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Climate & Global Dynamics
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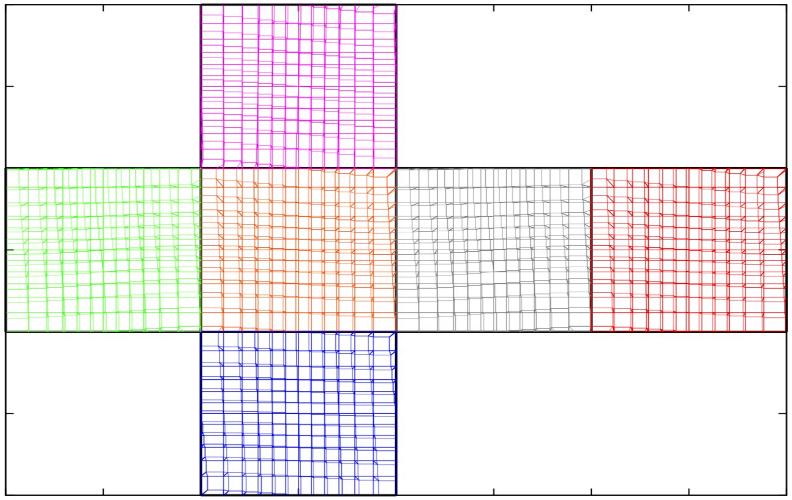
CAM-SE-CSLAM: Consistent finite-volume transport with spectral-element dynamics



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Climate and Global Dynamics Laboratory (CGD)
National Center for Atmospheric Research (NCAR)

PDEs on the Sphere, KIAPS, Seoul, October 17-23, 2015

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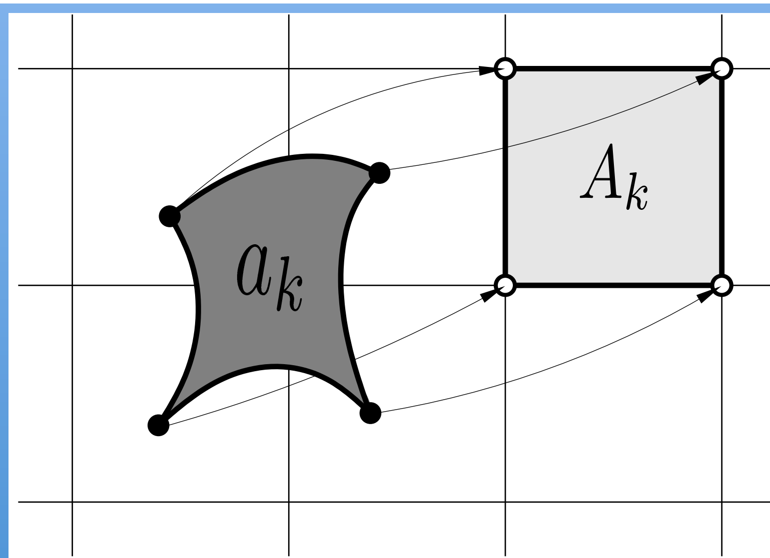
CAM-SE-CSLAM: Consistent finite-volume transport with ... ctral-

Consider the finite-volume Lagrangian form of continuity equation for air (pressure level thickness, Δp), and tracer (mixing ratio, q):

$$\int_{A_k} \psi_k^{n+1}\, dA = \int_{a_k} \psi_k^{n}\, dA, \qquad \psi = \Delta p,\ \Delta p\, q, \tag{1}$$

where n time-level.



No sources/ sinks

Some requirements (desirable properties) for transport schemes intended for global climate/climate-chemistry applications:

1. Global (and local) Mass-conservation

If Δp is pressure-level thickness and q is mixing ratio, then the total mass

$$M(t) = \int_{\Omega} \Delta p \, q \, dA,$$

is invariant in time: $M(t) = M(t=0)$ (no sources/sinks)

2. Shape-preservation

Scheme does not produce new extrema (in particular negatives) in q

3. Preservation of pre-existing functional relations between tracers

Transport scheme preserves $q_2 = f(q_1)$ (no sources/sinks)

4. Consistency

The continuity equations for air and tracers are coupled:

$$\int_{A_k} \Delta p_k^{n+1} \, dA = \int_{a_k} \delta p_k^n \, dA, \quad (2)$$

$$\int_{A_k} (\Delta p \, q)_k^{n+1} \, dA = \int_{a_k} (\delta p_k q)^n \, dA. \quad (3)$$

If $q = 1$ then (3) should reduce to (2).

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Some requirements (desirable properties) for transport schemes i[illegible]

Consistency is trivial if (2) and (3) are solved with the same numerical method, however, that is not always the case:

- "Off-line" chemistry: prescribed wind and mass fields from , e.g., re-analysis.
- "Online" applications where (3) is solved with a different numerical method than (2)

3. Prese[illegible] functional relations between tracers

Transpor[illegible] preserves $q_2 = f(q_1)$ (no sources/sinks)

4. Consistency

The continuity equations for air and tracers are coupled:

$$\int_{A_k} \Delta p_k^{n+1}\, dA = \int_{a_k} \delta p_k^n\, dA, \tag{2}$$

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If $q = 1$ then (3) should reduce to (2).

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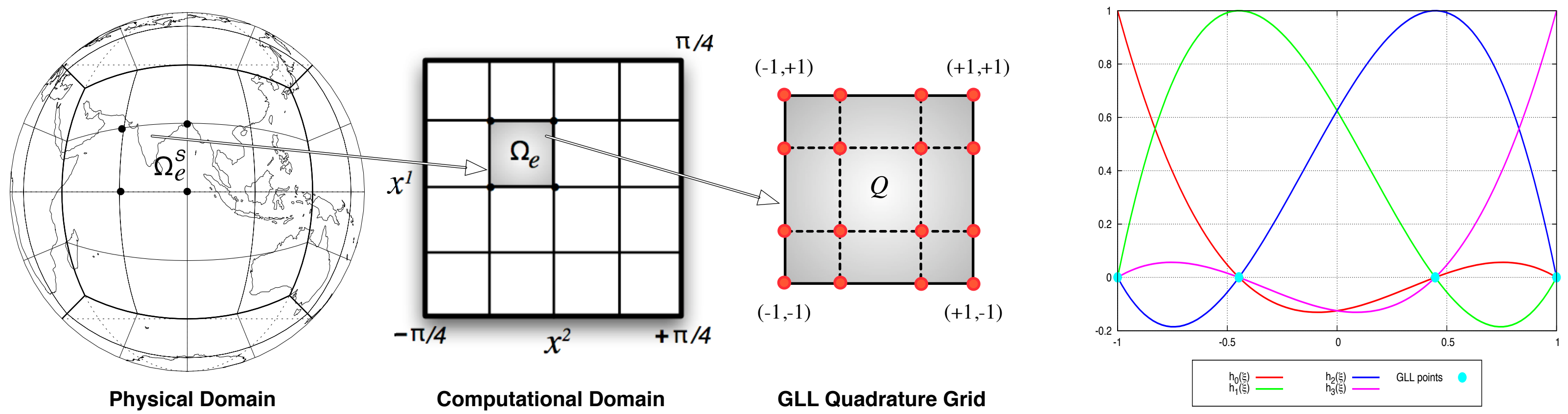
CAM-SE: NCAR Community Atmosphere Model with Spectral Elements dynamical core

Continuous Galerkin finite-element method (Taylor et al., 1997) on a cubed-sphere:

Nair et al., 2011



- Discretization is mimetic => mass-conservation & total energy conservation
- Conserves axial angular momentum very well (Lauritzen et al., 2014)
- Support static mesh-refinement and retains formal order of accuracy!
- Highly scalable to at least O(100K) processors (Dennis et al., 2012)
- Competitive "AMIP-climate" (Evans et al., 2012)
- **Lower computational throughput for many-tracer applications**

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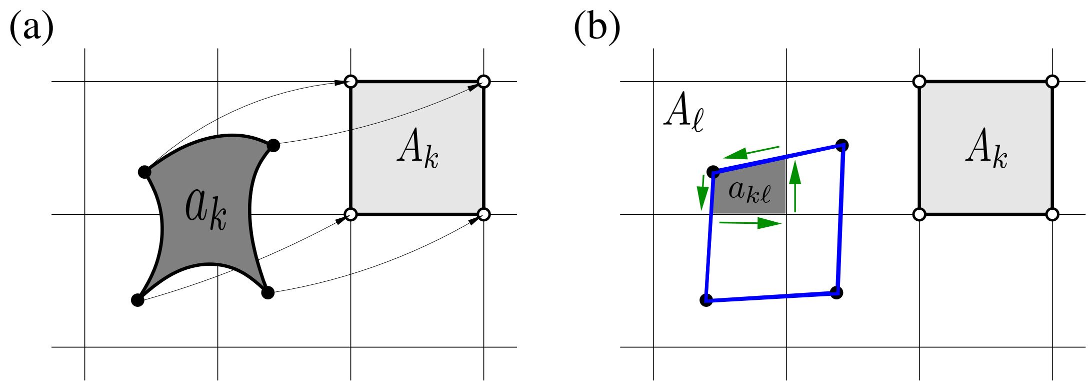
A way to accelerate tracer transport:

Conservative Semi-LAgrangian Multi-tracer (CSLAM)



Finite-volume Lagrangian form of continuity equation for air (pressure level thickness, Δp), and tracer (mixing ratio, q):

$$\int_{A_k} \psi_k^{n+1}\, dA = \int_{a_k} \psi_k^n\, dA \quad = \quad \sum_{\ell=1}^{L_k} \left[\sum_{\imath+\jmath \le 2} c_\ell^{(\imath,\jmath)} w_{k\ell}^{(\imath,\jmath)} \right], \quad \psi = \Delta p, \, \Delta p\, q,$$

where n time-level, $a_{k\ell}$ overlap areas, L_k #overlap areas, $c^{(\imath,\jmath)}$ reconstruction coefficients for ψ_k^n, and $w_{k\ell}^{(\imath,\jmath)}$ weights.

A way to accelerate tracer transport:

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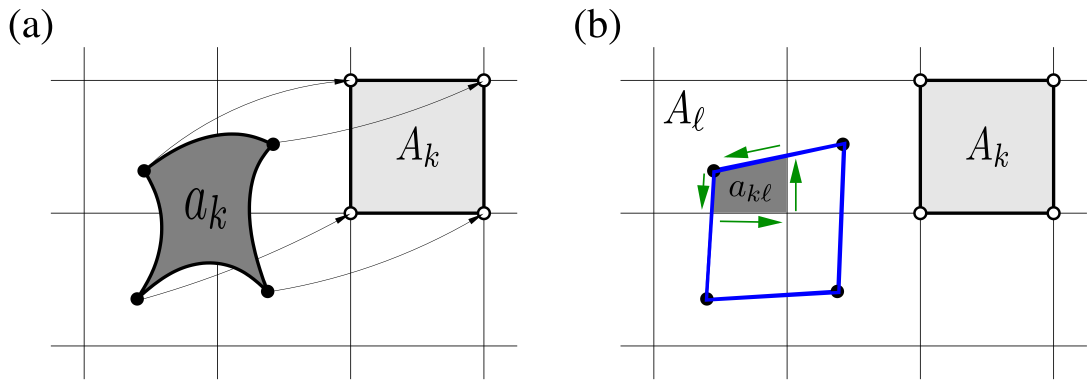
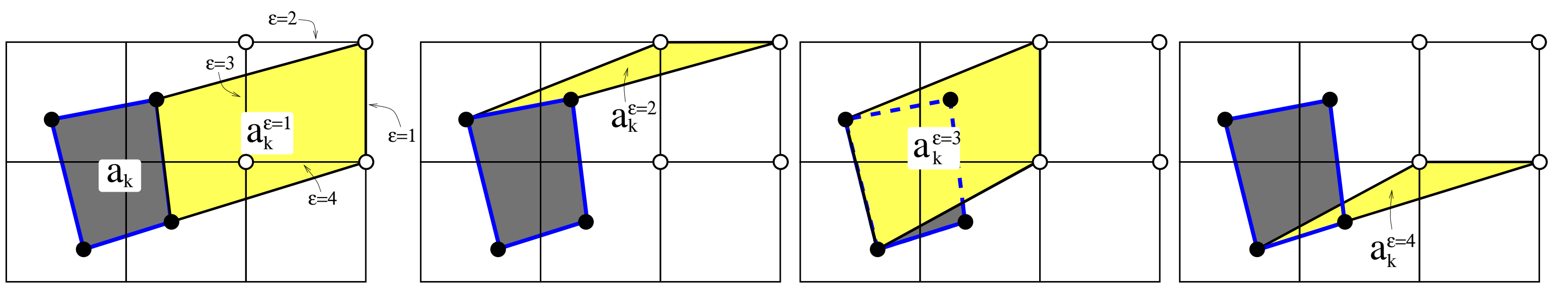


$$\int_{A_k} \psi_k^{n+1}\, dA = \int_{a_k} \psi_k^n\, dA \quad = \quad \sum_{\ell=1}^{L_k} \left[\sum_{i+j\le 2} c_\ell^{(i,j)} w_{k\ell}^{(i,j)} \right], \quad \psi = \Delta p,\, \Delta p\, q,$$

- Multi-tracer efficient: $w_{k\ell}^{(i,j)}$ re-used for each additional tracer (Dukowicz and Baumgardner, 2000).
- Scheme allows for large time-steps (flow deformation limited).
- Conserves mass, shape, linear correlations (Lauritzen et al., 2014).

A way to accelerate tracer transport:

Flux-form CSLAM ≡ Lagrangian CSLAM



$$\int_{A_k} \psi_k^{n+1}\, dA = \int_{A_k} \psi_k^n\, dA - \sum_{\epsilon=1}^{4} s_{k\ell}^{\epsilon} \int_{a_k^{\epsilon}} \psi\, dA, \quad \psi = \Delta p,\, \Delta p\, q.$$

where

- a_k^{ϵ} = 'flux-area' (yellow area) = area swept through face ϵ
- $s_{k\ell}^{\epsilon}$ = 1 for outflow and -1 for inflow.

Flux-form and Lagrangian forms of CSLAM are equivalent (Lauritzen et al., 2011).

Coupling finite-volume semi-Lagrangian transport with spectral element dynamics

4. Consistency

The continuity equations for air and tracers are coupled:

$$\int_{A_k} \Delta p_k^{n+1}\, dA = \int_{a_k} \delta p_k^n\, dA, \quad (2)$$

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If $q = 1$ then (3) should reduce to (2).

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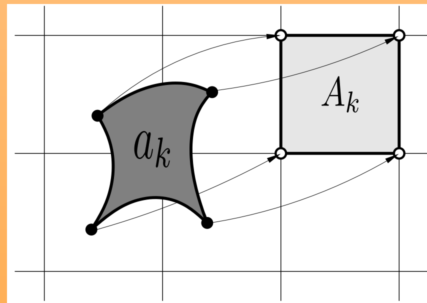
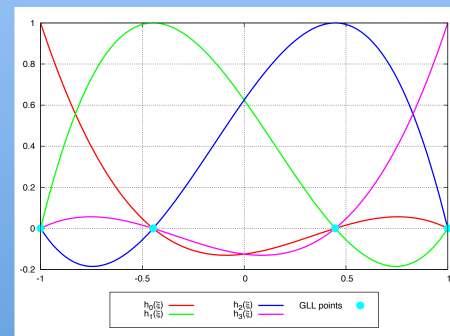
Coupling finite- ... an tra ... leme

Solved with semi-Lagrangian scheme (CSLAM)
(max Courant number < 1)



Solved with spectral-element Eulerian advection operator
(max Courant number < 0.3)



The continuity equ[...] for air and tracers [...] coupled:

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If $q = 1$ then (3) should reduce to (2).

We need to couple without violating mass-conservation, shape-preservation, and consistency

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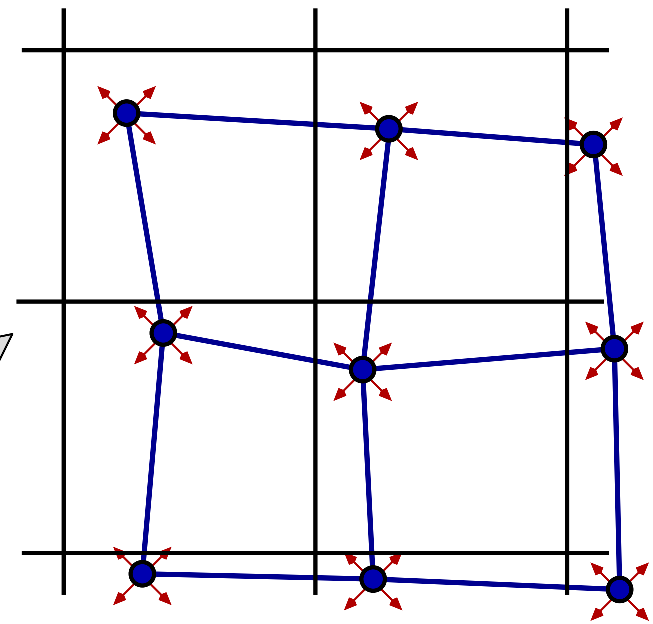
4. Consistency

Find upstream area, a_k, so that CSLAM predicted mass field is equal to CAM-SE predicted mass field:

$$\Delta p_k^{n+1}(\text{CAM-SE}) \quad = \quad \frac{1}{\Delta A}\int_{a_k} \delta p_k^n \, dA \;(\text{CSLAM}) \;,. \qquad (4)$$

If we choose to move departure points around so that (4) is fulfilled a global iteration problem results!

(and I am not sure it is well-posed!)



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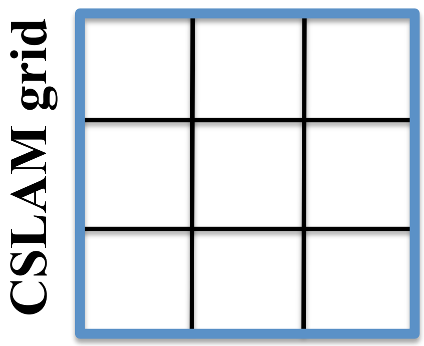
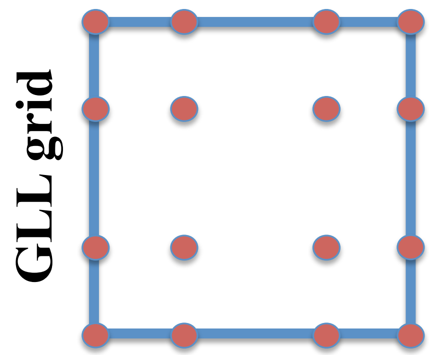
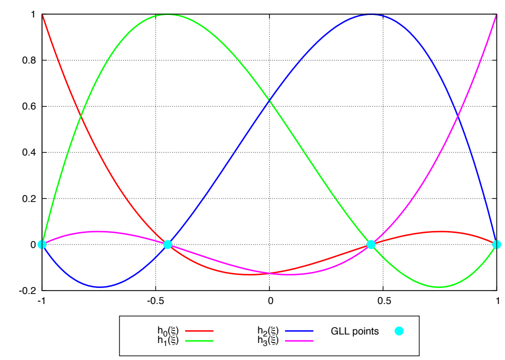
Solution: Cast problem in flux-form

- Spectral-element method does not operate with fluxes: Taylor et al. have derived a method to compute fluxes, $\mathcal{F}^{(SE)}$, through the CSLAM control volume faces! presented at ICMS conference in March, 2015.







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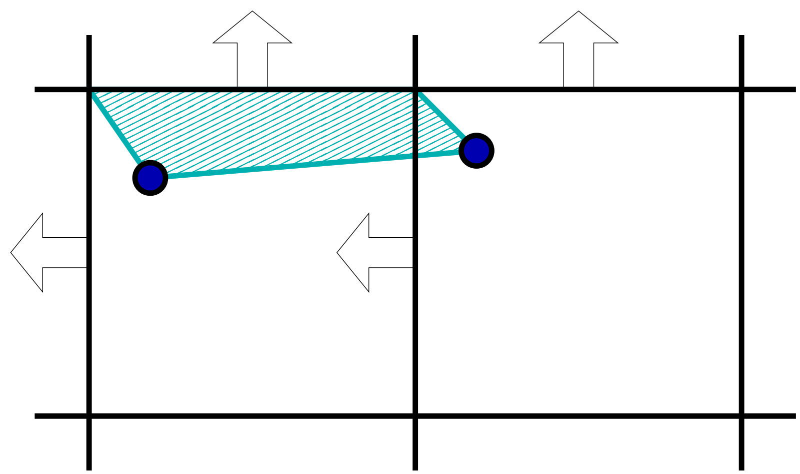
Solution: Cast problem in flux-form

Given $\mathcal{F}^{(SE)}$ find swept areas, $\delta\Omega$, so that:

1.
$$\mathcal{F}^{(CSLAM)} = \int_{\delta\Omega} \Delta p(x,y)\, dA = \mathcal{F}^{(SE)} \qquad \forall\ \delta\Omega.$$

2. The sum of all the swept areas, $\delta\Omega$, span the domain without cracks or overlaps



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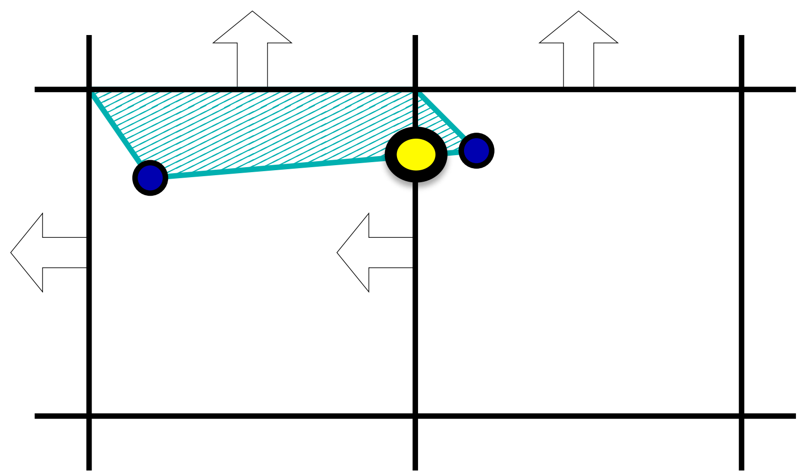
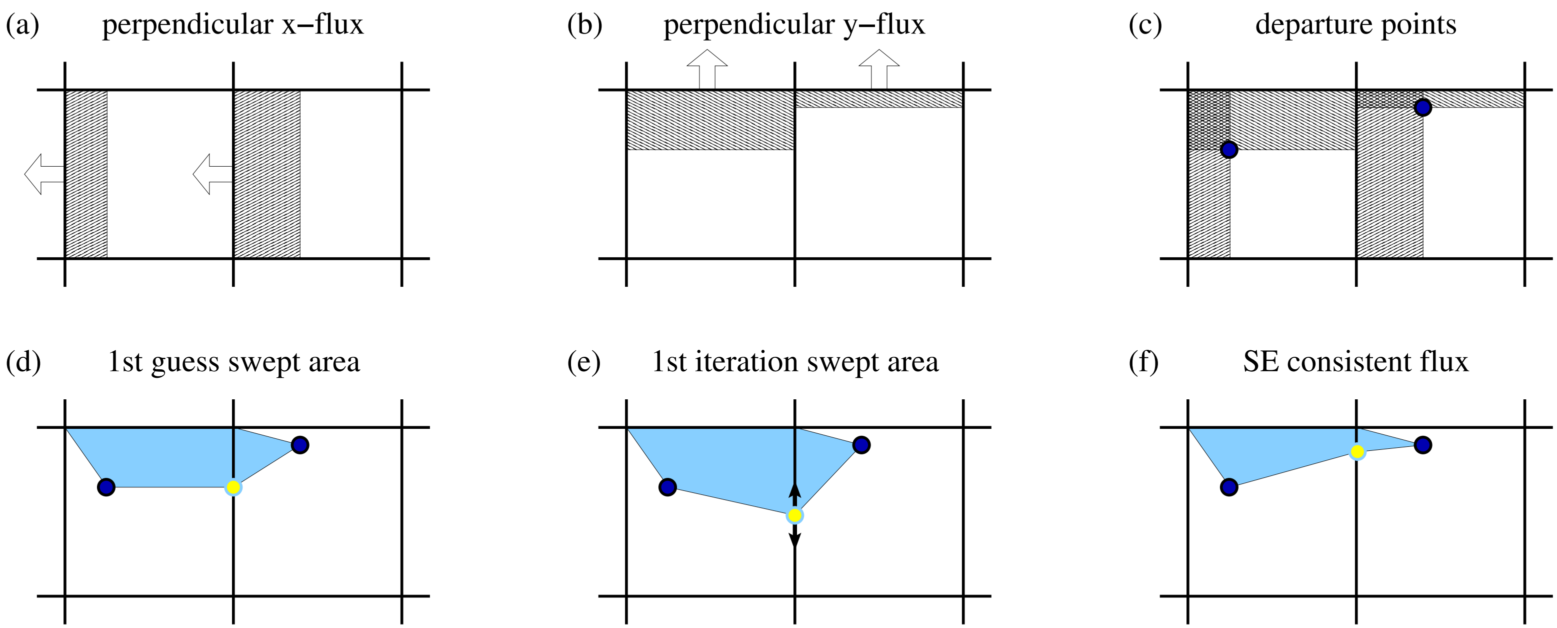


Solution: Cast problem in flux-form

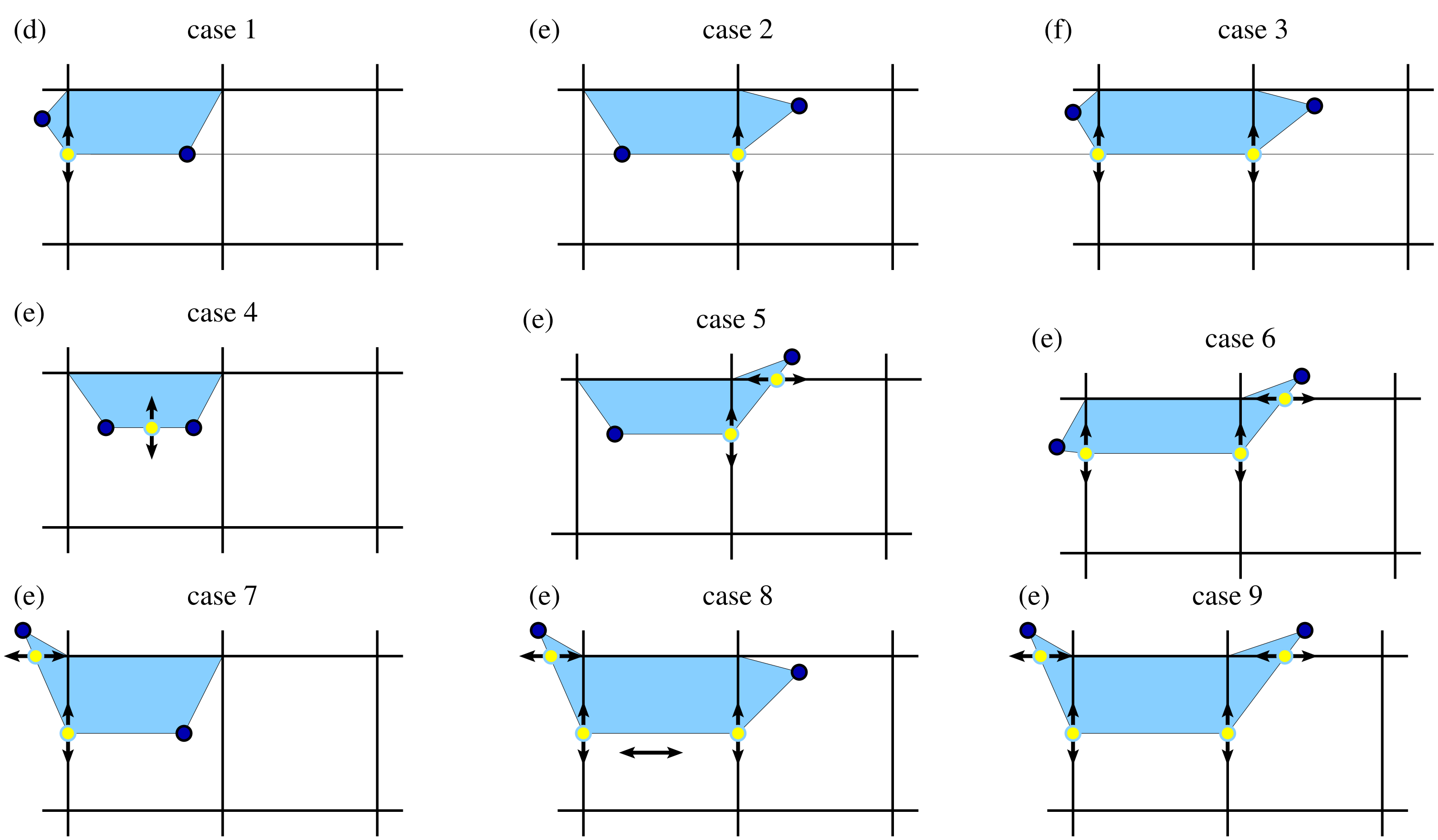
Consistent SE-CSLAM algorithm: step-by-step example



Well-posed? As long as flow deformation $\left|\frac{\partial u}{\partial x}\right| \Delta t \lesssim 1$ (Lipschitz criterion)

Solution: Cast problem in flux-form

Consistent SE-CSLAM algorithm: flow cases



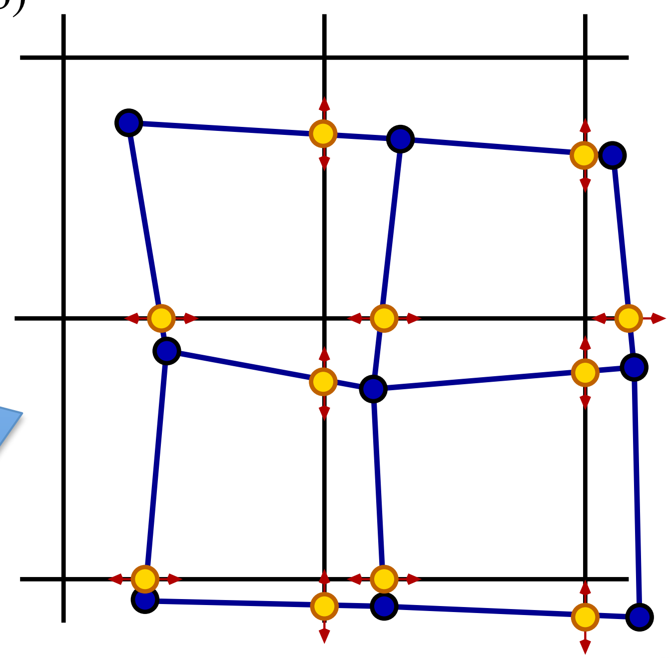
4. Consistency

Find upstream area, a_k, so that CSLAM predicted mass field is equal to CAM-SE predicted mass field:

$$\Delta p_k^{n+1}(\text{CAM-SE}) \;=\; \frac{1}{\Delta A}\int_{a_k} \delta p_k^n \, dA \; (\text{CSLAM}) \; , . \tag{4}$$

Local iteration problem to find equivalent upstream areas:

(b)



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Lauritzen, Taylor, Overfelt, Ullrich and Goldhaber (2016, IN PREP)

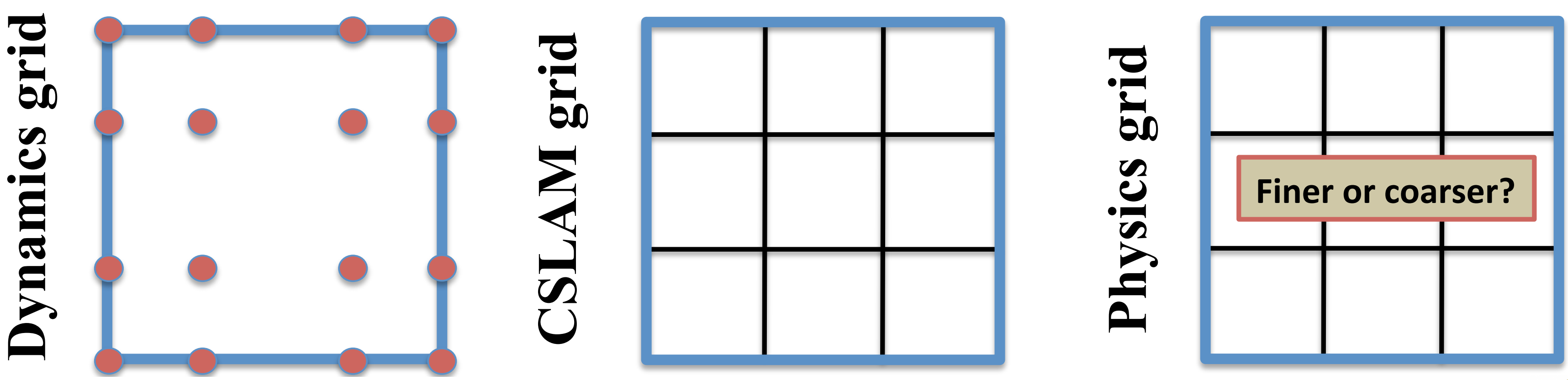
CAM-SE-CSLAM

A new model configuration based on CAM-SE:

- **SE**: Spectral-element dynamical core solving for $\vec{v}$, T, p_s

  (Dennis et al., 2012; Evans et al., 2012; Taylor and Fournier, 2010; Taylor et al., 1997)
- **CSLAM**: Semi-Lagrangian finite-volume transport scheme for tracers

  (Lauritzen et al., 2010; Erath et al., 2013, 2012; Harris et al., 2010)
- **Phys-grid**: Separating physics and dynamics grids, i.e. ability to compute physics tendencies based on cell-averaged values within each element instead of quadrature points



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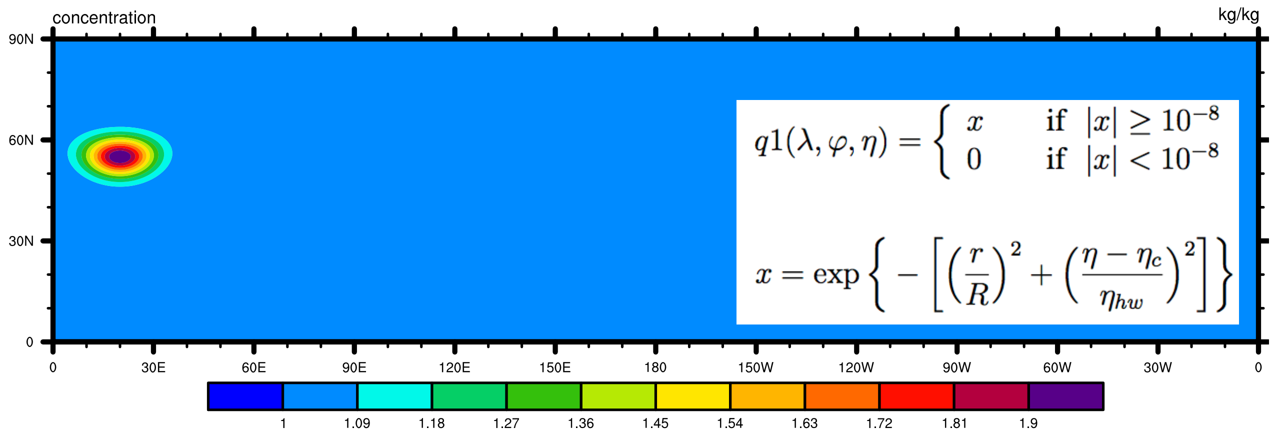
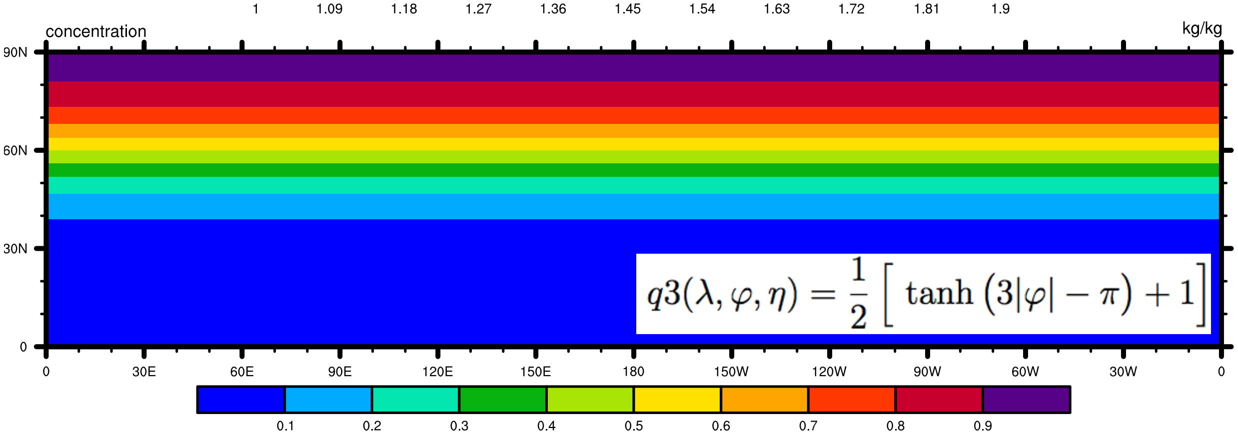
Lauritzen, Taylor, Overfelt, Ullrich and Goldhaber (2016, IN PREP)

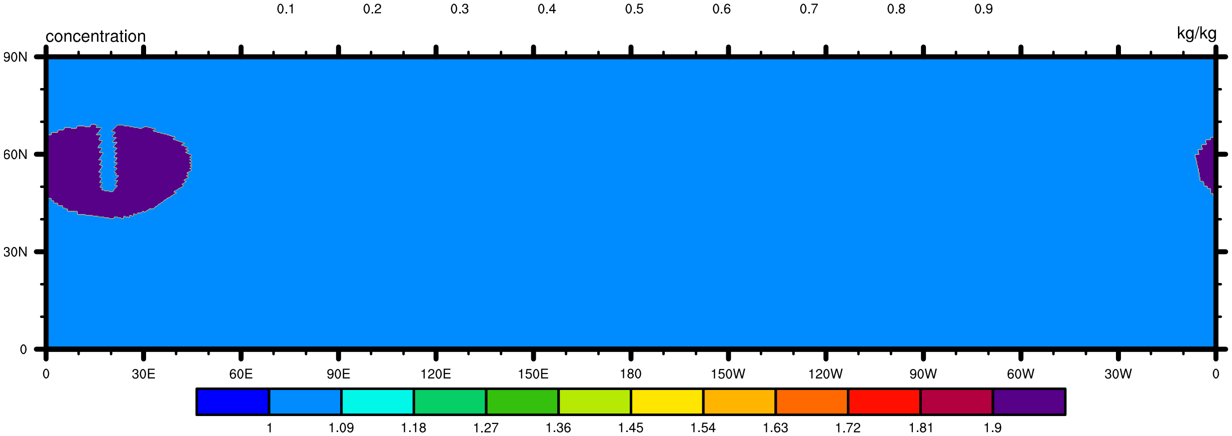
3 tracers: initial conditions



Gaussian "ball"



Zonally symmetric (smooth)



Slotted cylinder

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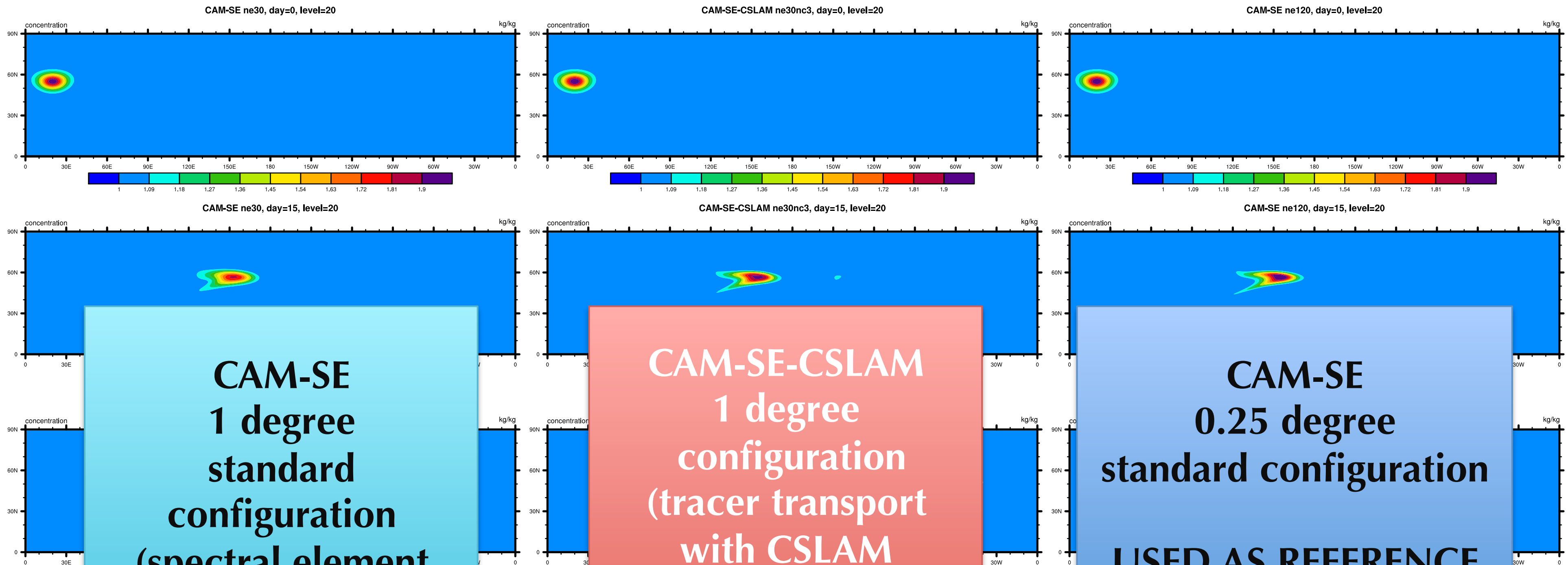
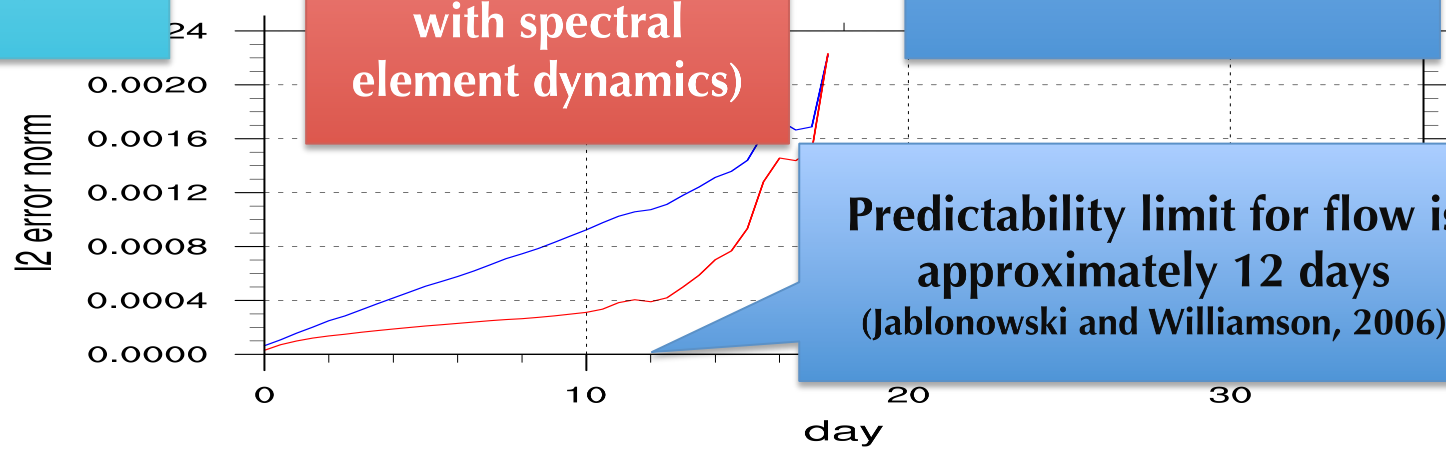


CAM-SE
1 degree
standard configuration
(spectral element advection)

CAM-SE-CSLAM
1 degree
configuration
(tracer transport with CSLAM consistently coupled with spectral element dynamics)

CAM-SE
0.25 degree
standard configuration

USED AS REFERENCE SOLUTION ("TRUTH")



Predictability limit for flow is approximately 12 days
(Jablonowski and Williamson, 2006)

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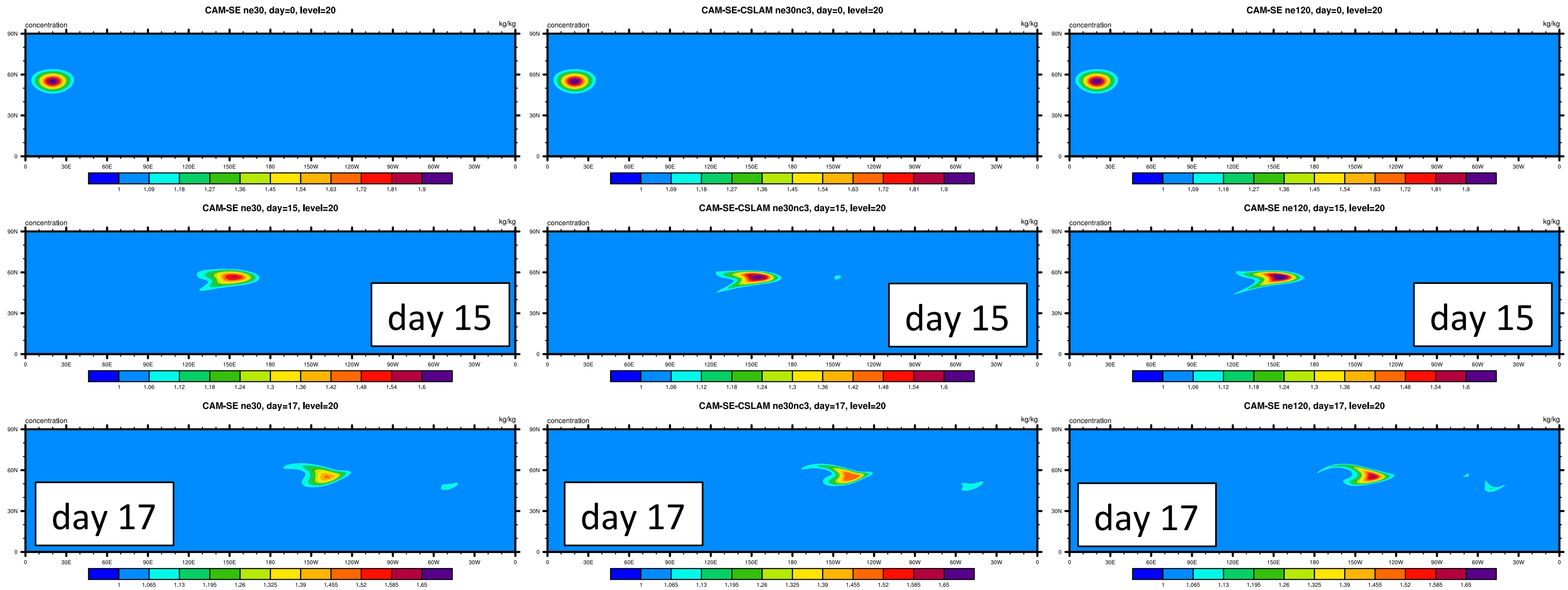
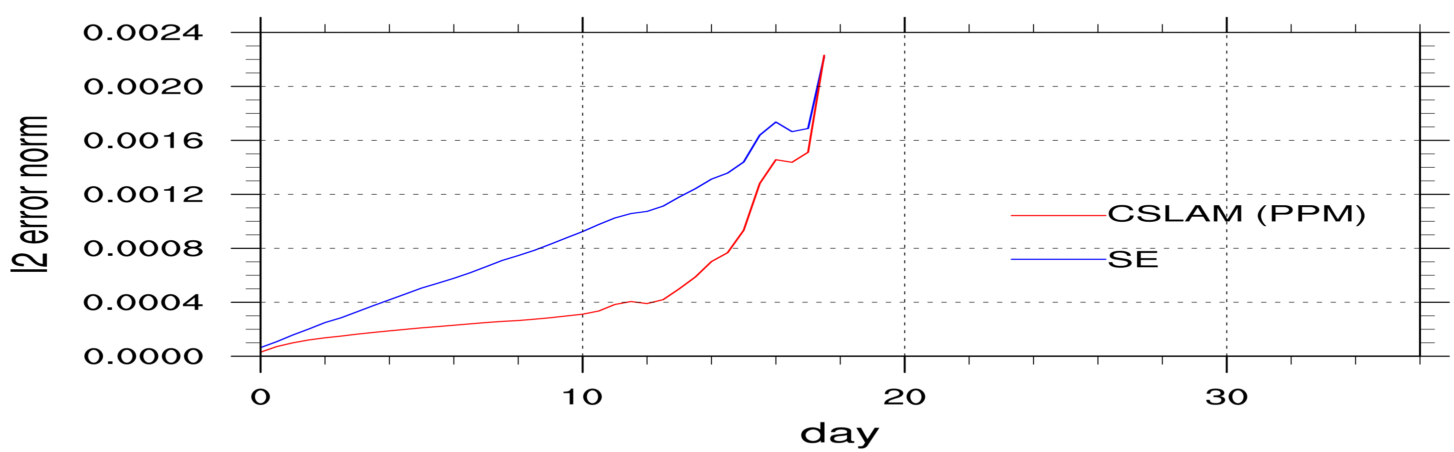
CAM-SE | **CAM-SE-CSLAM** | **CAM-SE reference**





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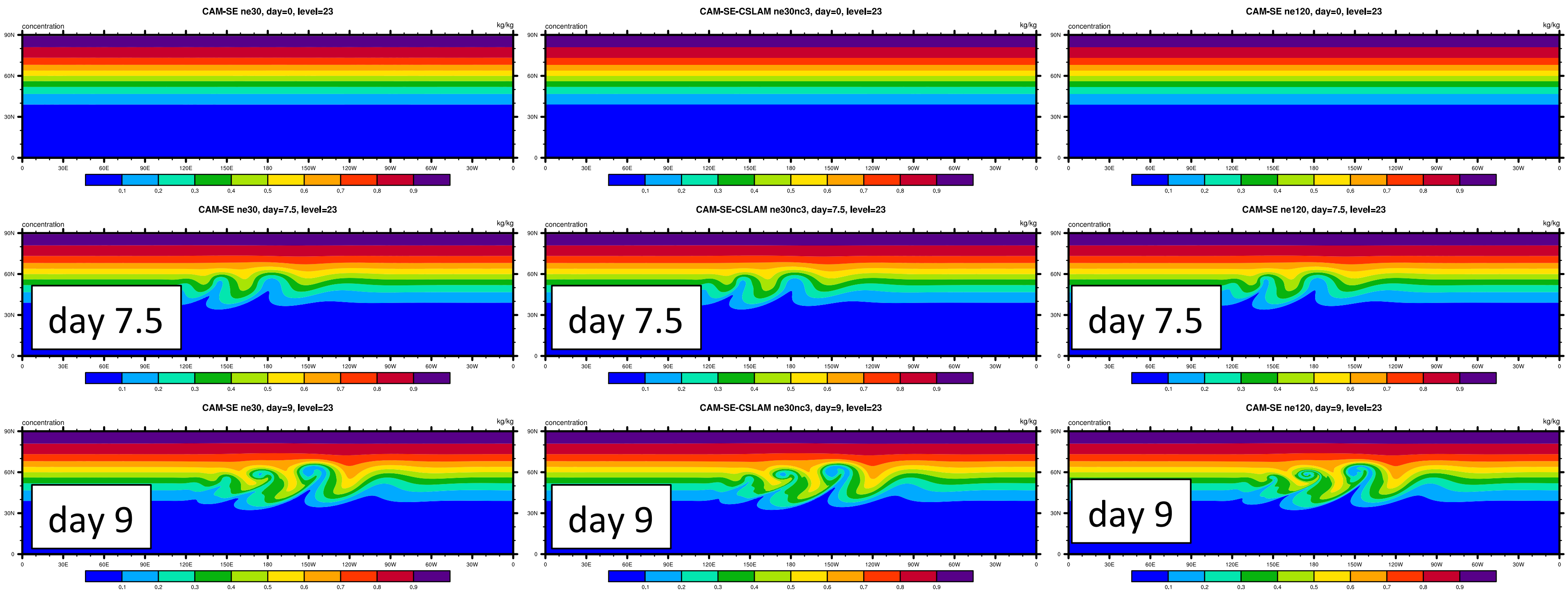
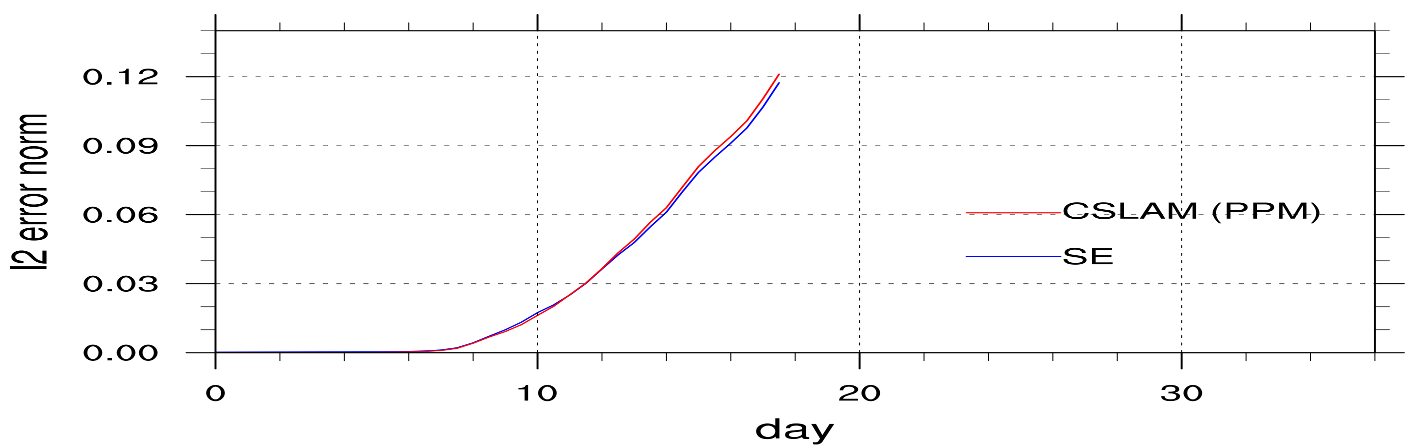
CAM-SE   CAM-SE-CSLAM   CAM-SE reference





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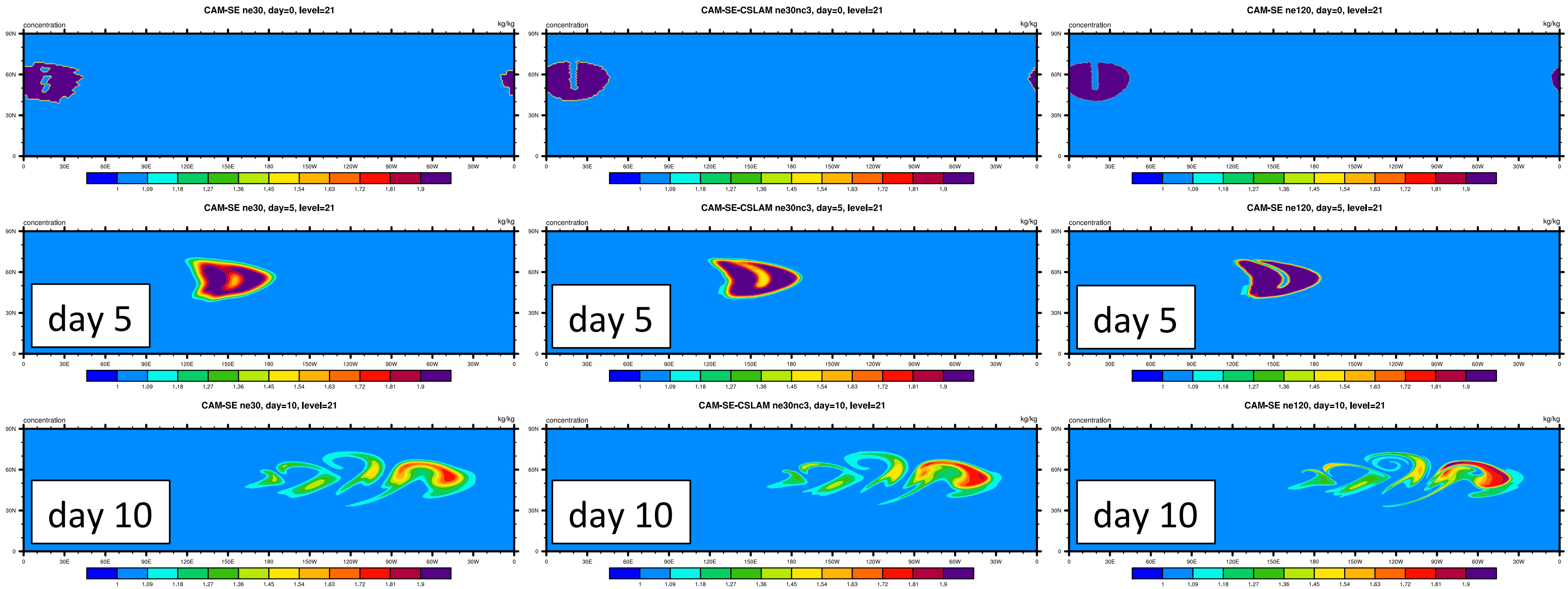
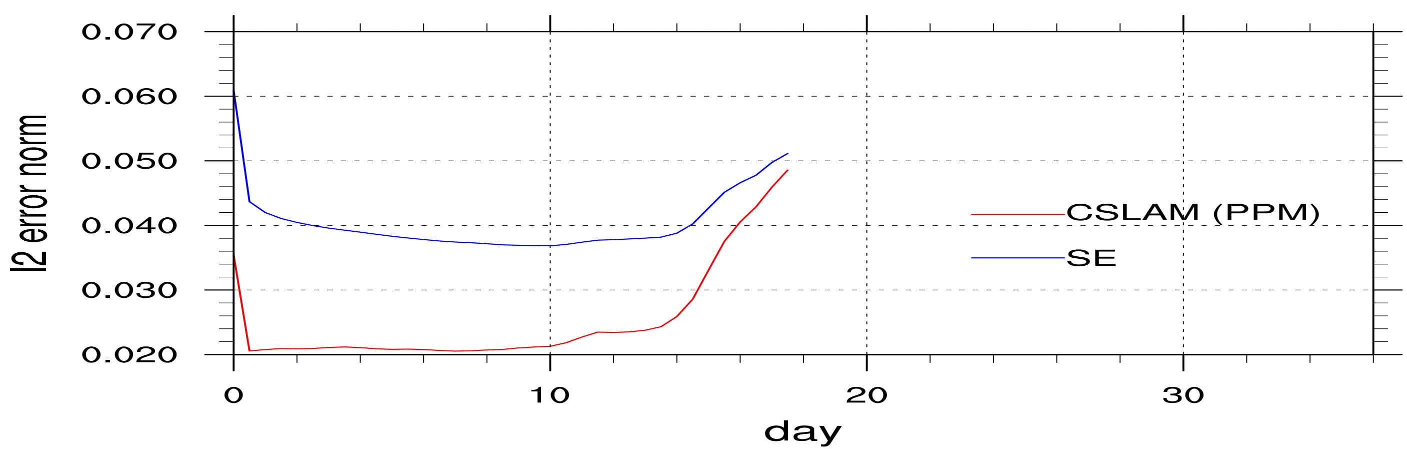
CAM-SE | CAM-SE-CSLAM | CAM-SE reference





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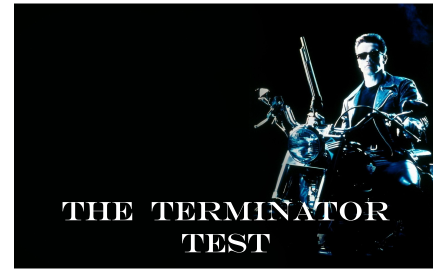
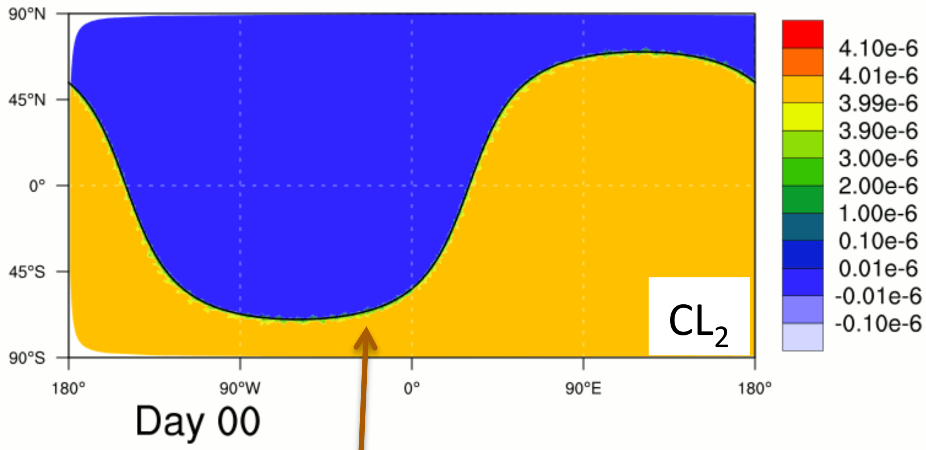
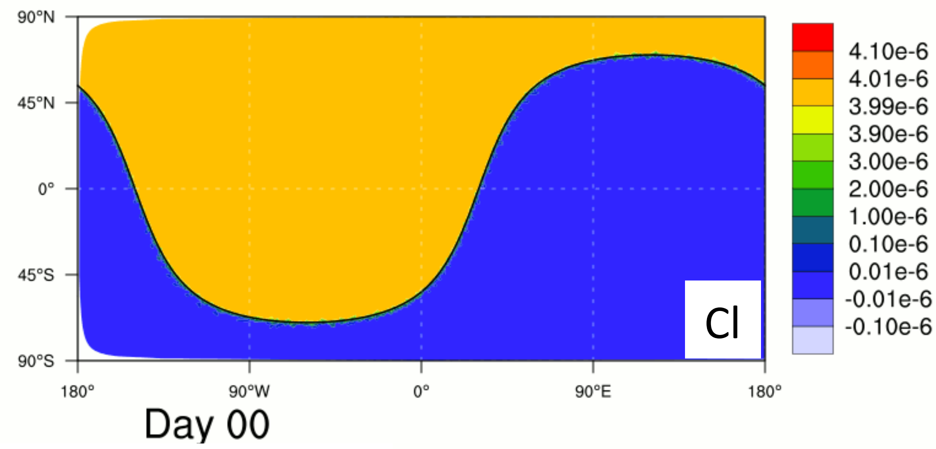
The terminator 'toy'-chemistry test: A simple tool to assess errors in transport schemes

(Lauritzen et al., 2015, GMD)

See: http://www.cgd.ucar.edu/cms/pel/terminator.html







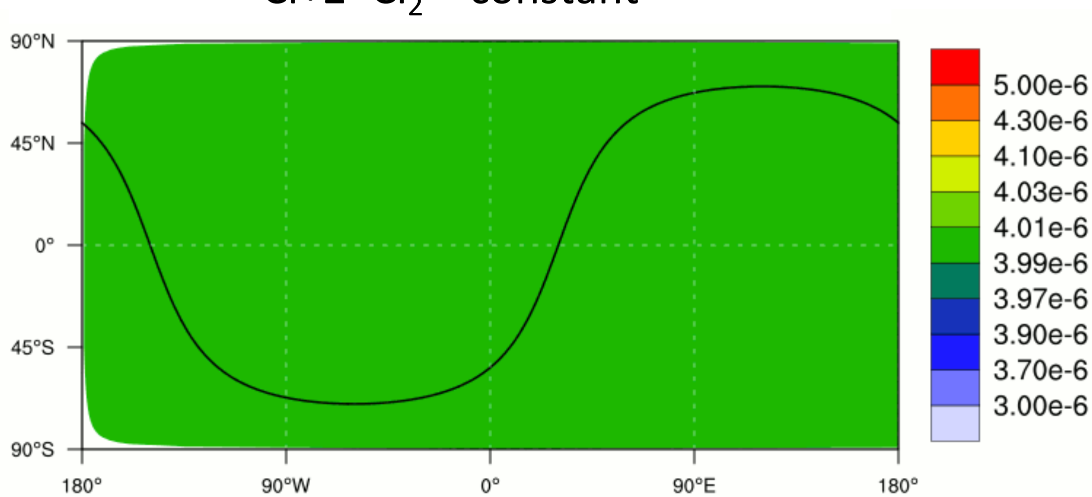
Non-linear Terminator 'toy' chemistry:

$$Cl_2 \rightarrow Cl + Cl : k_1$$

$$Cl + Cl \rightarrow Cl_2 : k_2$$

Exact solution: $Cl+2*Cl_2$ = constant

$Cl+2*Cl_2$ = constant



Wind field: Nair and Lauritzen deformational flow

Errors are due to non-conservation of linear correlations usually caused by the limiter/filter and/or physics-dynamics coupling!

`Toy' terminator chemistry code:

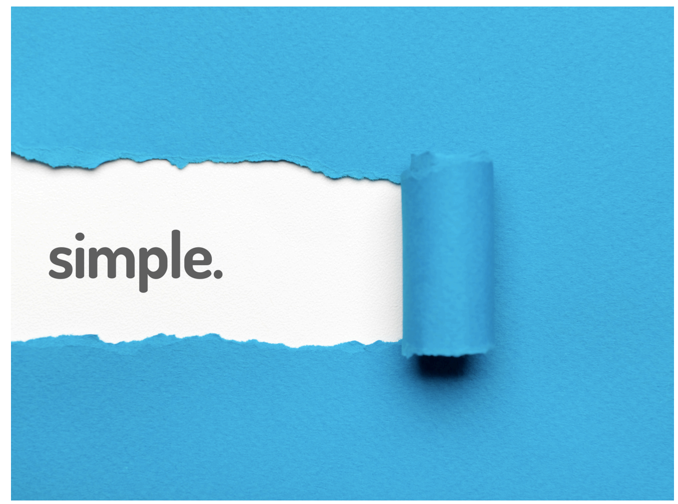
In terms of Fortran code the analytical forcing is given by:

```
! dt is size of physics time step
cly = cl + 2.0*cl2

r = k1 / (4.0*k2)
d = sqrt( r*r + 2.0*r*cly )
e = exp( -4.0*k2*d*dt )

if( abs(d*k2*dt) .gt. 1e-16 )
  el = (1.0-e) / (d*dt)
else
  el = 4.0*k2
endif

f_cl  = -el * (cl-d+r) * (cl+d+r) / (1.0 + e + dt*el*(cl+r))
f_cl2 = -f_cl / 2.0
```



3D version of terminator test

The terminator test setup can be used in any flow field and the analytical solution for Cly is always known!

- Use baroclinic wave setup (a variation of Ullrich et al., 2015)
- Initialize with same mixing ratio distribution in each layer (same as 2D test)
- As a diagnostic we use average column integrated mixing ratios (q) so that diagnostics are independent of vertical coordinate:

$$< q >= \frac{\int_{z=0}^{z_{top}} q\, dz}{\int_{z=0}^{z_{top}} dz}.$$

- Since exact solution is known (q_{Cly}=4E-6 kg/kg) we can compute error norms:

$\ell_2(t), \ell_\infty(t)$ and relative mass change $\Delta M(t)$ error norms for Cl_y:

$$\ell_2(t) = \frac{\sqrt{\int_{z=0}^{z_{top}} (< q_{Cly} > -4.0 \times 10^{-6})^2\, dz}}{\sqrt{\int_{z=0}^{z_{top}} (4.0 \times 10^{-6})^2\, dz}},$$

$$\ell_\infty(t) = \frac{\max_{\text{all } \lambda,\theta} |< q_{Cly} > -4.0 \times 10^{-6}|}{4.0 \times 10^{-6}},$$

$$\Delta M(t) = \frac{\int_{z=0}^{z_{top}} q_{Cly} dz - M_0}{M_0}$$

respectively, where

$$< q_{Cly} >=< q_{Cl} + 2q_{Cl2} > .$$

and M_0 is the initial mass of Cl_y

$$M_0 = \int_{z=0}^{z_{top}} 4.0 \times 10^{-6} dz.$$

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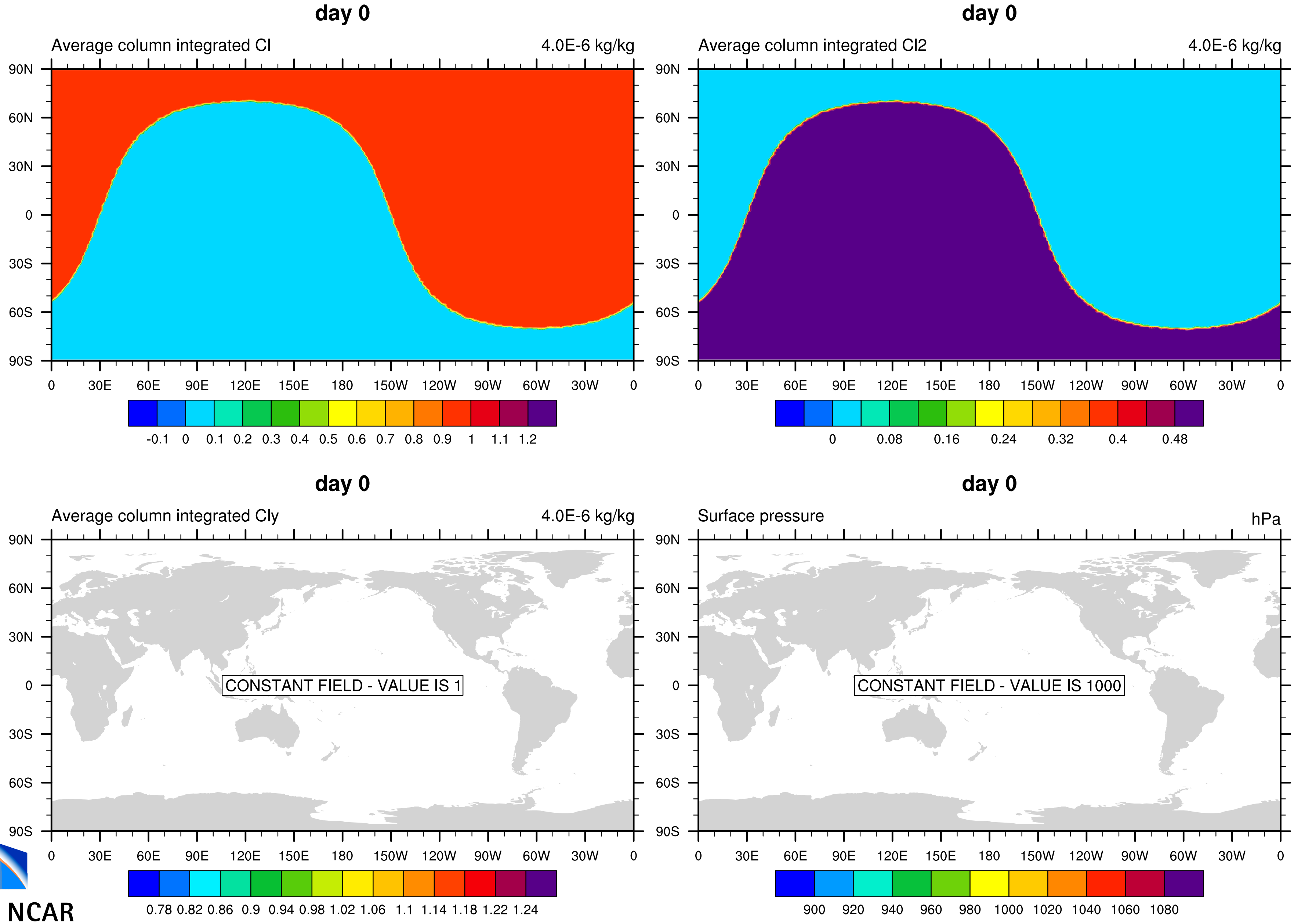
Initial condition



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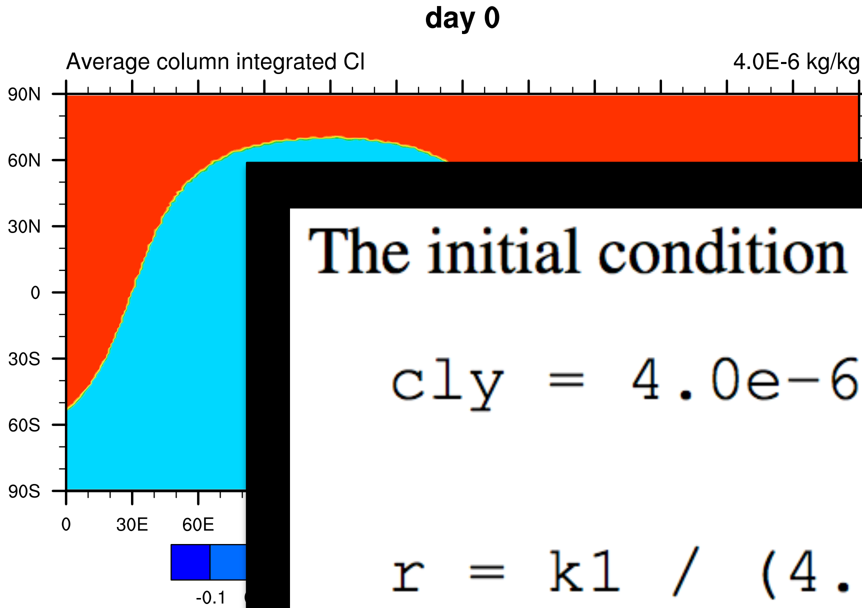
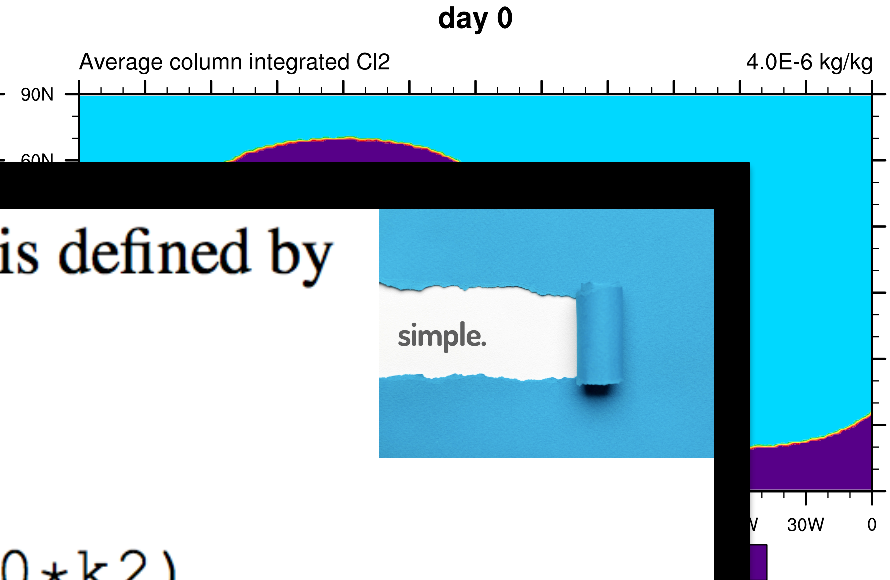
Initial condition



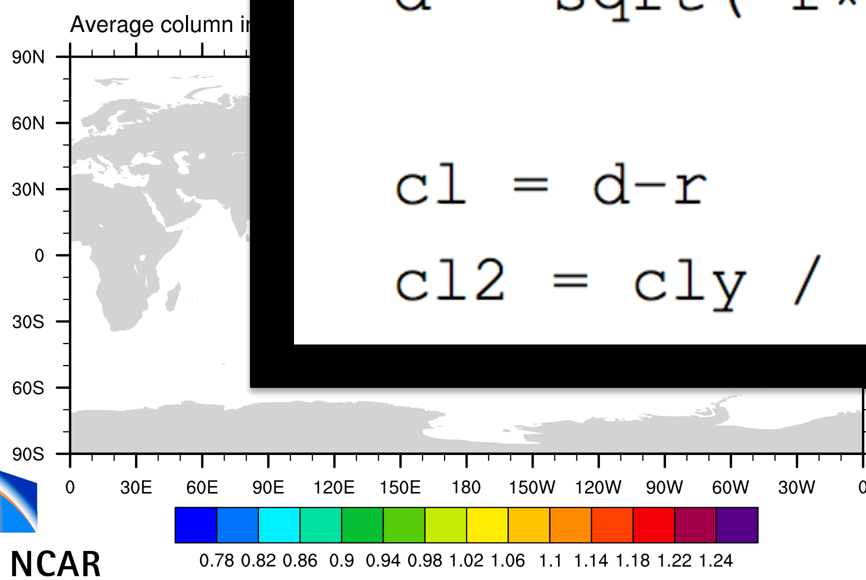


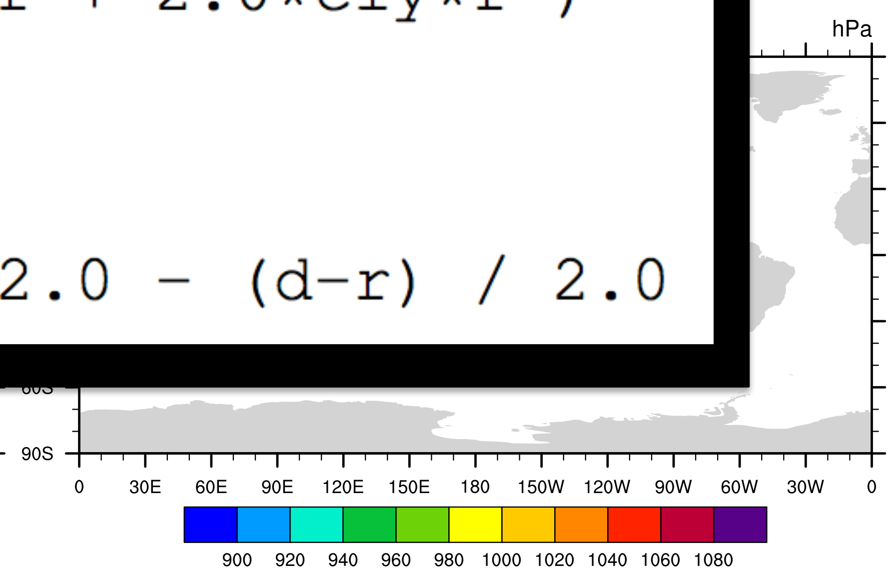
The initial condition is defined by

```
cly = 4.0e-6

r = k1 / (4.0*k2)
d = sqrt( r*r + 2.0*cly*r )

cl = d-r
cl2 = cly / 2.0 - (d-r) / 2.0
```





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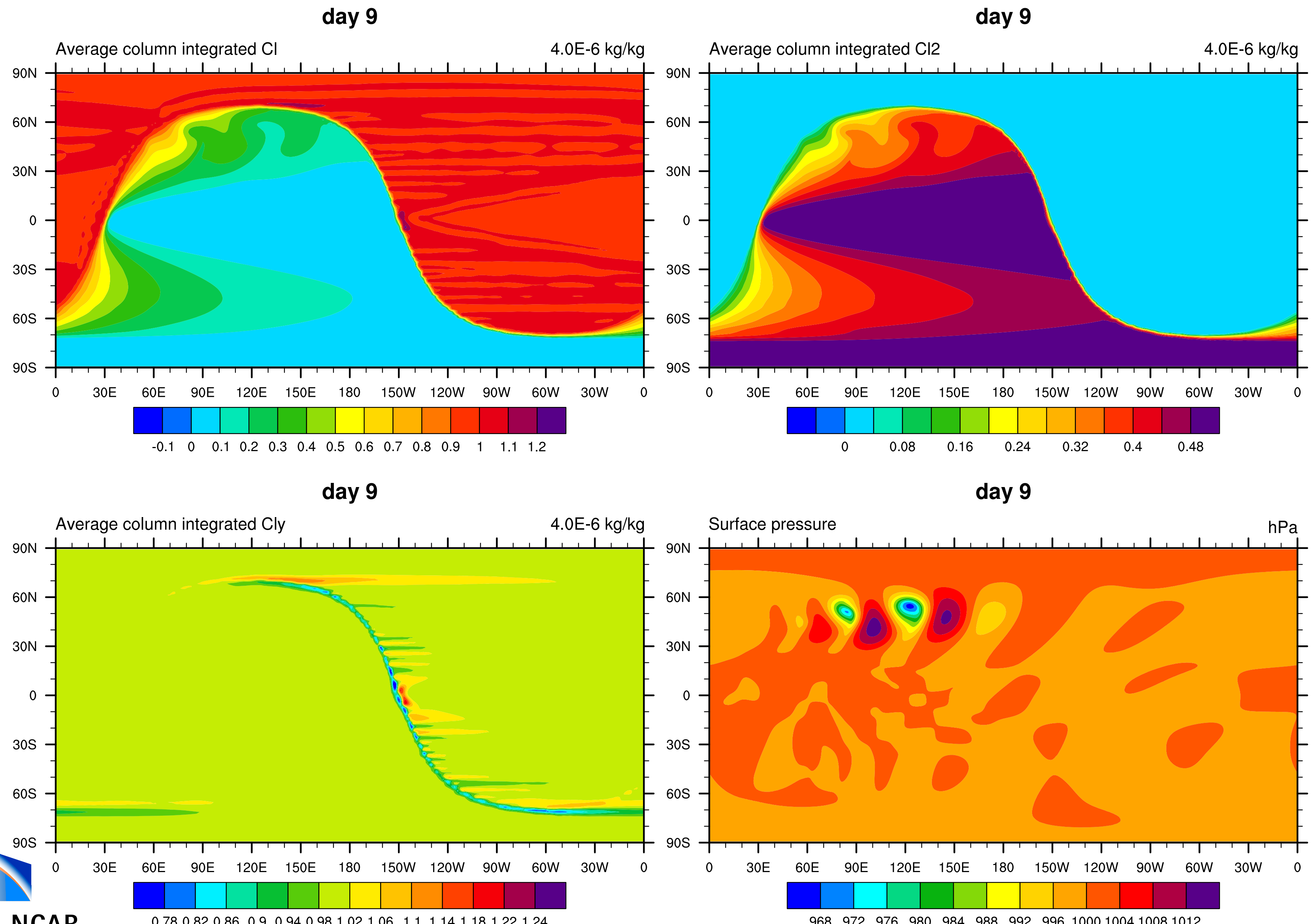
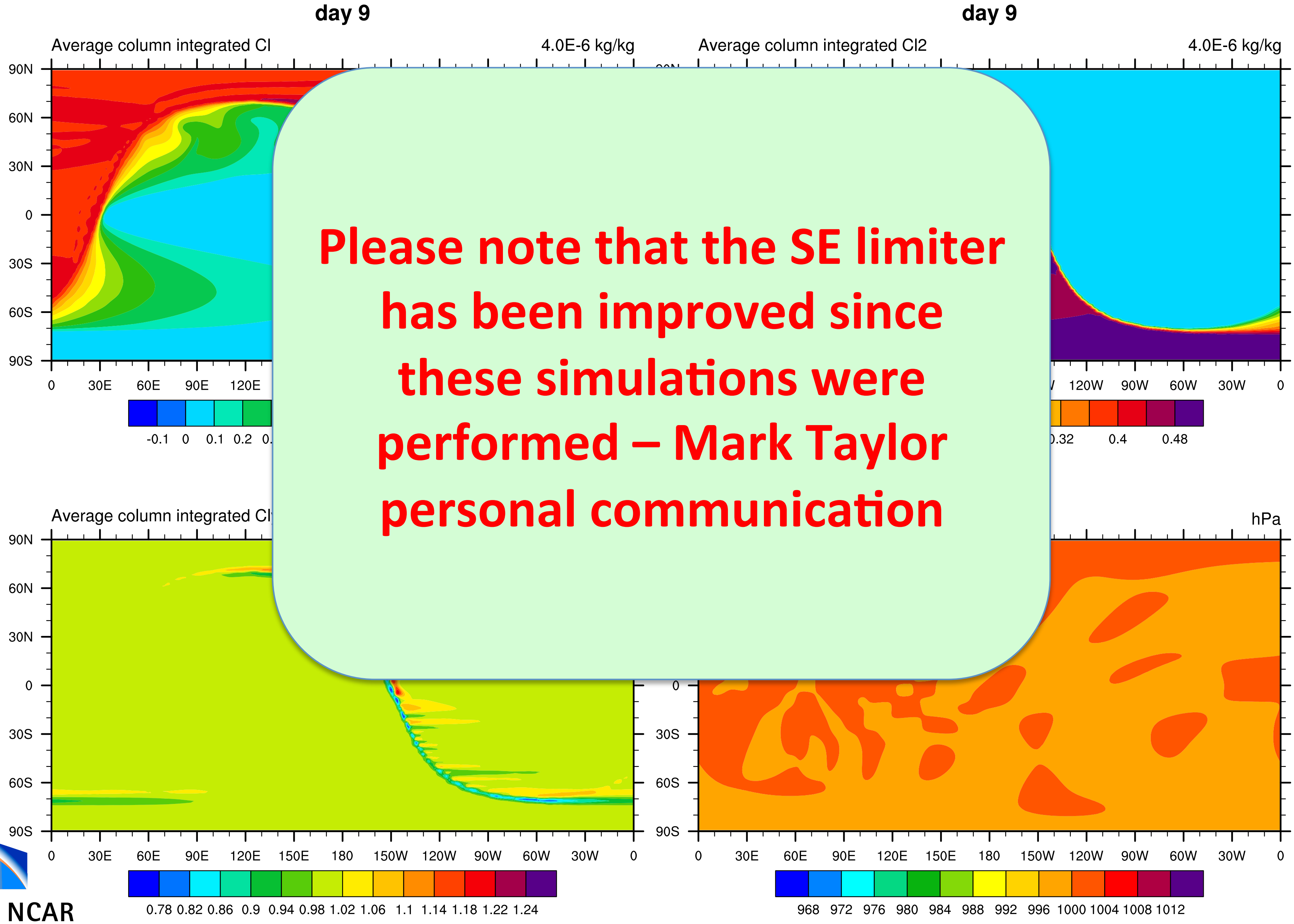
CAM-SE


day 9
Average column integrated Cl
4.0E-6 kg/kg
90N 60N 30N 0 30S 60S 90S
0 30E 60E 90E 120E 150E 180 150W 120W 90W 60W 30W 0
-0.1 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2
day 9
Average column integrated Cl2
4.0E-6 kg/kg
90N 60N 30N 0 30S 60S 90S
0 30E 60E 90E 120E 150E 180 150W 120W 90W 60W 30W 0
0 0.08 0.16 0.24 0.32 0.4 0.48
day 9
Average column integrated Cly
4.0E-6 kg/kg
90N 60N 30N 0 30S 60S 90S
0 30E 60E 90E 120E 150E 180 150W 120W 90W 60W 30W 0
0.78 0.82 0.86 0.9 0.94 0.98 1.02 1.06 1.1 1.14 1.18 1.22 1.24
day 9
Surface pressure
hPa
90N 60N 30N 0 30S 60S 90S
0 30E 60E 90E 120E 150E 180 150W 120W 90W 60W 30W 0
968 972 976 980 984 988 992 996 1000 1004 1008 1012

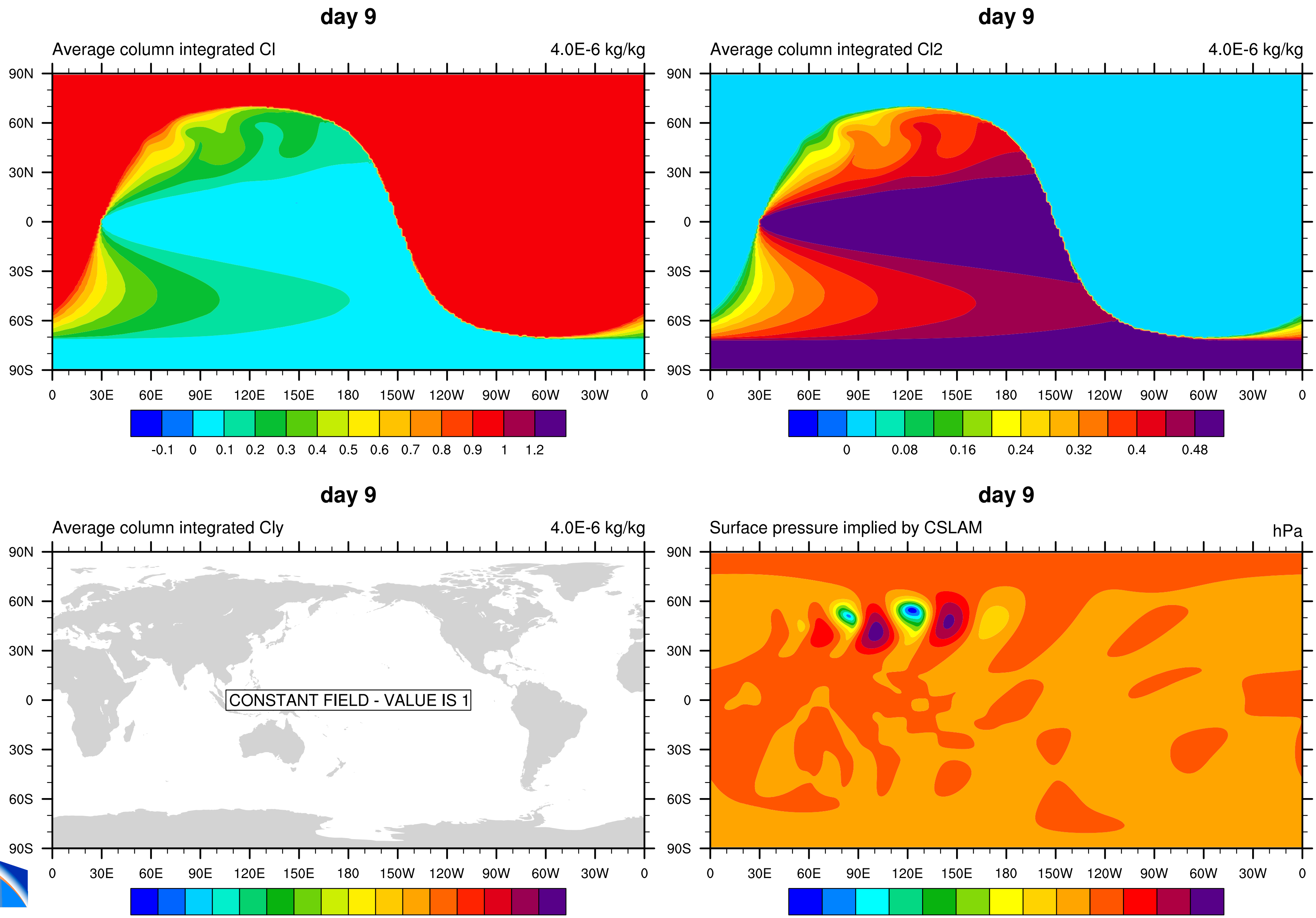
NCAR

CAM-SE



Please note that the SE limiter has been improved since these simulations were performed – Mark Taylor personal communication

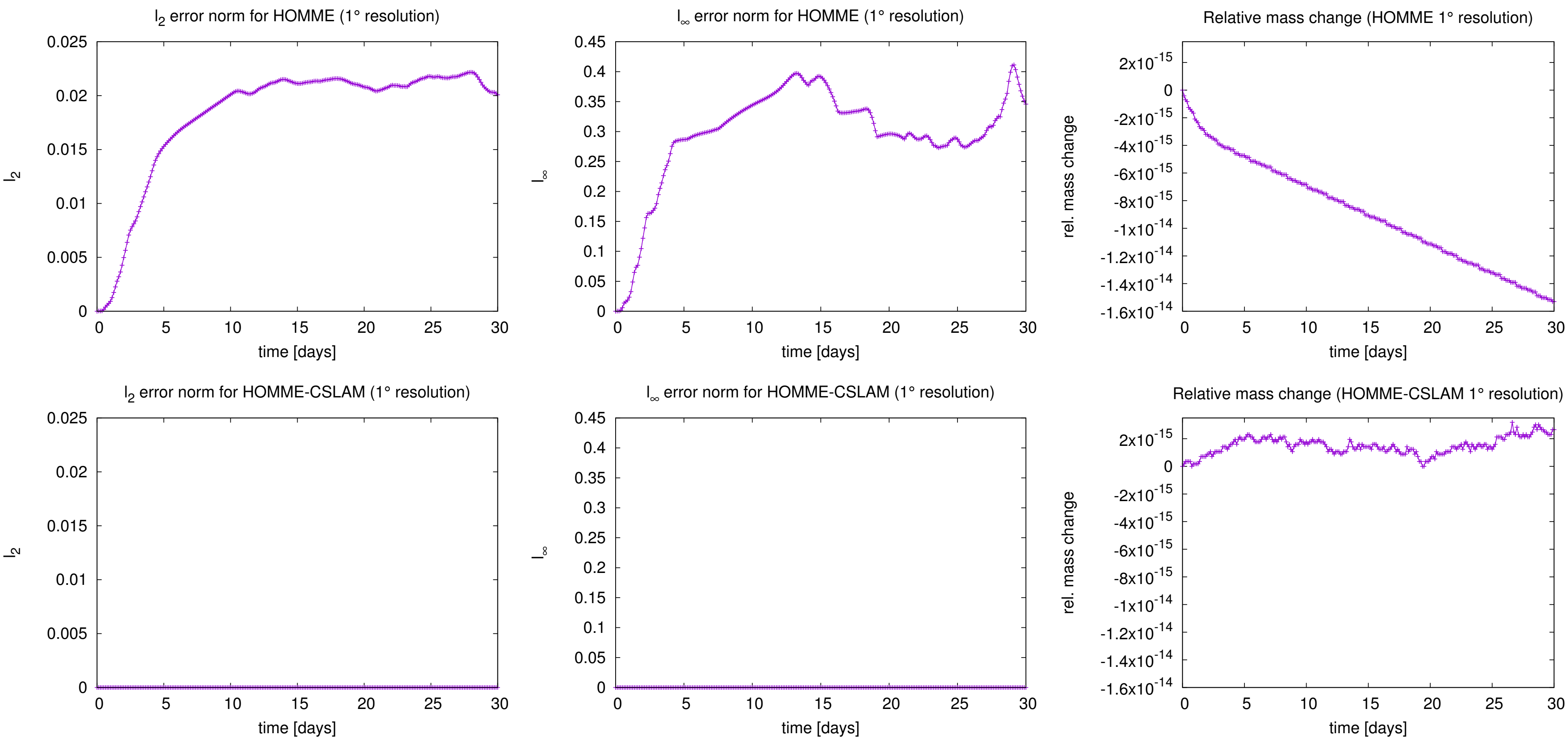
CAM-SE-CSLAM



NCAR

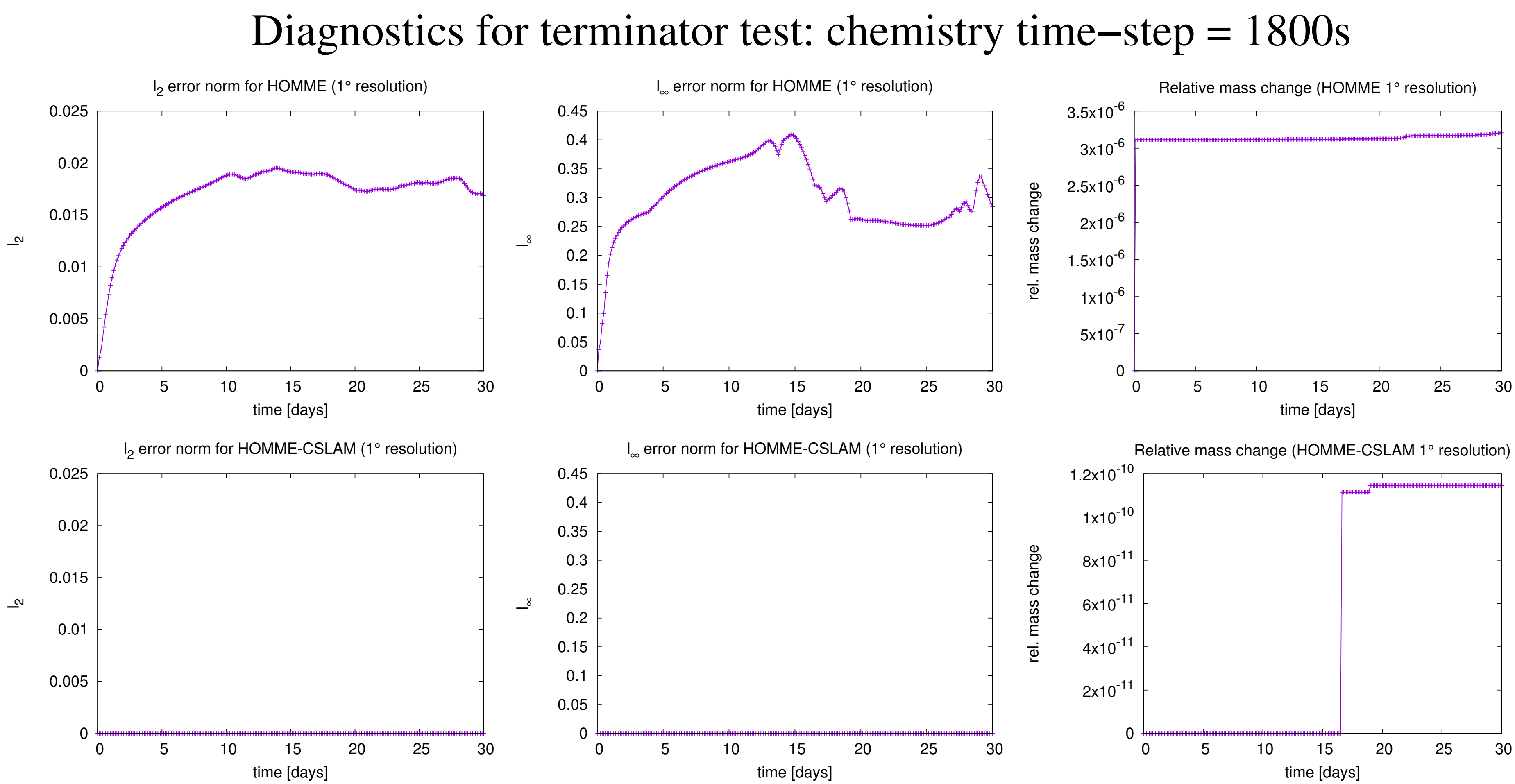
CAM-SE-CSLAM

Diagnostics for terminator test: chemistry time–step = 900s



NCAR

CAM-SE and CAM-SE-CSLAM



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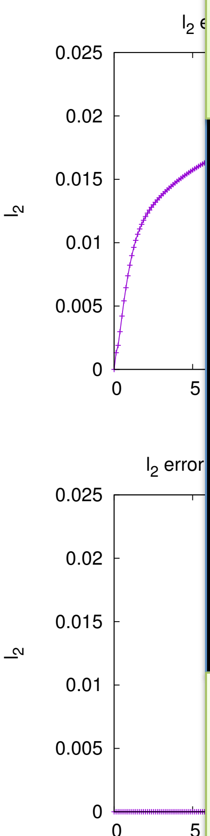
CAM-SE and CAM-SE-CSLAM

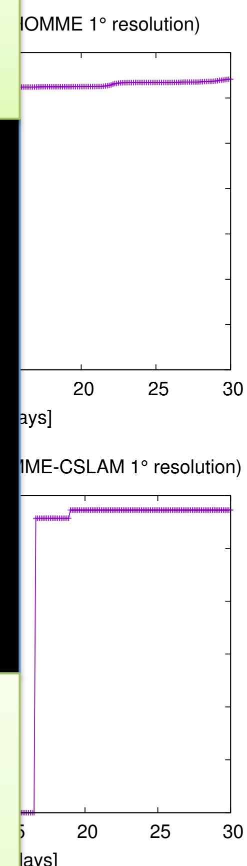


Mass-conservation of Cly violated by physics-dynamics coupling:

```
do i=Cl,Cl2
   ! dt is size of physics time step
   F=dt*F_i
   if (q_i+F<0) then
      if (q_i<0) then
         !don’t make negate q more negative
         F=0
      else
         ! make forcing as close as possible to f_q without making q negative
         F=-q_i
     end if
  end if
  q_i=q_i+F
end do
```

Example: if forcing for q_{Cl} is modified by if-statement then $F_{Cl} \neq -2F_{Cl2}$ and preservation of linear correlations by physics-dynamics coupling is violated and thereby mass-conservation of Cl_y is violated!

Occurs at a point 2 times for CAM-SE-CSLAM and frequently (in time and space) for CAM-SE.



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