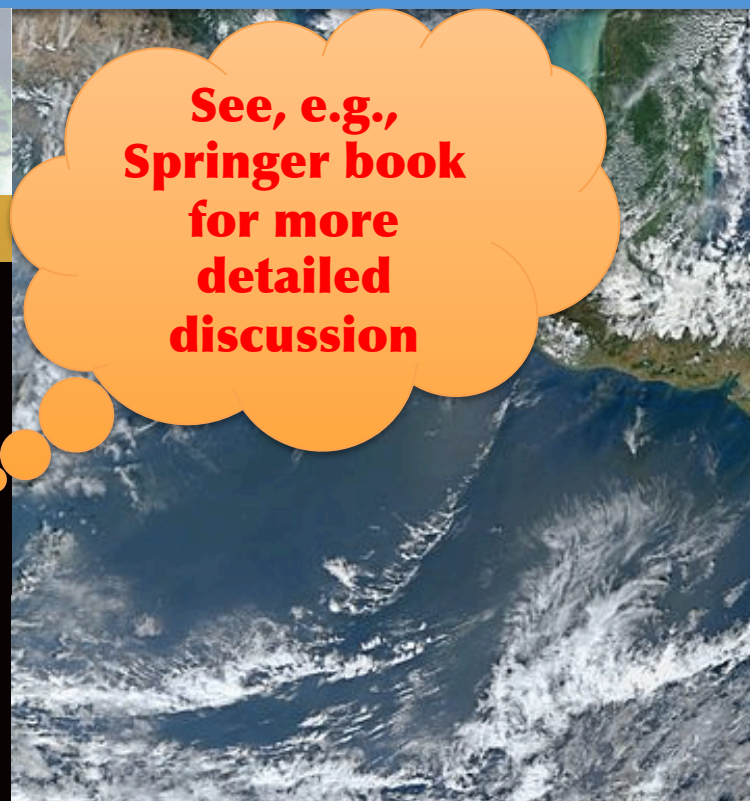
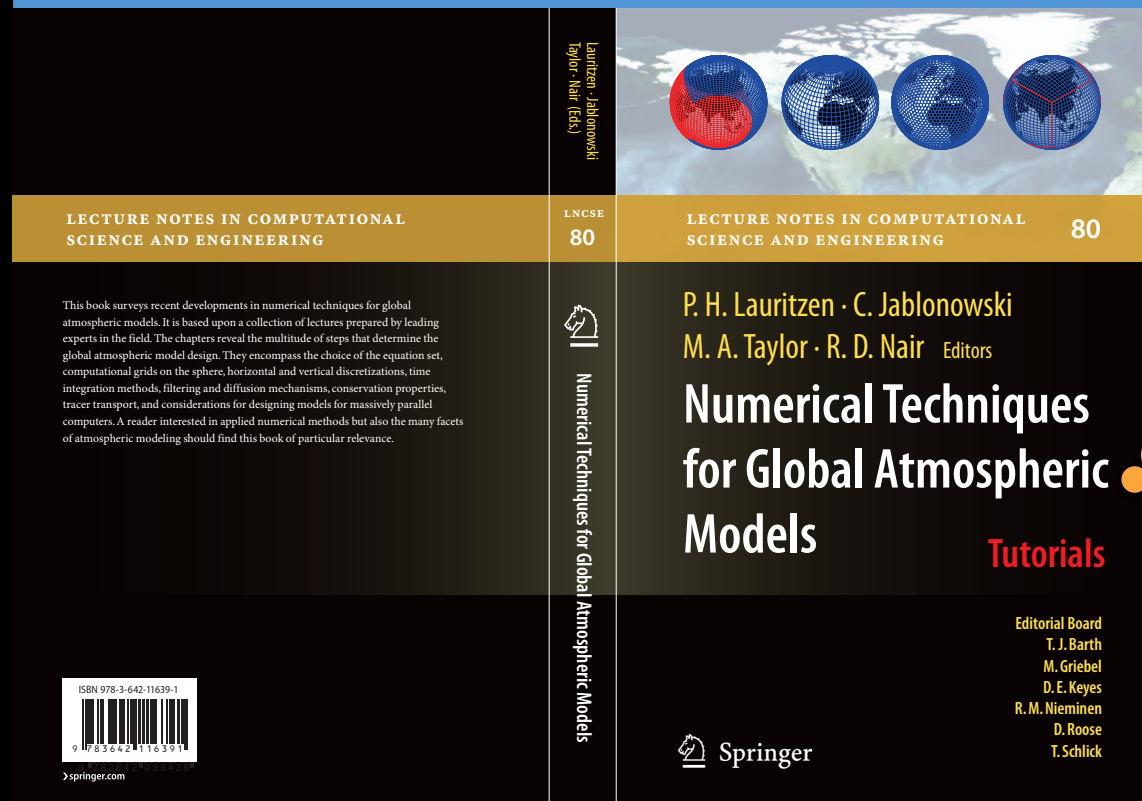
- **Air (either dry or moist)**
- **Water species: water vapor, cloud liquid water and cloud ice water. (rain, snow, hail, graupel, ...)**
- **Microphysics: aerosol mass and number concentration.**
- **Chemistry: 100+ tracers in chemistry version of CAM. E.g. ozone, chlorine compounds, bromine, ... (some highly reactive; some long lived)**

Tracer transport is the most computationally costly part of a dynamical core!

Overall, tracer transport needs to be:

B. Accurate:

Accuracy requirements in the context climate/chemistry-climate applications are much more stringent than conventional root-mean-square error convergence for smooth problems ...



Requirements (desirable properties) for transport schemes intended for global climate/climate-chemistry applications:

1. Global (and local) Mass-conservation

If Δp is pressure-level thickness and q is mixing ratio, then the total mass

$$M(t) = \int_{\Omega} \Delta p q dA,$$

is invariant in time: $M(t) = M(t = 0)$ (no sources/sinks)

2. Shape-preservation

Scheme does not produce new extrema (in particular negatives) in q

3. Preservation of pre-existing functional relations between tracers

Transport scheme preserves $q_2 = f(q_1)$ (no sources/sinks)

4. Consistency

The continuity equations for air and tracers are coupled:

$$\int_{A_k} \Delta p_k^{n+1} dA = \int_{a_k} \delta p_k^n dA, \quad (2)$$

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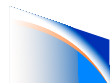
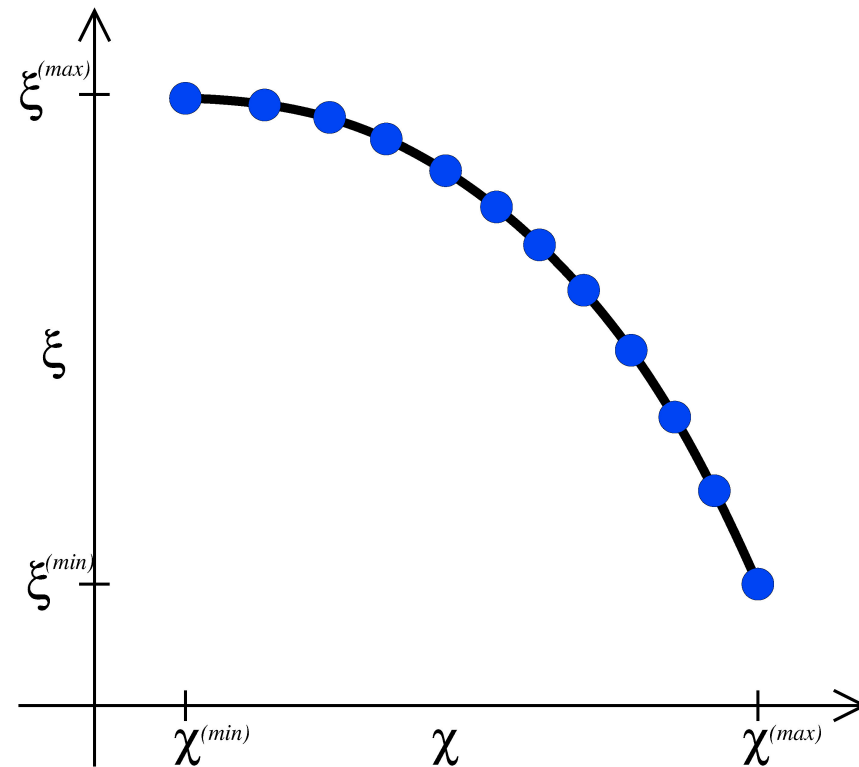
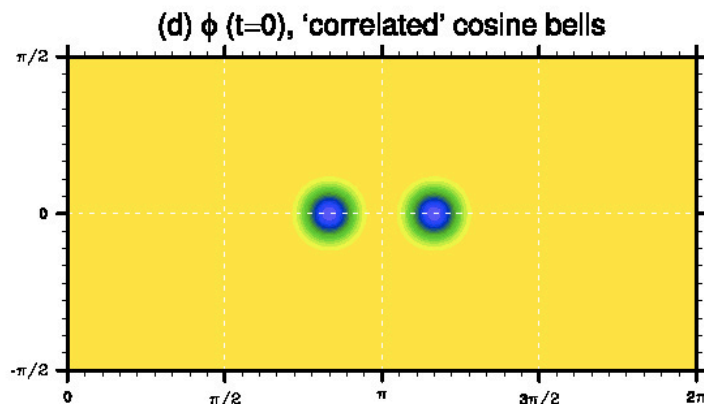
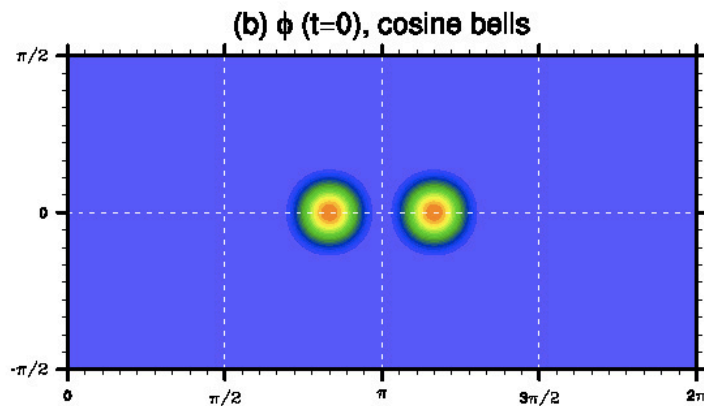
If $q = 1$ then (3) should reduce to (2).



Preserving pre-existing functional relation between tracers under challenging flow conditions

Initial conditions

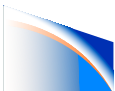
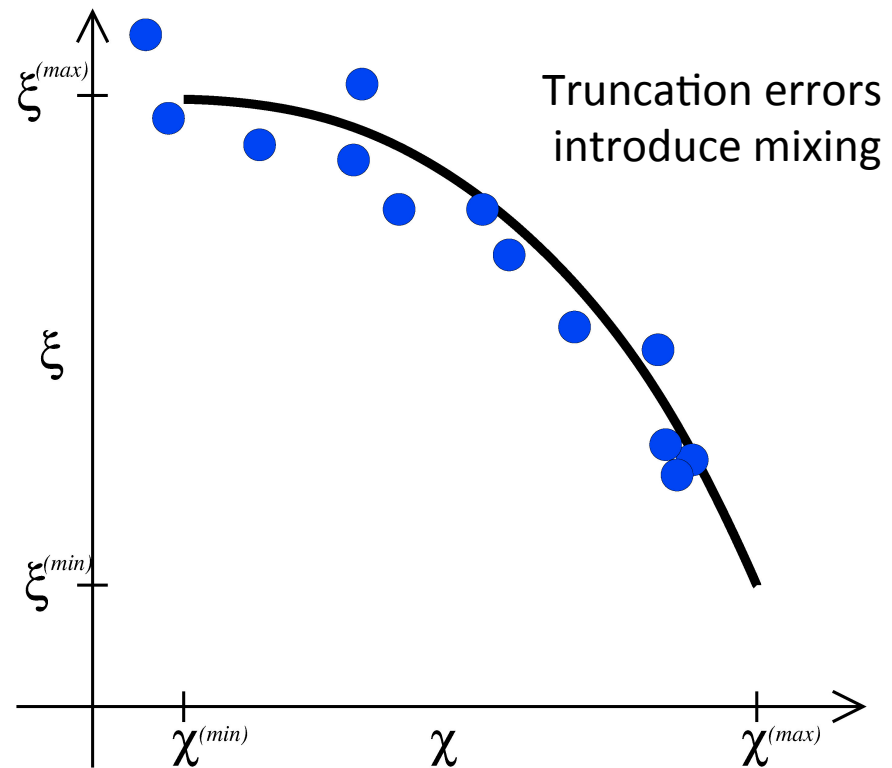
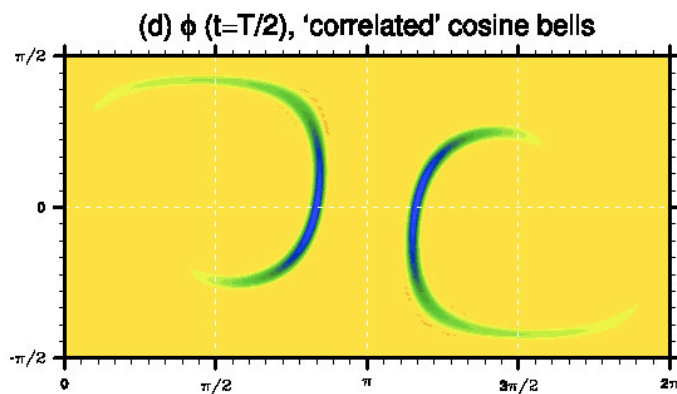
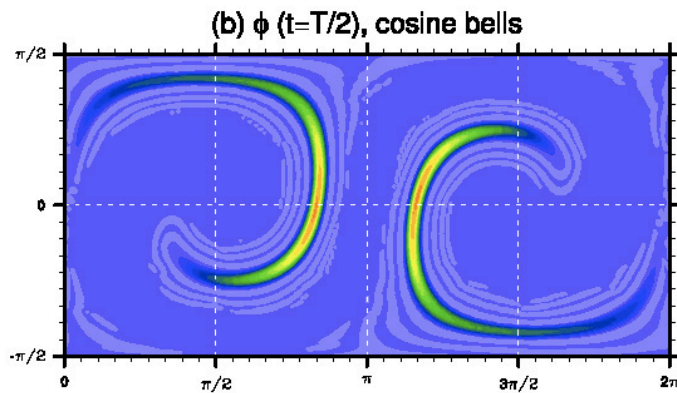
tracer 1: cosine bells tracer 2: correlated cosine bells $\Psi(\chi) = a\chi^2 + b$



Preserving pre-existing functional relation between tracers under challenging flow conditions

Initial conditions

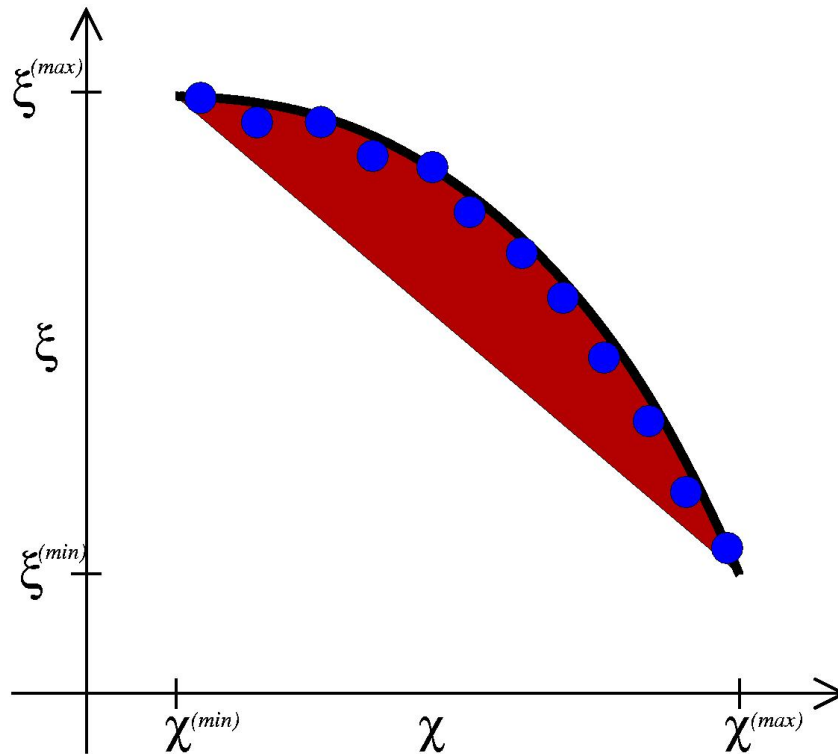
tracer 1: cosine bells tracer 2: correlated cosine bells $\Psi(\chi) = a\chi^2 + b$



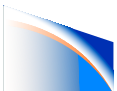
Preserving pre-existing functional relation between tracers under challenging flow conditions

Classification of mixing on scatter plot:

- Mixing that resembles 'real' mixing – convex hull (red area)
- Everything else is spurious unmixing

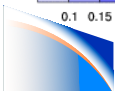
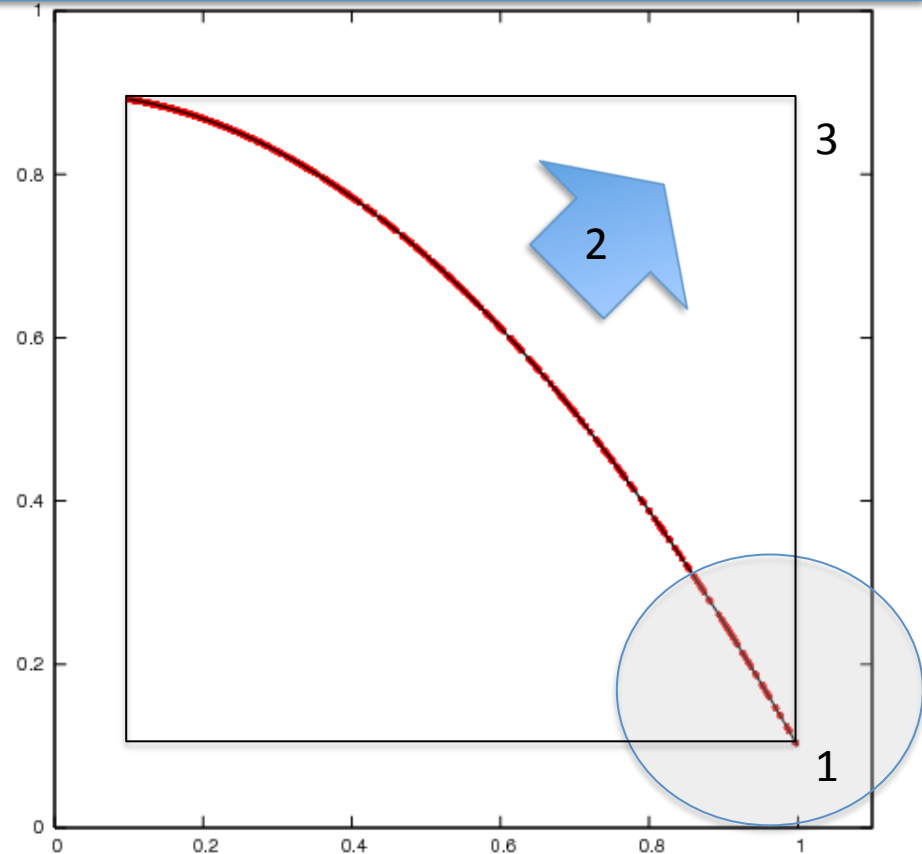
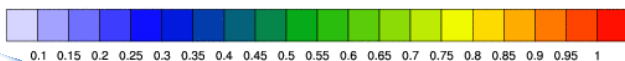
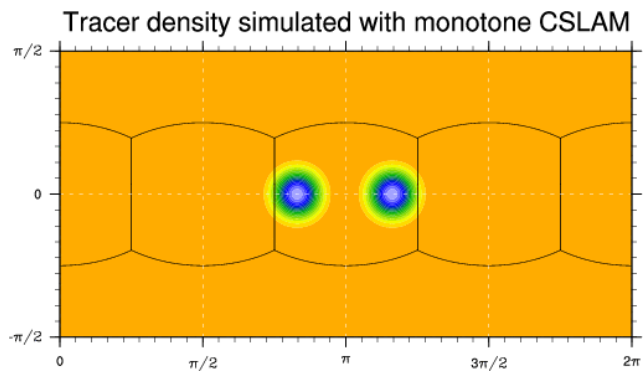
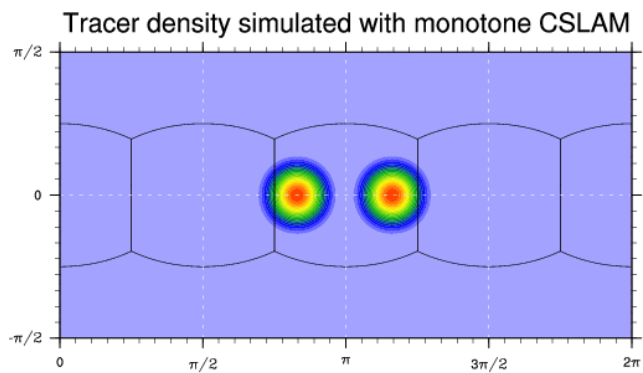


Thuburn and McIntyre (1997, JGR)



Preserving pre-existing functional relation between tracers under challenging flow conditions

Note: 1. Max value decrease, 2. Unmixing even if scheme is shape-preserving, 3. No expanding range unmixing





A standard test case suite for two-dimensional linear transport on the sphere

P. H. Lauritzen¹, W. C. Skamarock¹, M. J. Prather², and M. A. Taylor³

¹National Center for Atmospheric Research, Boulder, Colorado, USA

²Earth System Science Department, University of California, Irvine, California, USA

³Sandia National Laboratories, Albuquerque, New Mexico, USA

<http://www.geosci-model-dev.net/5/887/2012/gmd-5-887-2012.pdf>

A standard test case suite for two-dimensional linear transport on the sphere: results from a collection of state-of-the-art schemes

P. H. Lauritzen¹, P. A. Ullrich¹¹, C. Jablonowski², P. A. Bosler², D. Calhoun³, A. J. Conley¹, T. Enomoto⁴, L. Dong⁵, S. Dubey⁶, O. Guba⁷, A. B. Hansen¹⁴, E. Kaas⁸, J. Kent², J.-F. Lamarque¹, M. J. Prather⁹, D. Reinert¹⁰, V. V. Shashkin^{12,13}, W. C. Skamarock¹, B. Sørensen⁹, M. A. Taylor⁷, and M. A. Tolstykh^{12,13}

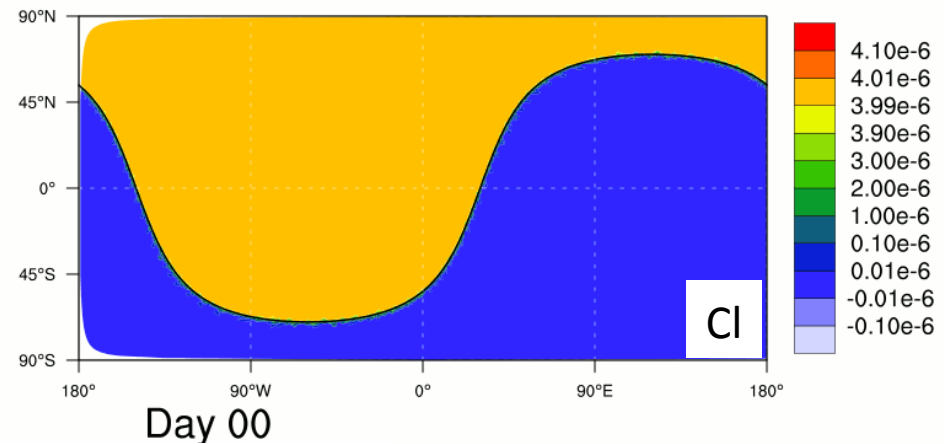
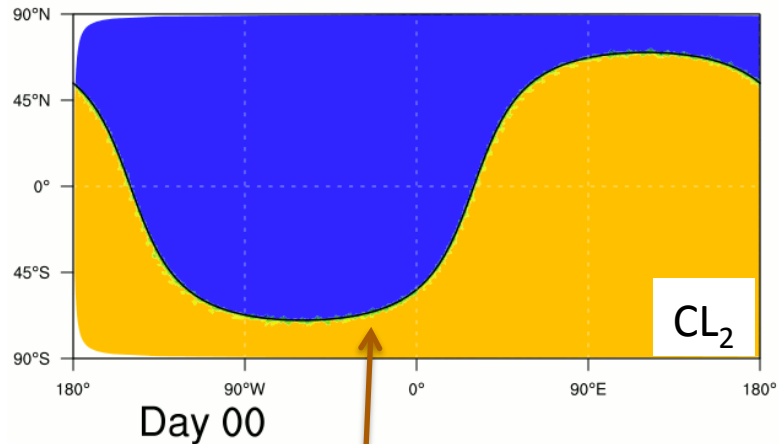
<http://www.geosci-model-dev.net/7/105/2014/gmd-7-105-2014.html>



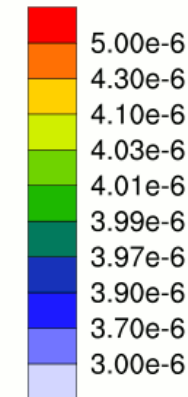
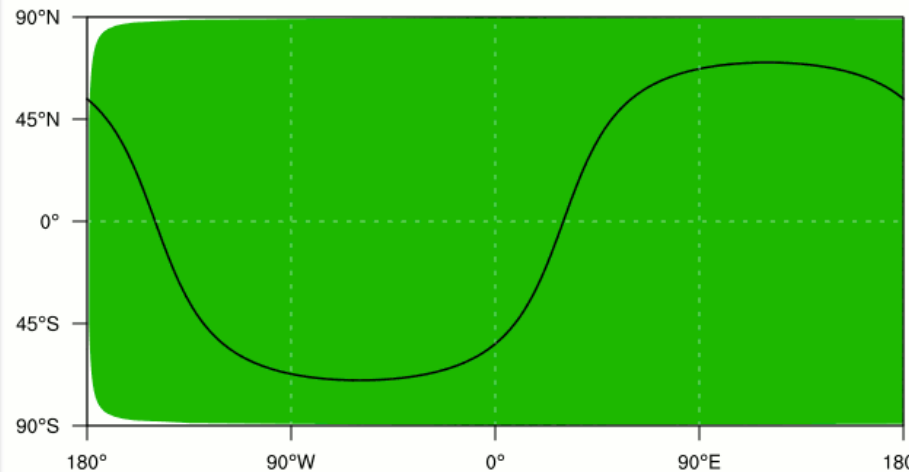
The terminator 'toy'-chemistry test: A simple tool to assess errors in transport schemes

(Lauritzen et al., 2015, GMD)

See: <http://www.cgd.ucar.edu/cms/pel/terminator.html>

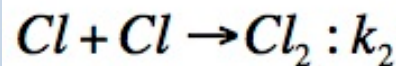
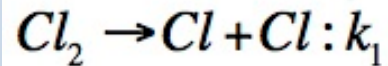


$$Cl + 2 * Cl_2 = \text{constant}$$



Wind field:
Nair and
Lauritzen
deformational
flow

Non-linear
Terminator 'toy'
chemistry:



Exact solution:

$$Cl + 2 * Cl_2 = \text{constant}$$

Errors are due to non-conservation of linear correlations usually caused by the limiter/filter and/or physics-dynamics coupling!



Requirements (desirable properties) for transport schemes intended for global climate/climate-chemistry applications:

1. Global (and local) Mass-conservation

If Δp is pressure-level thickness and q is mixing ratio, then the total mass

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is invariant in time: $M(t) = M(t = 0)$ (no sources/sinks)

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$$\int_{A_k} (\Delta p q)_k^{n+1} dA = \int_{a_k} (\delta p_k q)^n dA. \quad (3)$$

If $q = 1$ then (3) should reduce to (2).



Requirements (desirable properties) for transport schemes intended for

Consistency is trivial if (2) and (3) are solved with the same numerical method, however, that is not always the case:

- “Off-line” chemistry: prescribed wind and mass fields from , e.g., re-analysis.
- “Online” applications where (3) is solved with a different numerical method than (2)

3. Preserving functional relations between tracers

Transport scheme preserves $q_2 = f(q_1)$ (no sources/sinks)

4. Consistency

The continuity equations for air and tracers are coupled:

$$\int_{A_k} \Delta p_k^{n+1} dA = \int_{a_k} \delta p_k^n dA, \quad (2)$$

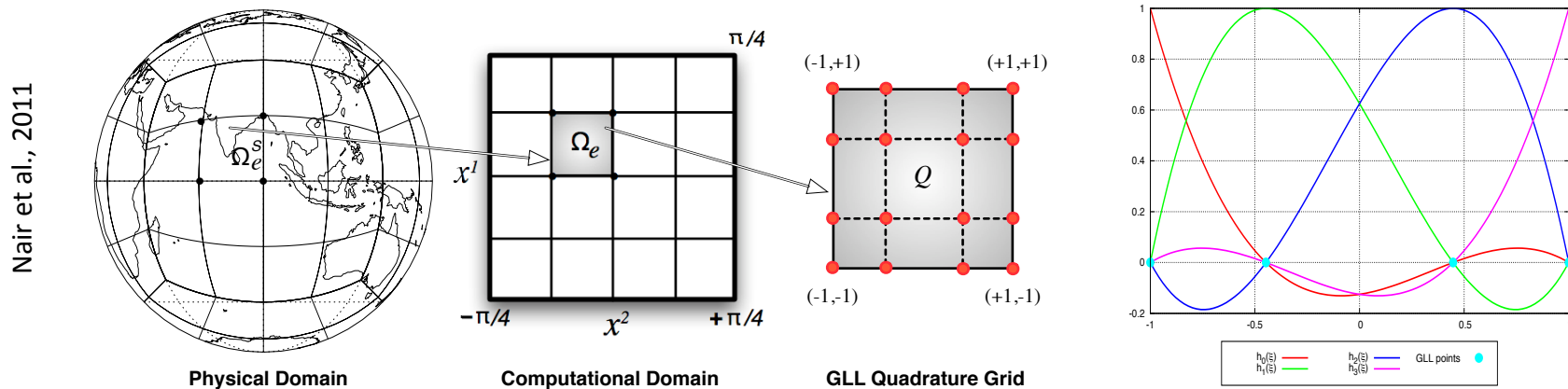
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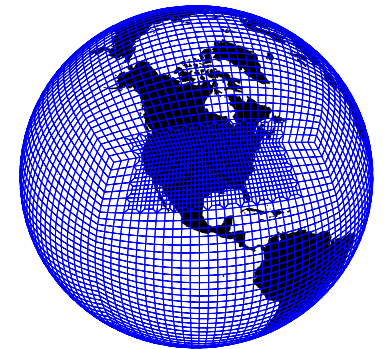


NCAR CAM-SE: NCAR Community Atmosphere Model with Spectral Elements dynamical core

Continuous Galerkin finite-element method (Taylor et al., 1997) on a cubed-sphere:



- 👍 Discretization is mimetic => mass-conservation & total energy conservation
- 👍 Conserves axial angular momentum very well (Lauritzen et al., 2014)
- 👍 Support static mesh-refinement and retains formal order of accuracy!
- 👍 Highly scalable to at least $O(100K)$ processors (Dennis et al., 2012)
- 👍 Competitive “AMIP-climate” (Evans et al., 2012)
- 👎 **Lower computational throughput for many-tracer applications**

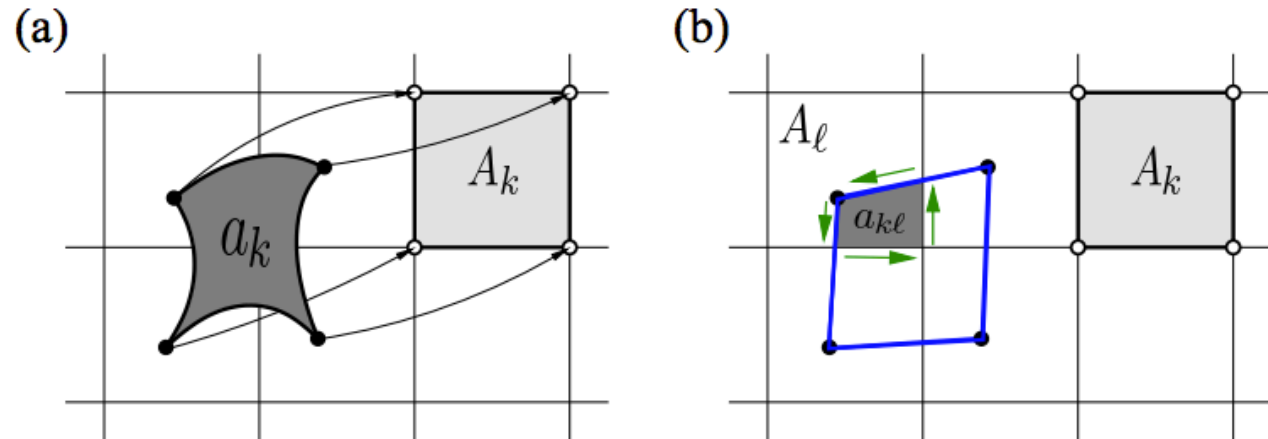




A way to accelerate tracer transport:



CSLaM scheme (Conservative Semi-Lagrangian Multi-tracer)



Finite-volume Lagrangian form of continuity equation for $\psi = \rho, \rho \phi$:

$$\int_{A_k} \psi_k^{n+1} dx dy = \int_{a_k} \psi_k^n dx dy = \sum_{\ell=1}^{L_k} \left[\sum_{i+j \leq 2} c_{\ell}^{(i,j)} w_{k\ell}^{(i,j)} \right],$$

where weights $w_{k\ell}^{(i,j)}$ are functions of the coordinates of the vertices of $a_{k\ell}$.

$w_{k\ell}^{(i,j)}$ can be re-used for each additional tracer (Dukowicz and Baumgardner, 2000)
 computational cost for each additional tracer is the reconstruction and limiting/filtering.
 CSLAM is stable for long time-steps (CFL>1)

Lauritzen, Nair and Ullrich (*J. Comput. Phys.*, 2010)

Coupling finite-volume semi-Lagrangian transport with spectral element dynamics

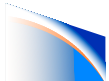
4. Consistency

The continuity equations for air and tracers are coupled:

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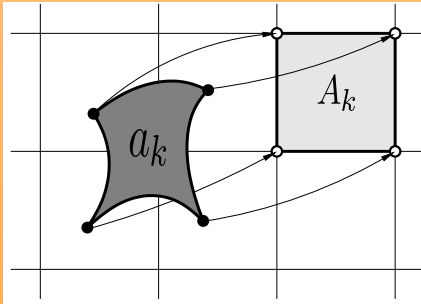
$$\int_{A_k} (\Delta p q)_k^{n+1} dA = \int_{a_k} (\delta p_k q)^n dA. \quad (3)$$

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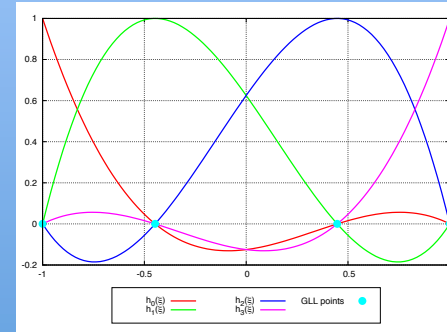


Coupling finite- in time element

Solved with semi-Lagrangian
scheme (CSLAM)
(max Courant number < 1)



Solved with spectral-element
Eulerian advection operator
(max Courant number < 0.3)



The continuity equation for air and tracers is coupled:

$$\int_{A_k} \Delta p_k^{n+1} dA = \int_{a_k} \delta p_k^n dA, \quad (2)$$

$$\int_{A_k} (\Delta p q)_k^{n+1} dA = \int_{a_k} (\delta p_k q)^n dA. \quad (3)$$

If $q = 1$ then (3) should reduce to (2).

We need to couple without violating mass-conservation,
shape-preservation, and consistency



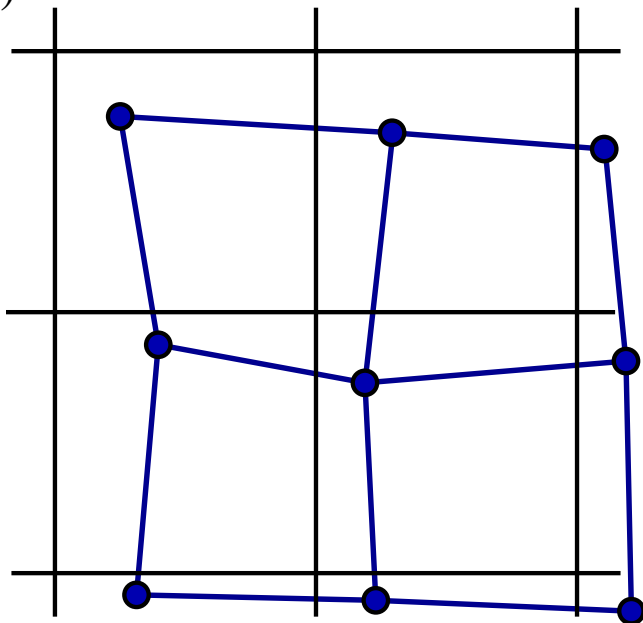
4. Consistency

Find upstream area, a_k , so that CSLAM predicted mass field is equal to CAM-SE predicted mass field:

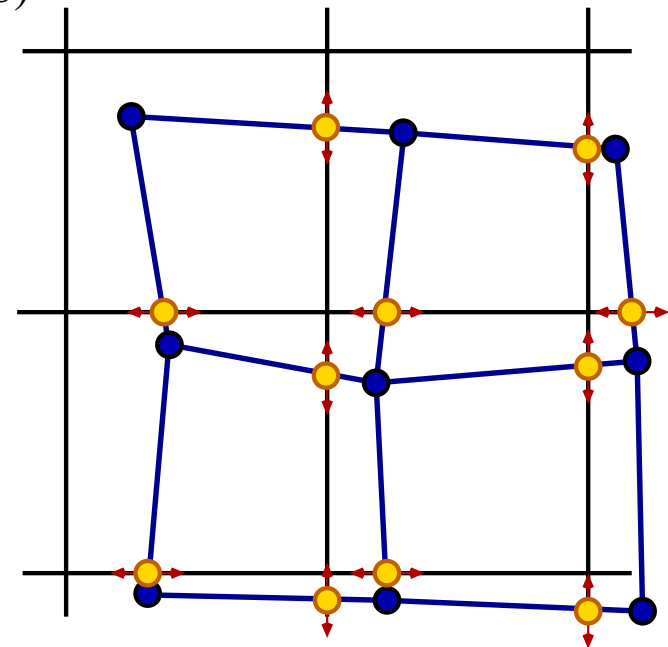
$$\Delta p_k^{n+1}(\text{CAM-SE}) = \frac{1}{\Delta A} \int_{a_k} \delta p_k^n dA (\text{CSLAM}), \quad (4)$$

Many details of algorithm (well-posedness, ...) are left out here ...

(a)



(b)



NCAR

Lauritzen, Taylor, Overfelt, Ullrich and Goldhaber (2015, IN PREP)

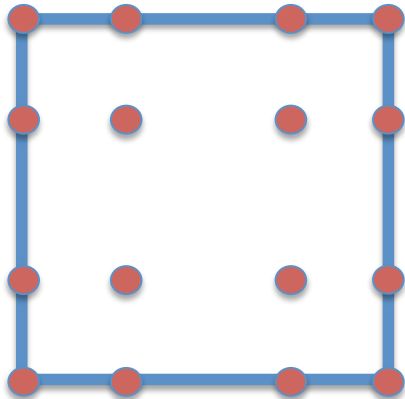


CAM-SE-CSLAM

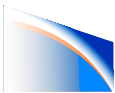
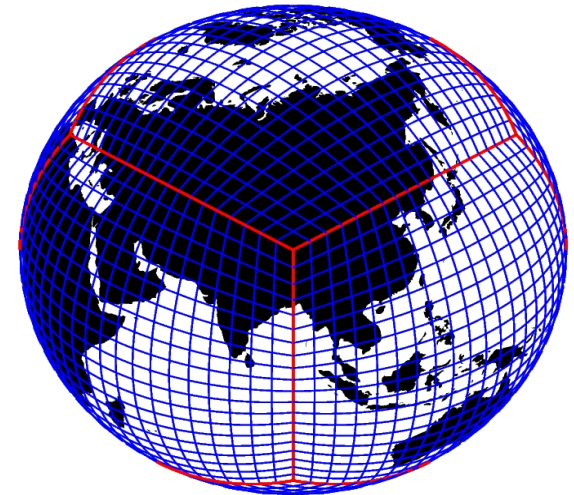
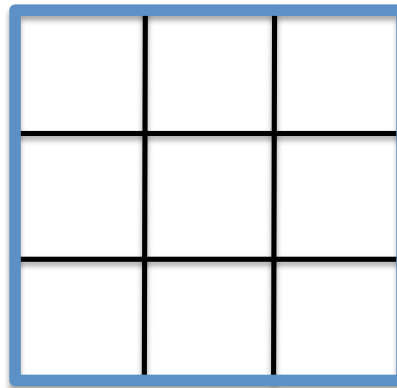
A new model configuration based on CAM-SE:

- **SE:** Spectral-element dynamical core solving for \vec{v} , T , p_s
(Dennis et al., 2012; Evans et al., 2012; Taylor and Fournier, 2010; Taylor et al., 1997)
- **CSLAM:** Semi-Lagrangian finite-volume transport scheme for tracers
(Lauritzen et al., 2010; Erath et al., 2013, 2012; Harris et al., 2010)
- **Phys-grid:** Separating physics and dynamics grids, i.e. ability to compute physics tendencies based on cell-averaged values within each element instead of quadrature points

Dynamics grid



CSLAM grid



NCAR

Lauritzen, Taylor, Overfelt, Ullrich and Goldhaber (2015, IN PREP)



Idealized baroclinic wave

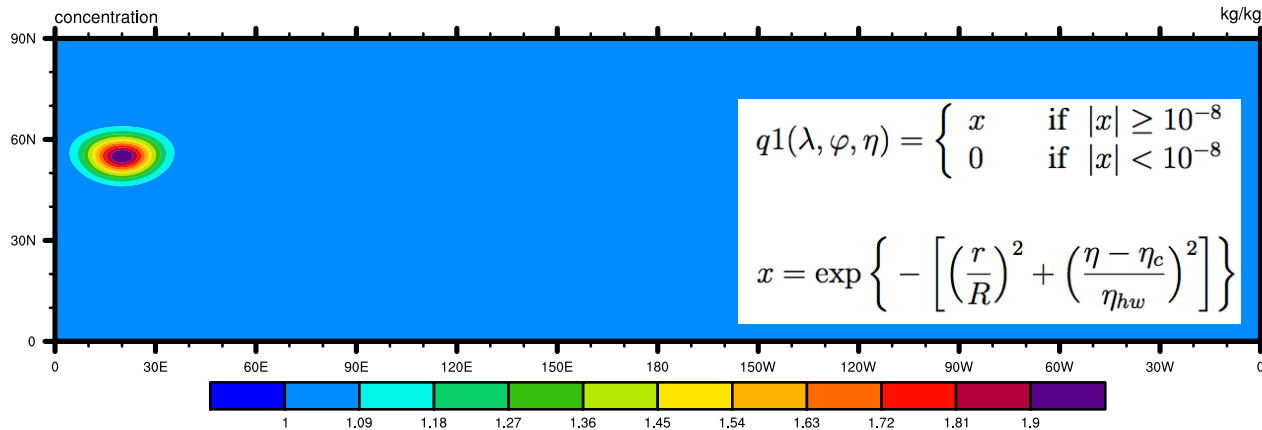
No sub-grid-scale forcing, dry.

Jablonowski and Williamson (2006) and Lauritzen et al. (2010).

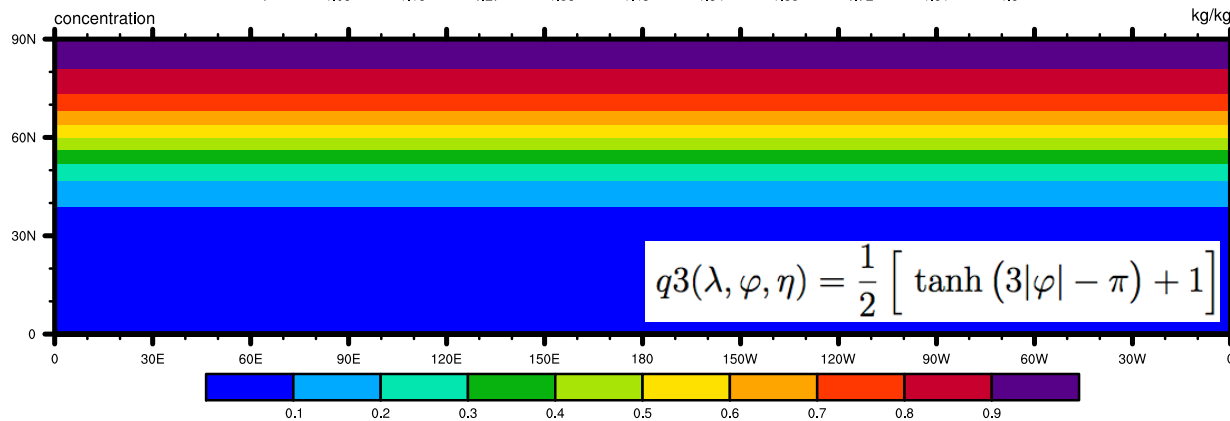


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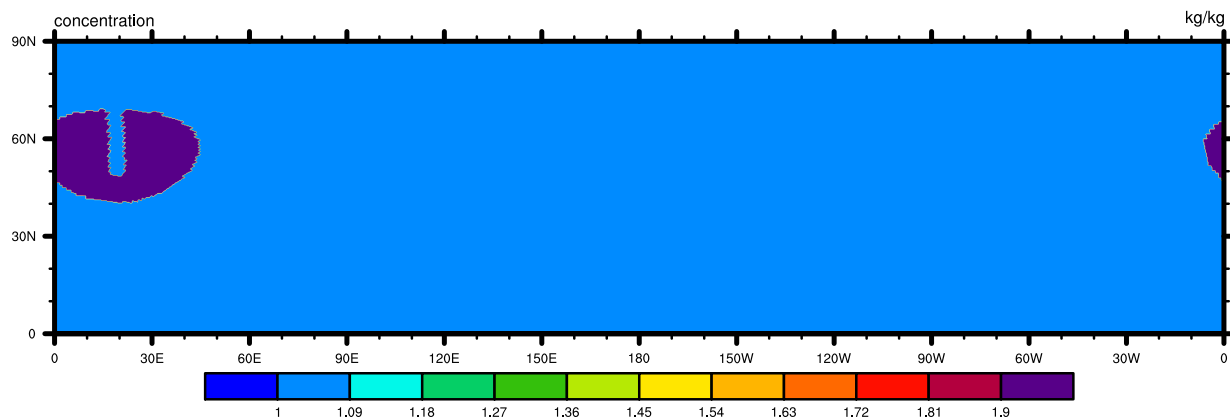
3 tracers: initial conditions



Gaussian
"ball"

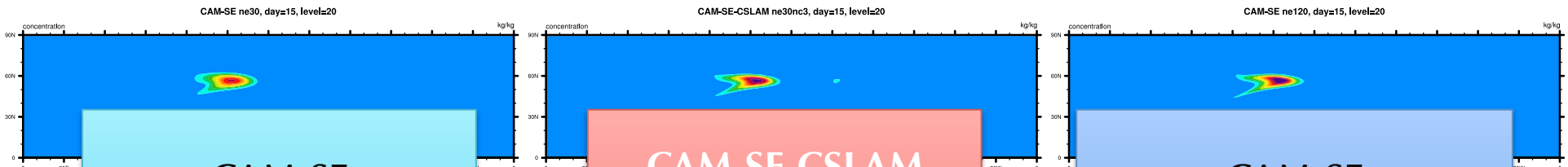
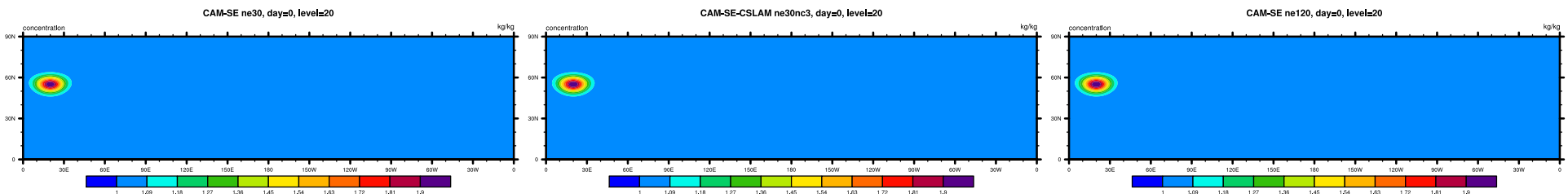


Zonally
symmetric
(smooth)



Slotted
cylinder



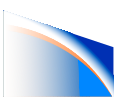
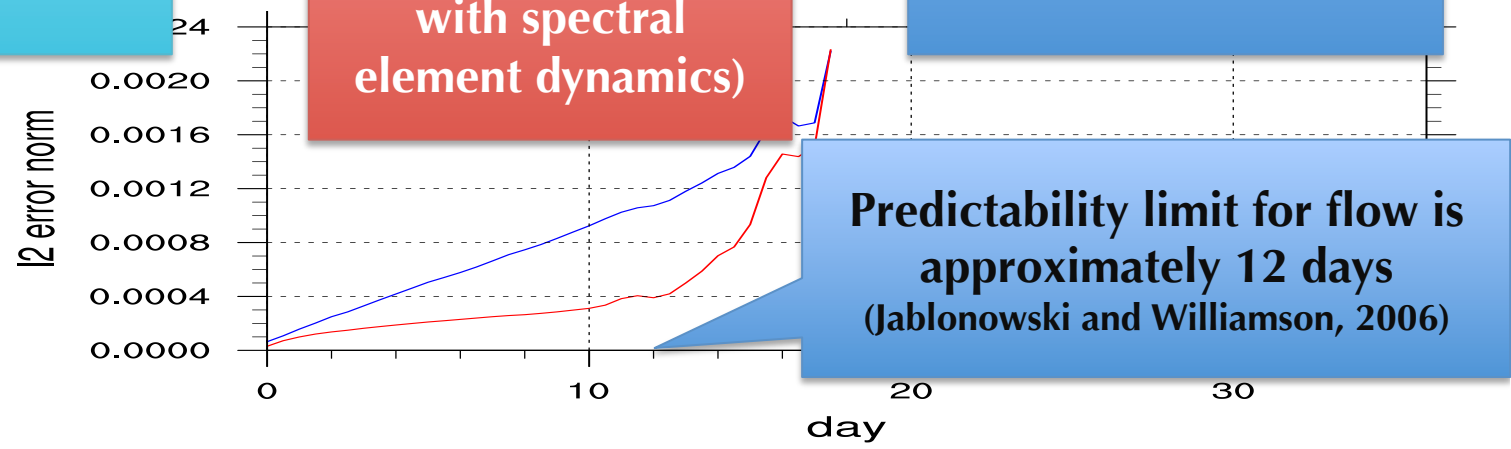


**CAM-SE
1 degree
standard
configuration
(spectral element
advection)**

**CAM-SE-CSLAM
1 degree
configuration
(tracer transport
with CSLAM
consistently coupled
with spectral
element dynamics)**

**CAM-SE
0.25 degree
standard configuration

USED AS REFERENCE
SOLUTION ("TRUTH")**

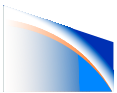
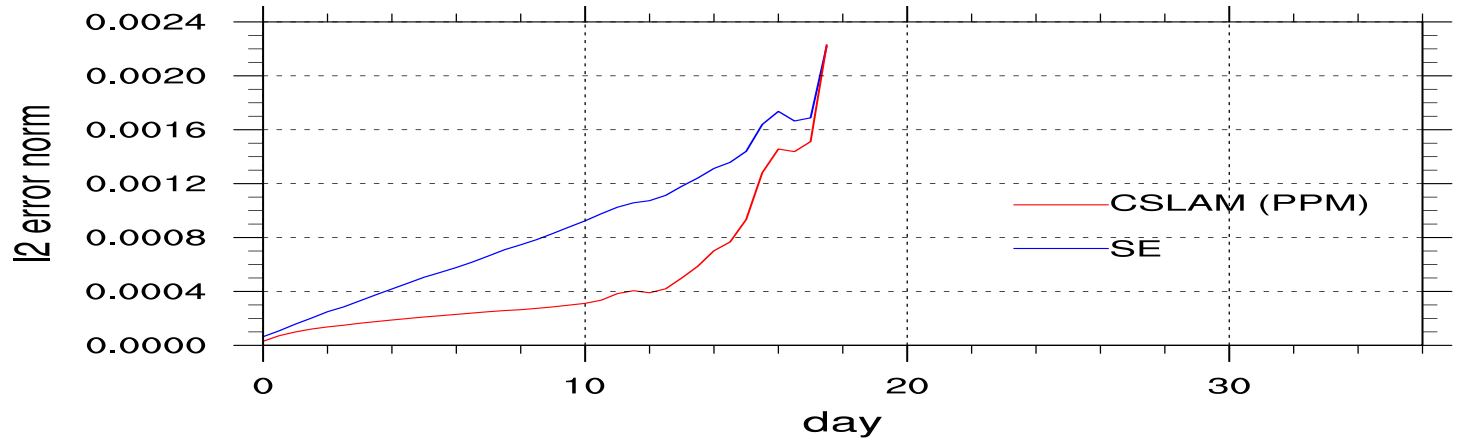
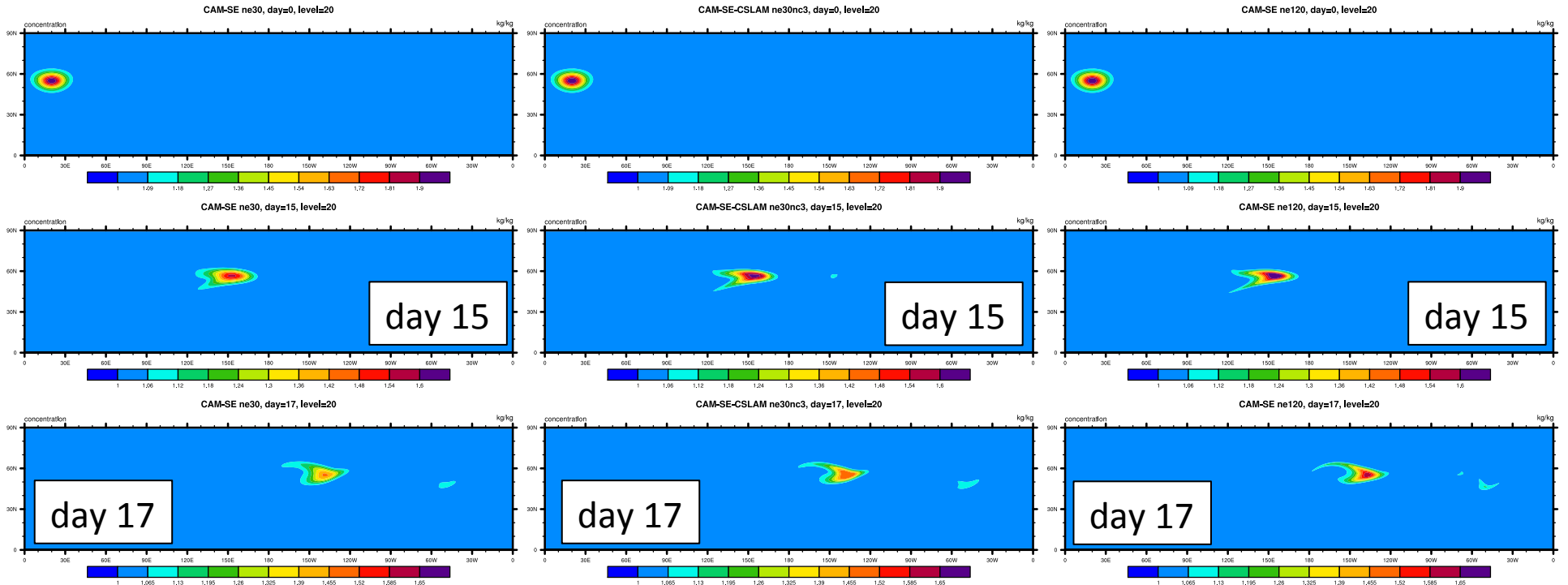


NCAR

CAM-SE

CAM-SE-CSLAM

CAM-SE reference

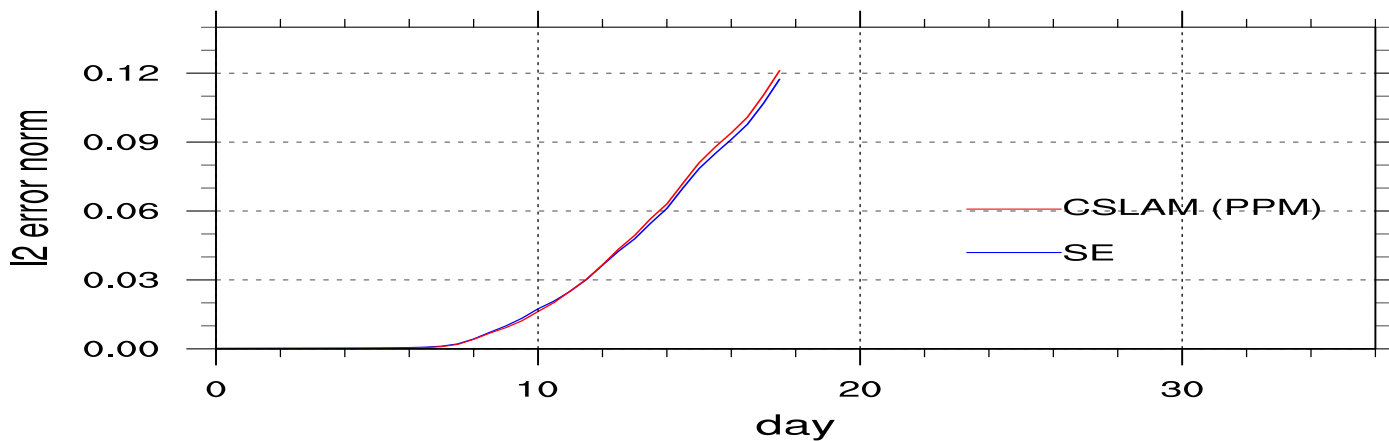
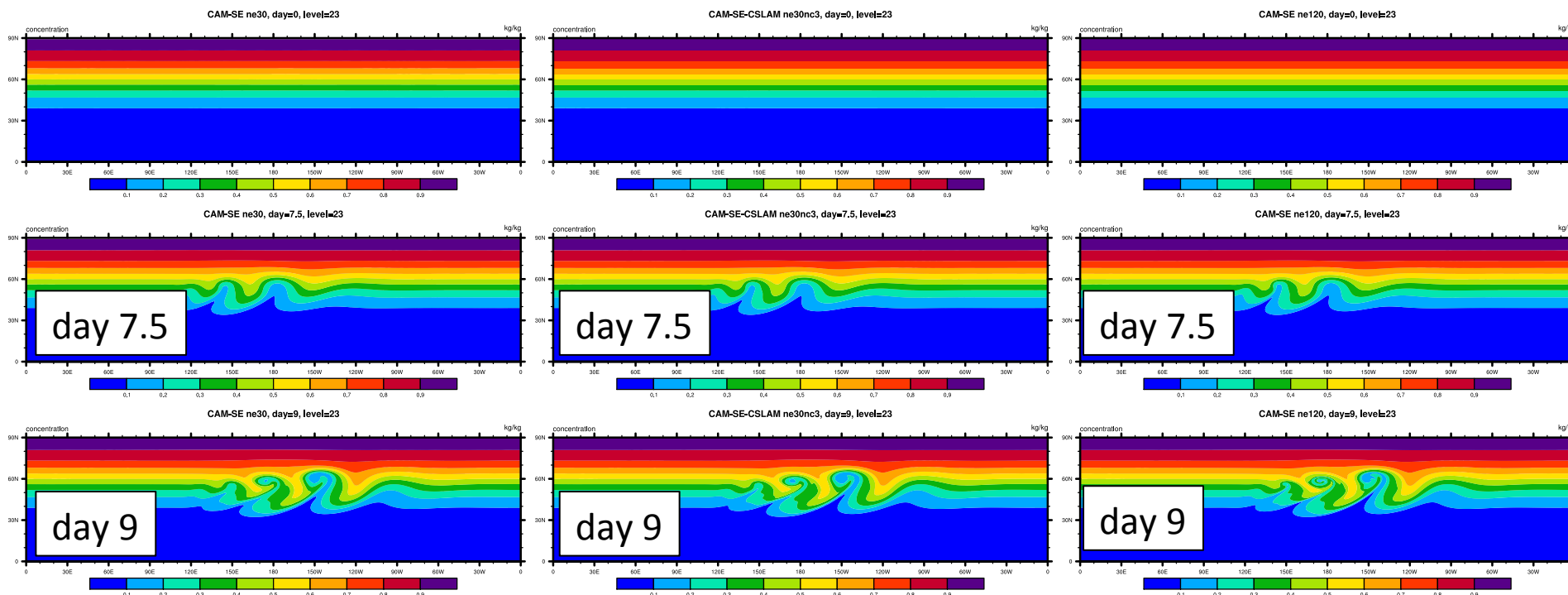


NCAR

CAM-SE

CAM-SE-CSLAM

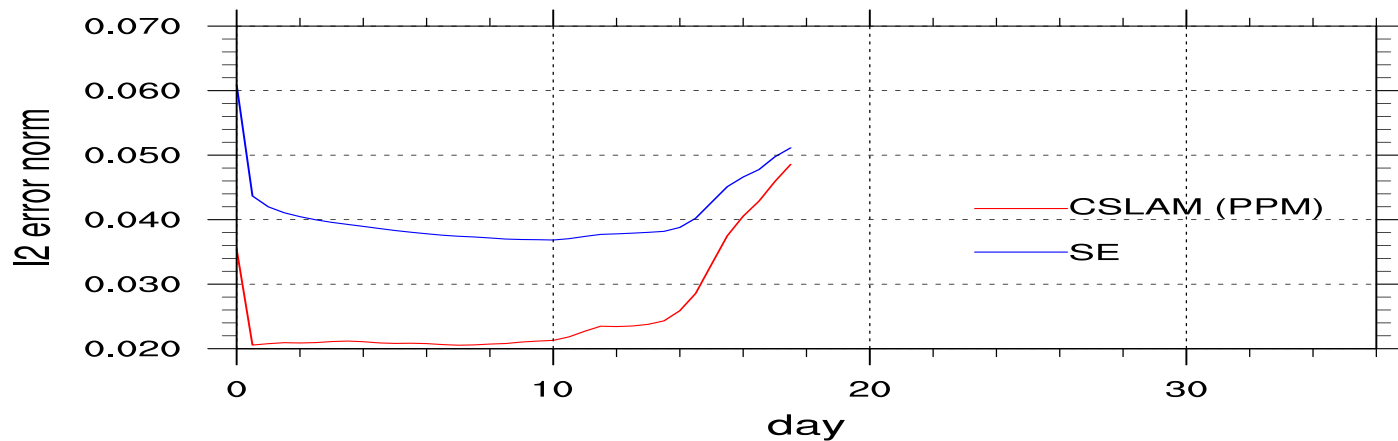
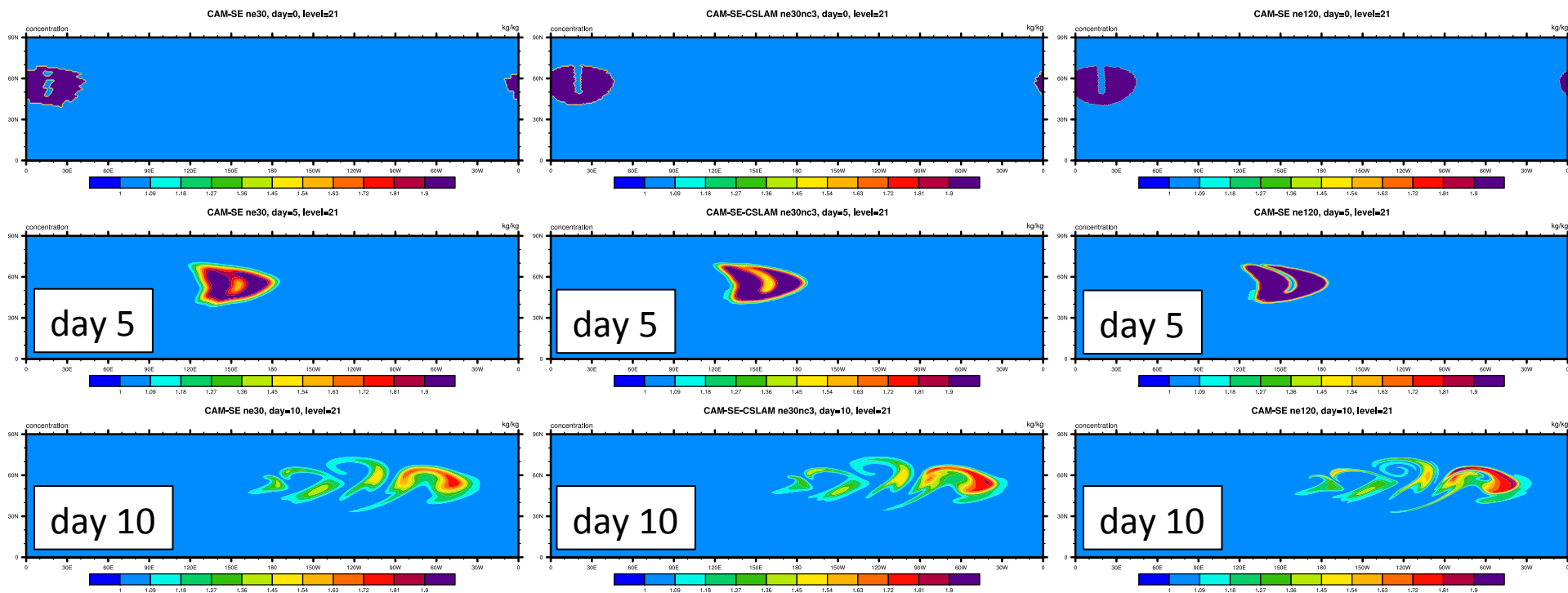
CAM-SE reference



CAM-SE

CAM-SE-CSLAM

CAM-SE reference



NCAR

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- Dukowicz, J. K. and Baumgardner, J. R. (2000). Incremental remapping as a transport/advection algorithm. *J. Comput. Phys.*, 160:318–335.
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