



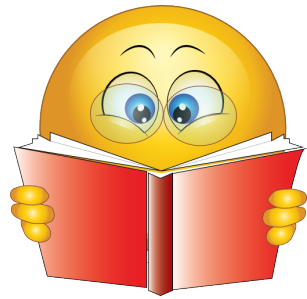
# Representation of topography in dynamical cores

**Peter Hjort Lauritzen**

**Atmospheric Modeling and Predictability Section  
Climate and Global Dynamics Laboratory  
National Center for Atmospheric Research**

**The Interaction of PRecipitation and Orography (IPRO)  
June 5-16, 2017  
NCAR, Boulder, Colorado**

# Career path



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Advanced Study Program  
Developing Scientific Leaders of the Future



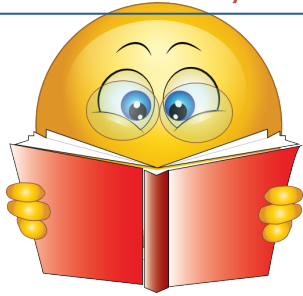
Dmi  
Vejr, klima og hav



2003 summer colloquium  
(organized by J.Tribbia)

# Career path

1994-6: Undergraduate studies in Mathematics and Physics



1996-7: Lead guitarist in funk-rock band; rock music school



1998: B.Sc. Physics (non-linear dynamics), Univ. of Aarhus, Denmark



1998-9: Firefighter & Rescue worker, Danish Emergency Service



2002: M.Sc. Geophysics, Univ. of Copenhagen, Denmark



**ASP**  
Advanced Study Program

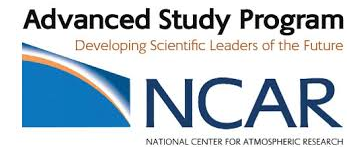
2003 summer colloquium (organized by J.Tribbia)



2005: Ph.D. Geophysics, Univ. of Copenhagen, Denmark



2006-8: ASP Postdoc, NCAR



2009-16: Scientist I, II, NCAR  
2016-...: Scientist III, NCAR



# Overview

## 1. Introduction: modeling the atmosphere

- **Resolved and un-resolved scales**
- **'Define' dynamical core and parameterizations**

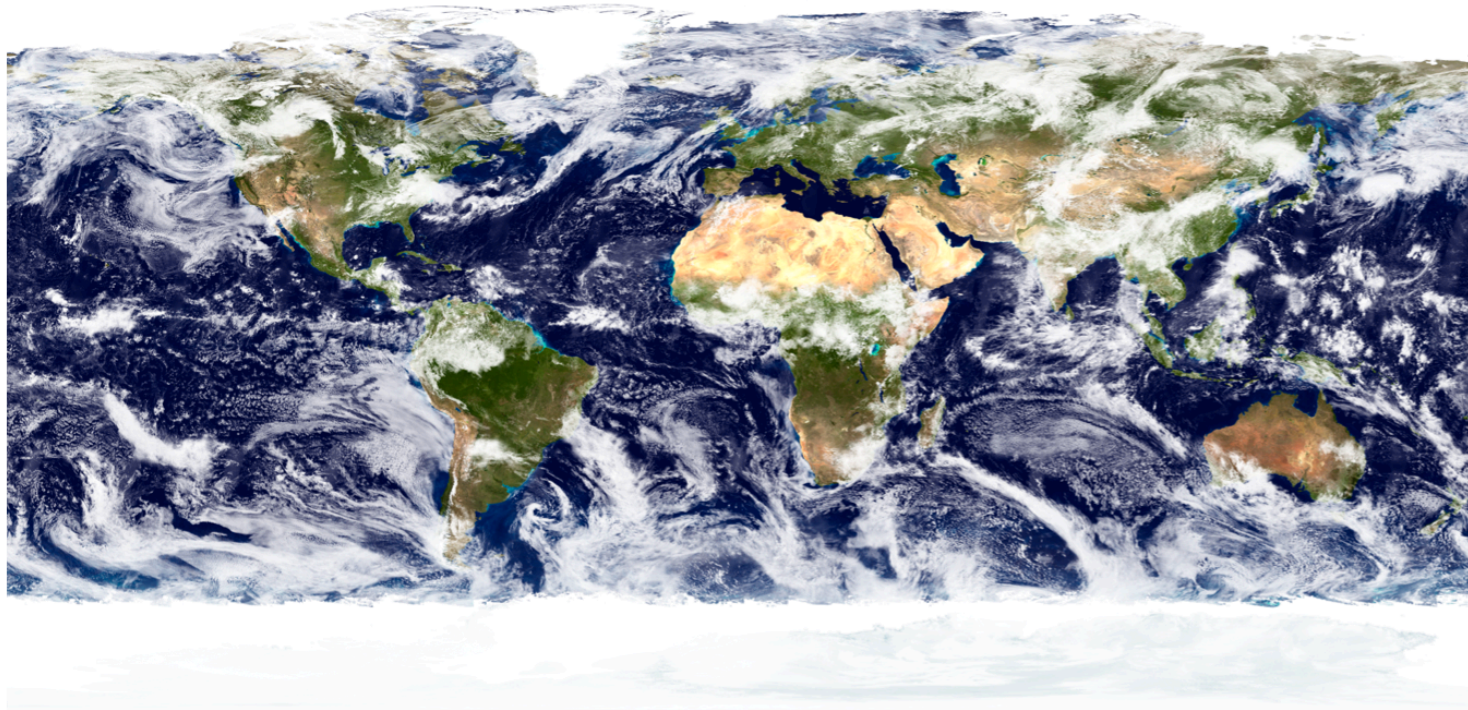
## 2. Representation of topography in models

- **From DEMs (Digital Elevation Models) to climate/weather model**
- **Resolved scale and sub-grid-scale topography**
- **Topography smoothing for the dynamical core**

## 3. Vertical coordinates and topography

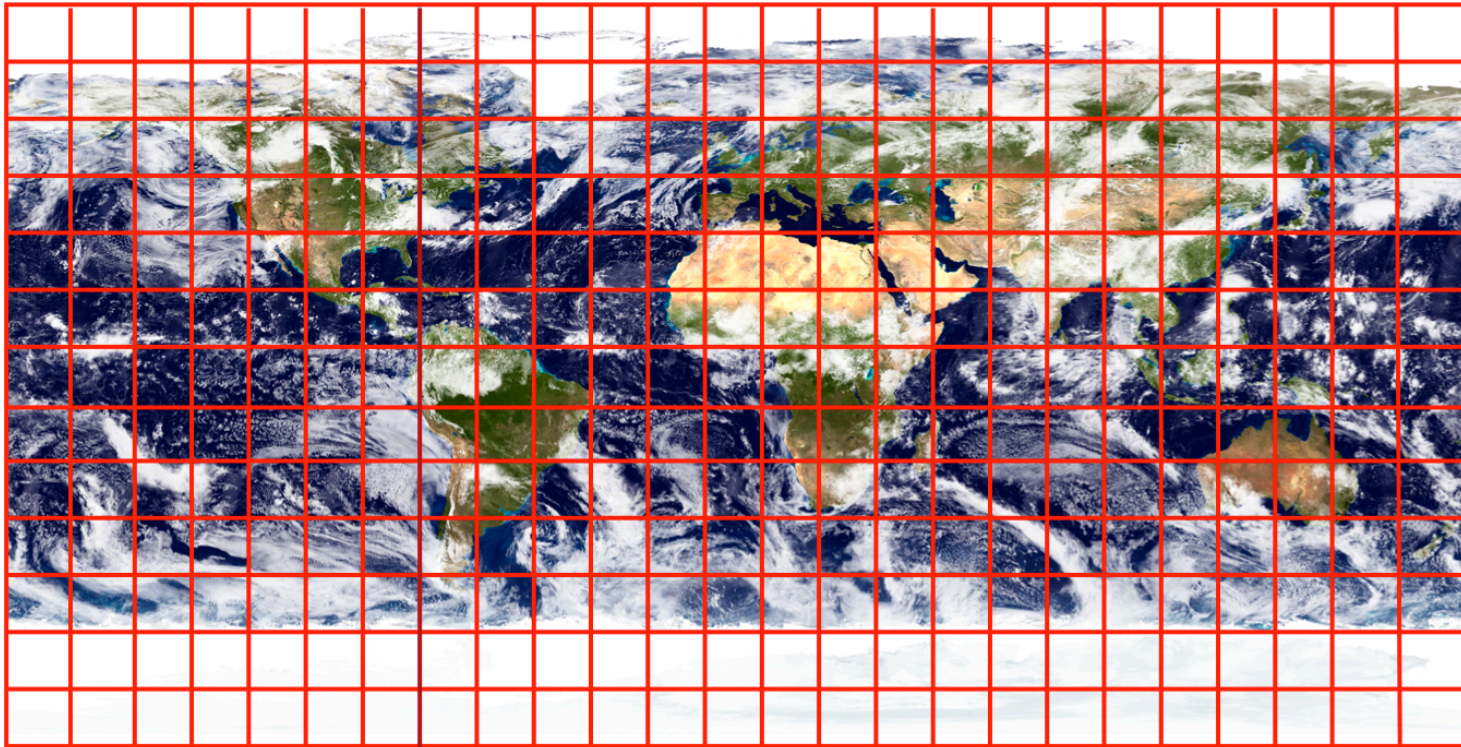


# Domain



Source: NASA Earth Observatory

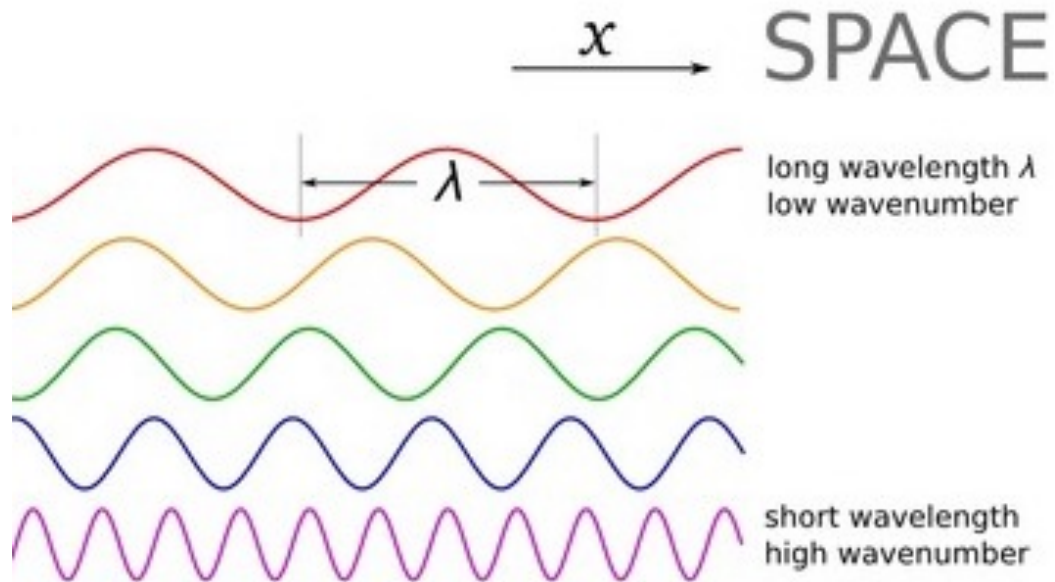
## Horizontal computational space



- Red lines: regular latitude-longitude grid
- Grid-cell size defines the smallest scale that can be resolved ( $\neq$  **effective resolution!**)
- Many important processes taking place sub-grid-scale that must be parameterized
- Loosely speaking, the parameterizations compute grid-cell average tendencies due to sub-grid-scale processes in terms of the (resolved scale) atmospheric state
- In modeling jargon parameterizations are also referred to as *physics* (what is unphysical about resolved scale dynamics?)

# Wavenumber analysis

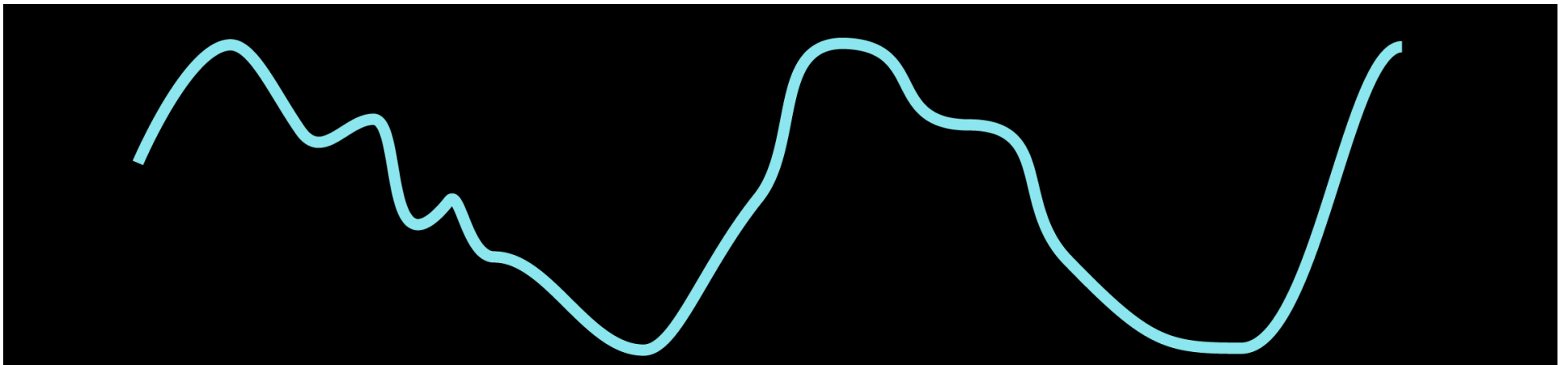
$$k = \frac{2\pi}{\lambda}$$

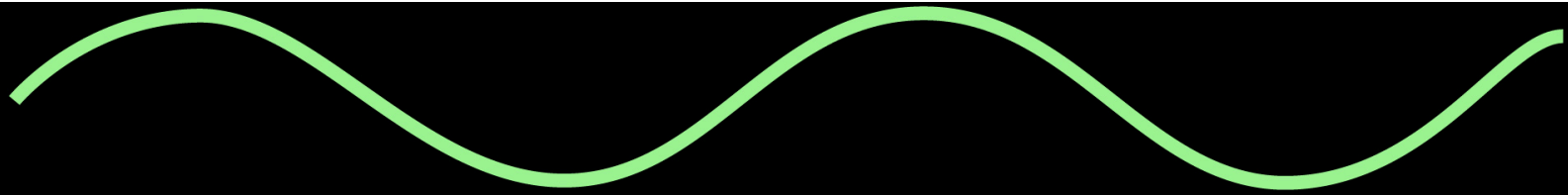




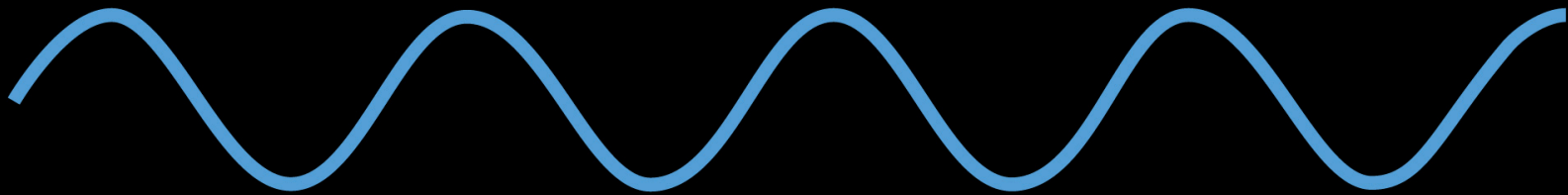
# Fourier decomposition (Fourier series)

- **Approximate distribution with sum of different amplitude sine waves**

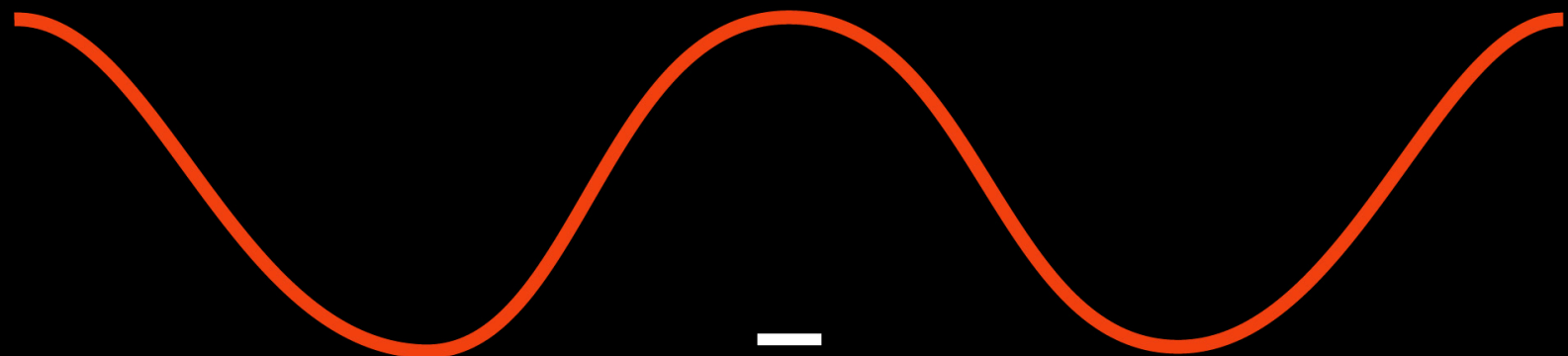




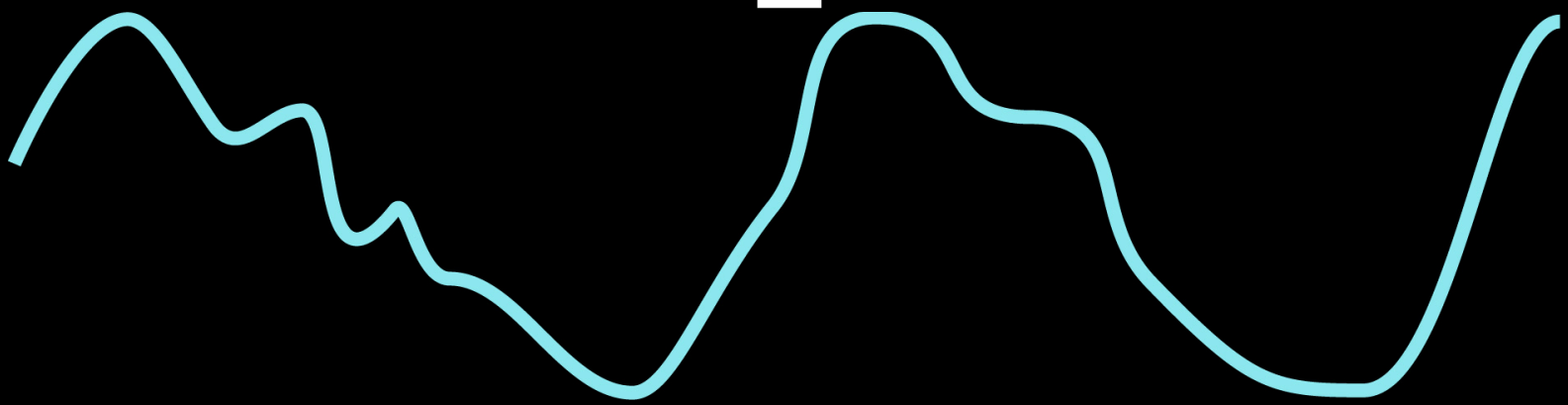
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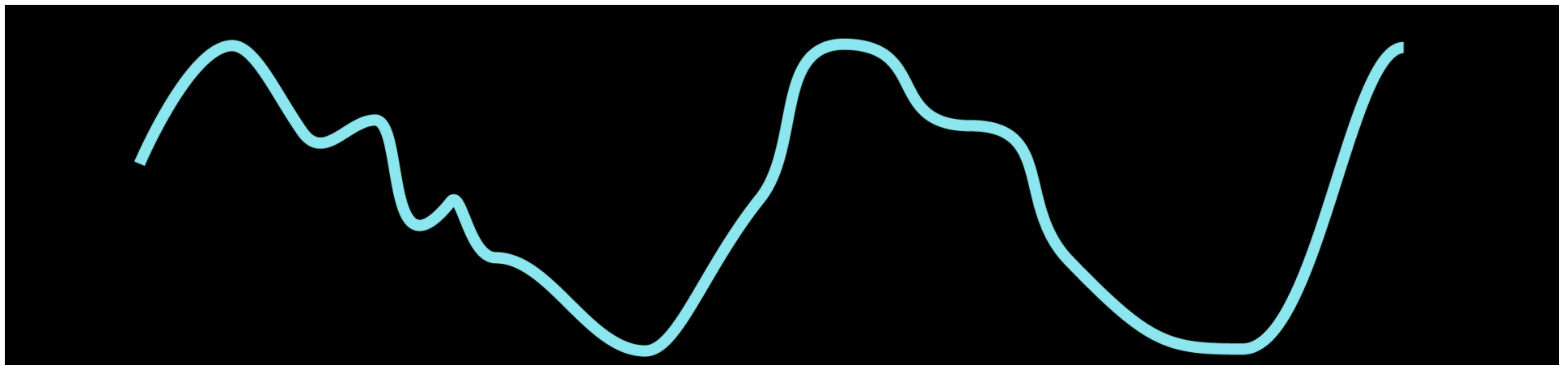
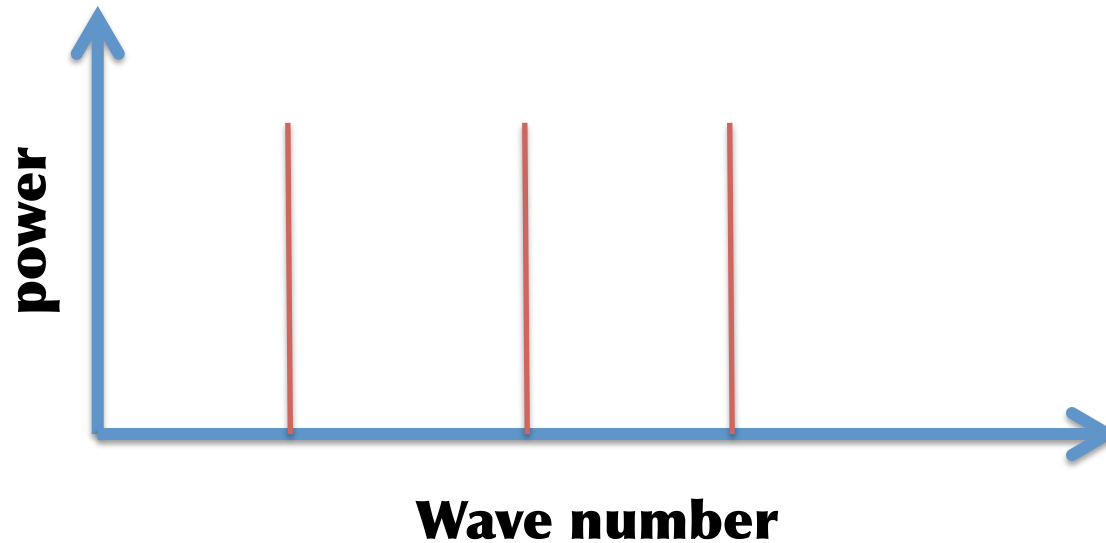
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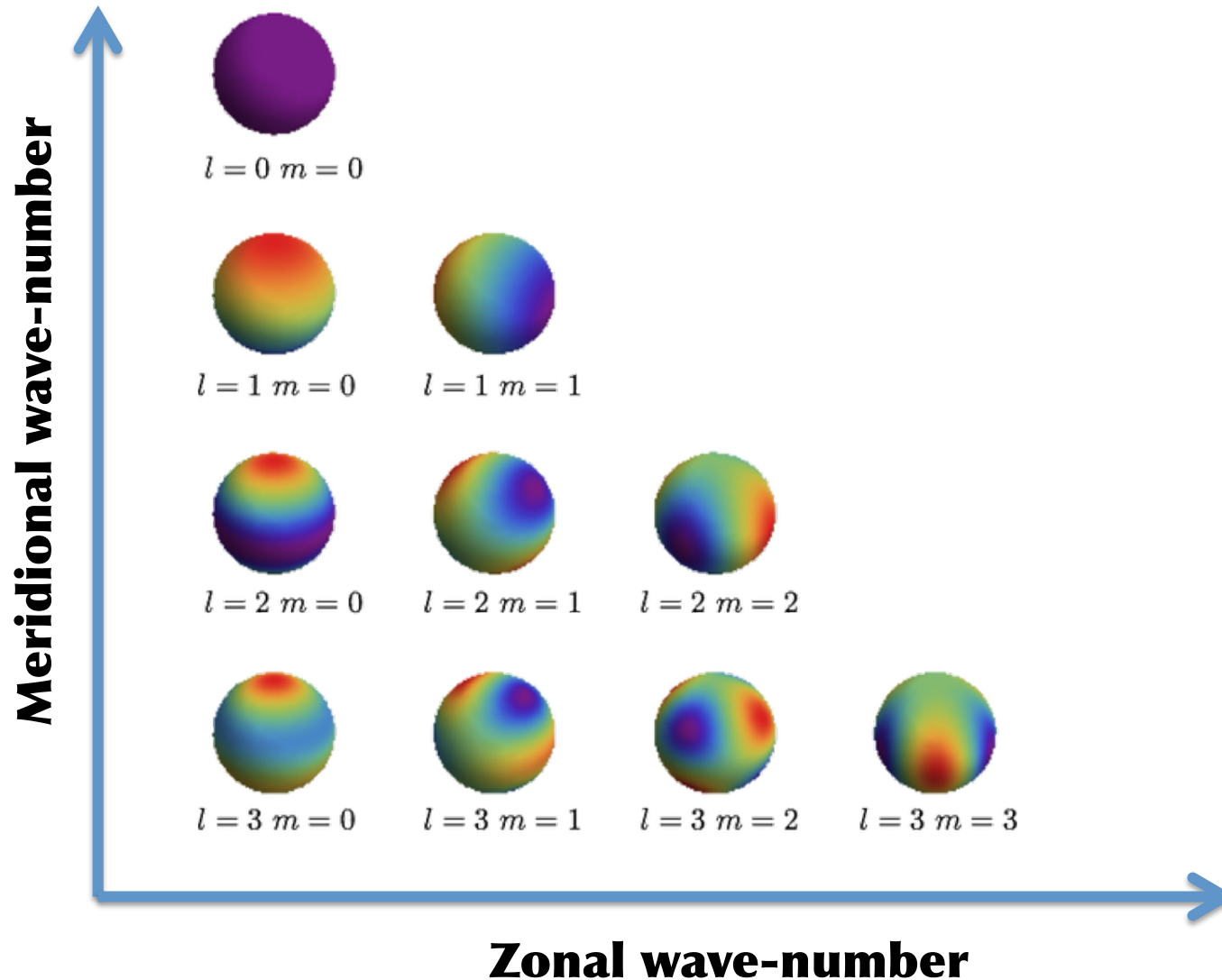


# Fourier decomposition (Fourier series)

- **Power spectrum: ~ amplitude of the sine waves**



# Wavenumber analysis on the sphere: Spherical Harmonics



**Effective resolution:** smallest scale ( highest wave-number  $k = k_{eff}$ ) that model can accurately represent

- $k_{eff}$  can be assessed analytically for linearized equations (Von Neumann analysis)
- In a full model one can assess  $k_{eff}$  using total kinetic energy spectra (TKE) of, e.g., horizontal wind  $\vec{v}$  (see Figure below)

**Effective resolution is typically 4-10 grid-lengths depending on numerical method!**

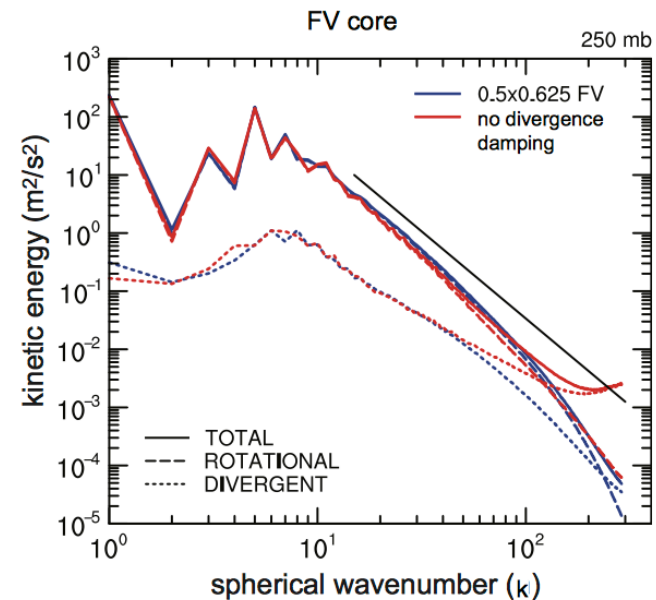
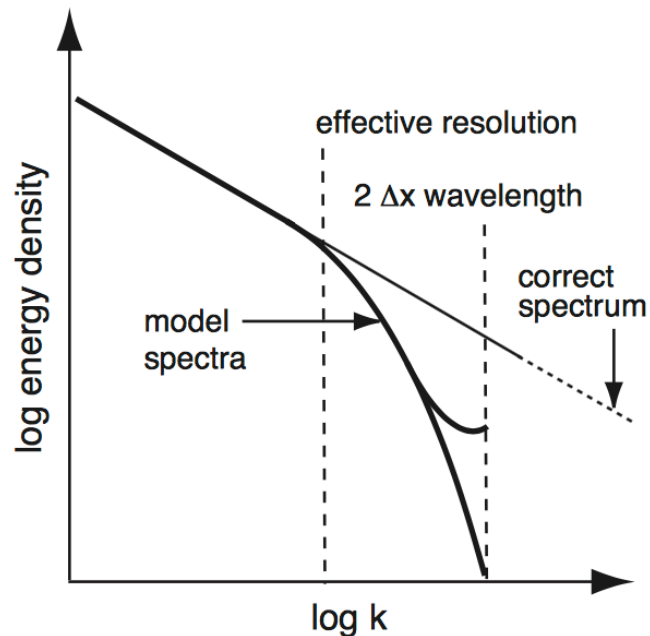
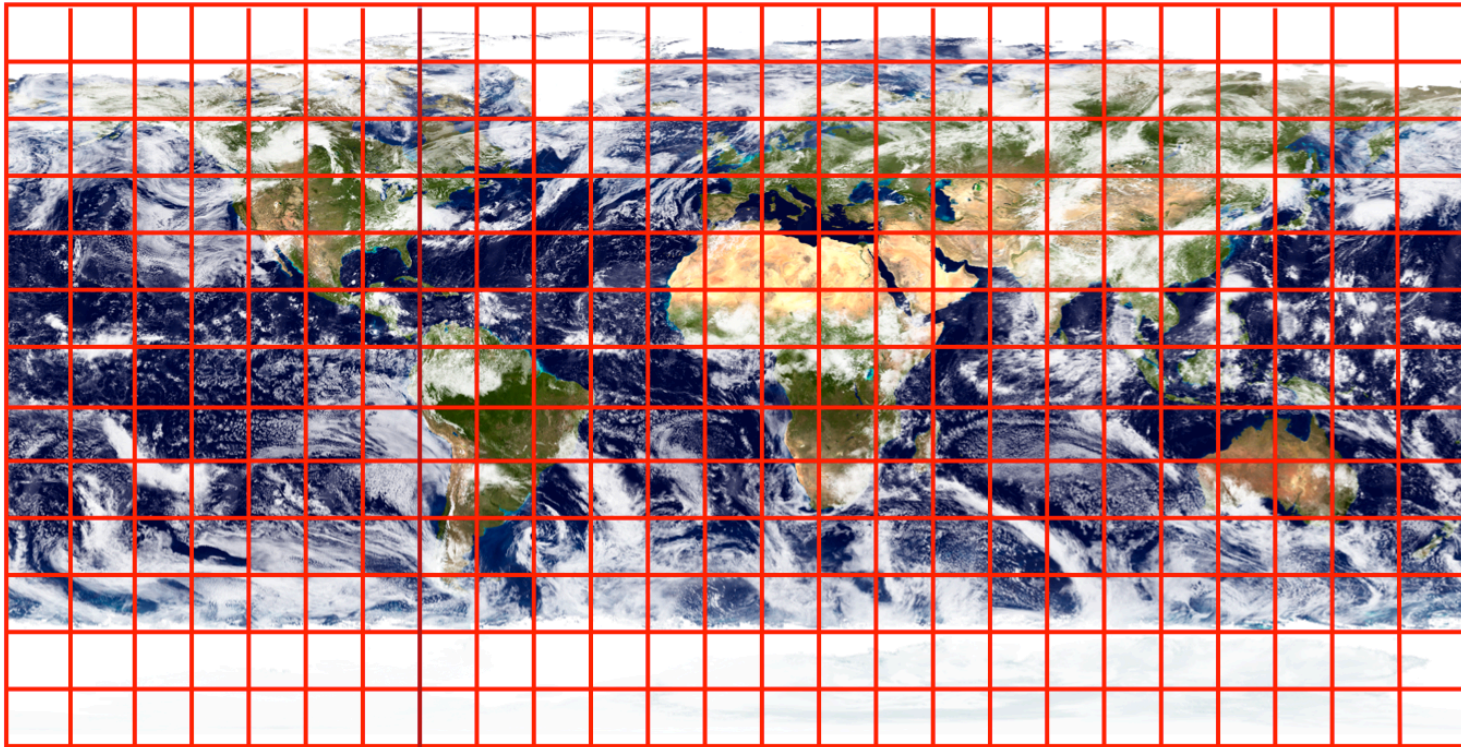


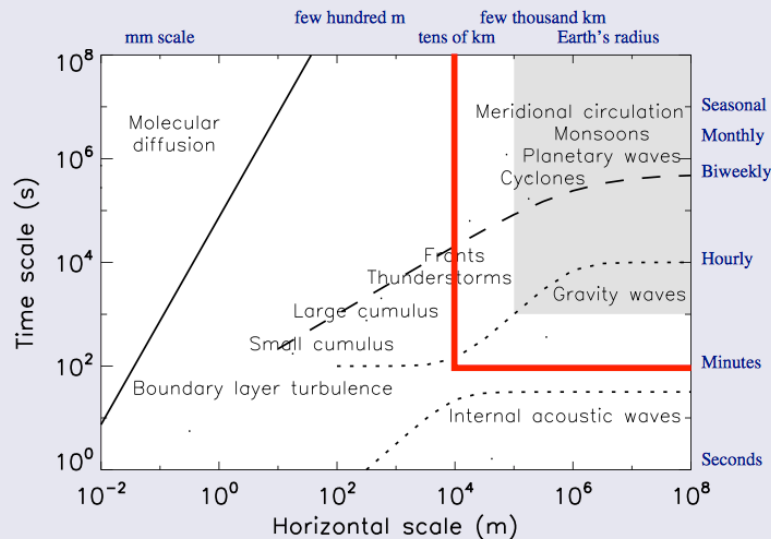
Figure from Skamarock (2011): (left) Schematic depicting the possible behavior of spectral tails derived from model forecasts. (right) TKE (solid lines) as a function of spherical wavenumber for the CCSM finite-volume dynamical core derived from aquaplanet simulations. The total KE is broken into divergent and rotational components (dashed lines) and the solid black lines shows the  $k^{-3}$  slope.

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# Multi-scale nature of atmosphere dynamics (from Thuburn 2011)



## Horizontal resolution:

- the shaded region shows the resolved space/time scales in typical current day climate models (approximately  $1^\circ - 2^\circ$  resolution)
- highest resolution at which CAM is run/developed is on the order of  $10 - 25\text{ km}$
- as the resolution is increased some 'large-scale' parameterizations may no longer be necessary (e.g., large scale convection) and we might need to redesign some parameterizations that were developed for horizontal resolutions of hundreds of km's

- $\mathcal{O}(10^4\text{ km})$ : large scale circulations (Asian summer monsoon).
- $\mathcal{O}(10^4\text{ km})$ : undulations in the jet stream and pressure patterns associated with the largest scale Rossby waves (called *planetary waves*)
- $\mathcal{O}(10^3\text{ km})$ : cyclones and anticyclones
- $\mathcal{O}(10\text{ km})$ : the transition zones between relatively warm and cool air masses can collapse in scale to form fronts with widths of a few tens of km
- $\mathcal{O}(10^3\text{ km} - 100\text{ m})$ : convection can be organized on a huge range of different scales (tropical intraseasonal oscillations; supercell complexes and squall lines; individual small cumulus clouds formed from turbulent boundary layer eddies)
- $\mathcal{O}(10\text{ m} - 1\text{ mm})$ : turbulent eddies in boundary layer (lowest few hundred m's of the atmosphere, where the dynamics is dominated by turbulent transports); range in scale from few hundred m's (the boundary layer depth) down to mm scale at which molecular diffusion becomes significant.



# Model code

## Parameterization suite

- Moist processes: deep convection, shallow convection, large-scale condensation
- Radiation and Clouds: cloud microphysics, precipitation processes, radiation
- Turbulent mixing: planetary boundary layer parameterization, vertical diffusion, gravity wave drag

### 2.5 Equations of motion

The  $\eta^{(d)}$ -coordinate adiabatic and frictionless atmospheric primitive equations assuming floating Lagrangian vertical coordinates [Starr, 1945; Lin, 2004] can be written as

$$\begin{aligned}\frac{\partial \vec{v}}{\partial t} + (\zeta + f) \hat{k} \times \vec{v} + \nabla_{\eta^{(d)}} \left( \frac{1}{2} \vec{v}^2 + \Phi \right) + \frac{1}{\rho} \nabla_{\eta^{(d)}} p &= 0, \\ \frac{\partial T}{\partial t} + \vec{v} \cdot \nabla_{\eta^{(d)}} T - \frac{1}{c_p \rho} \omega &= 0, \\ \frac{\partial}{\partial t} \left( \frac{\partial \mathcal{P}^{(d)}}{\partial \eta^{(d)}} m^{(\ell)} \right) + \nabla_{\eta^{(d)}} \cdot \left( \frac{\partial \mathcal{P}^{(d)}}{\partial \eta^{(d)}} m^{(\ell)} \vec{v} \right) &= 0, \quad \ell = 'd', 'v', 'cl', 'ci',\end{aligned}$$

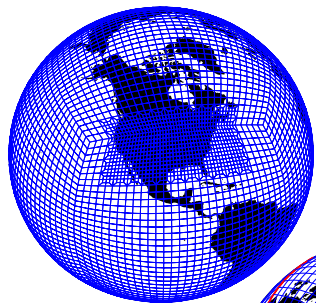
## 'Resolved' dynamics

'Roughly speaking, the **dynamical core** solves the governing fluid and thermodynamic equations on resolved scales, while the parameterizations represent sub-grid-scale processes and other processes not included in the dynamical core such as radiative transfer.' - Thuburn (2008)

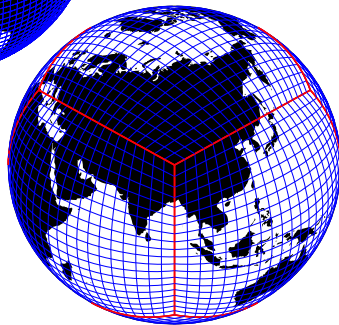
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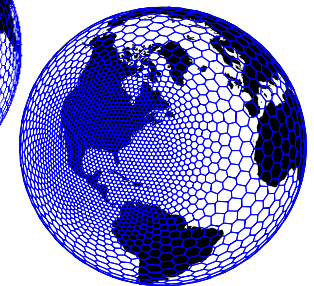
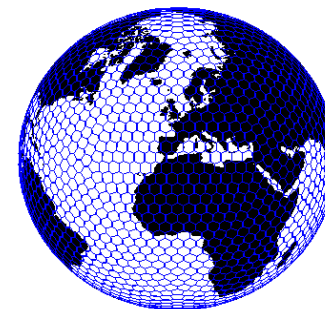
**CAM-SE:  
spectral-element  
dynamical core**



**CAM-FV: finite-  
volume  
dynamical core**



**MPAS: Model  
for Prediction  
Across Scales**



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# Representation of topography in models

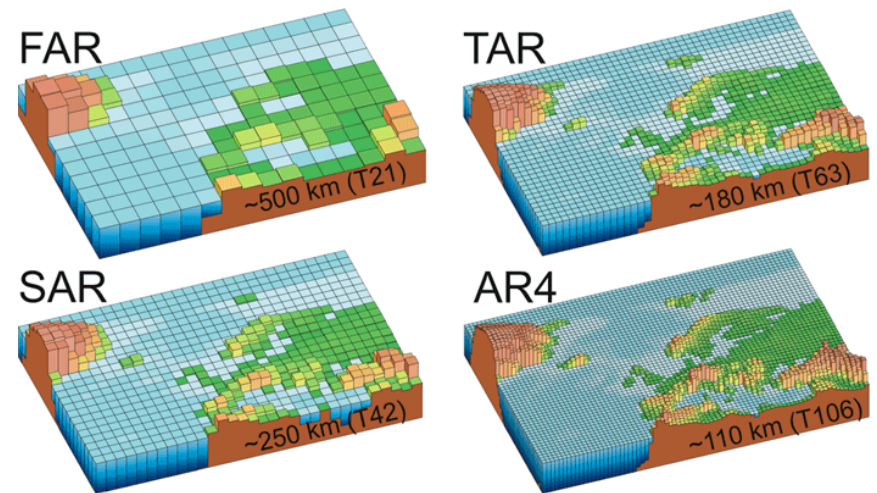
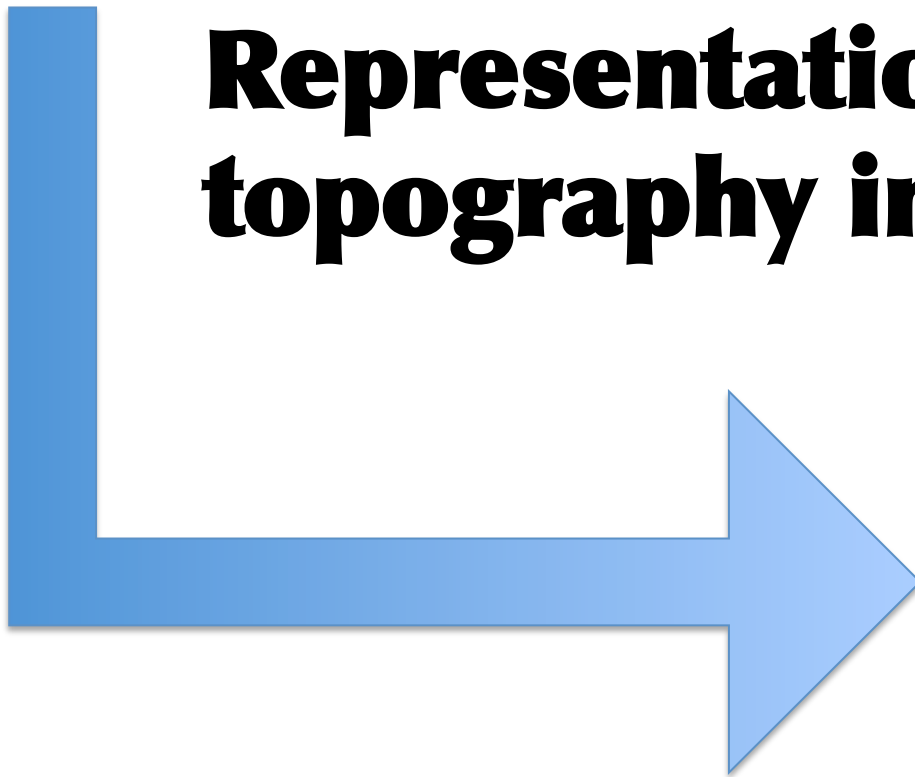
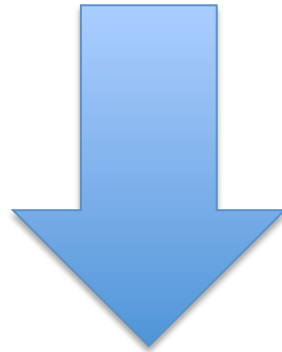
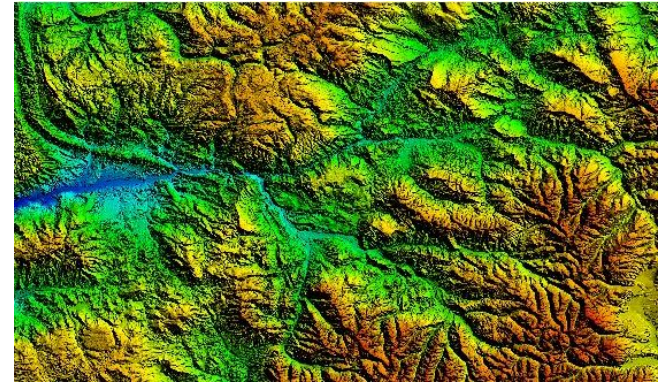


Figure courtesy of IPCC, AR4 WG Chapter 1

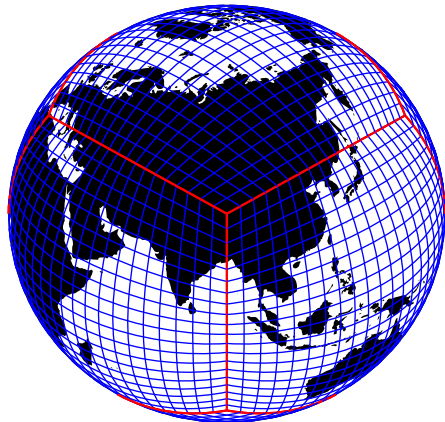
# From DEM to weather/climate model

**A digital elevation model (DEM) is a digital model or 3D representation of a terrain's surface created from terrain elevation data.**

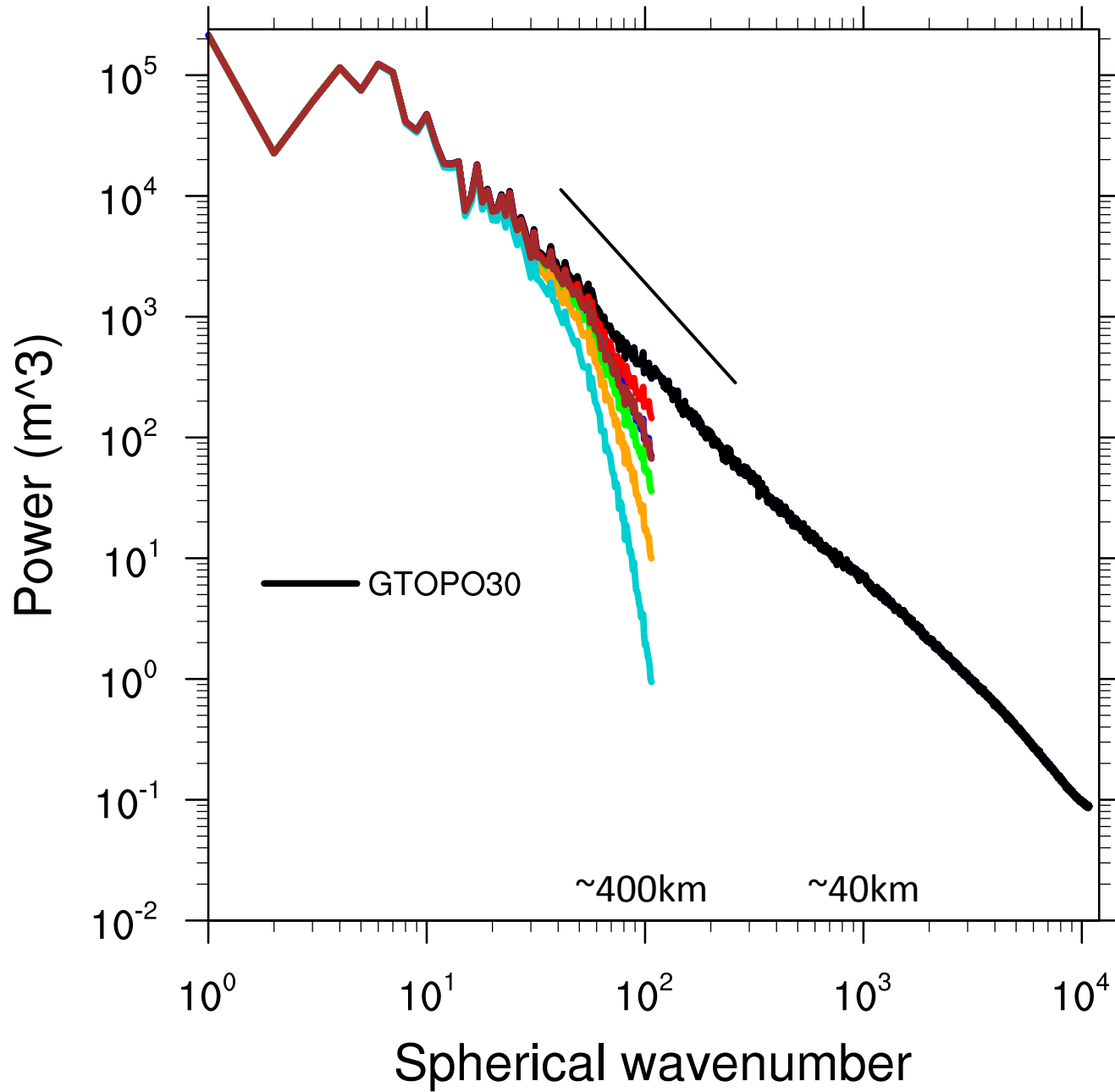
[https://en.wikipedia.org/wiki/Digital\\_elevation\\_model](https://en.wikipedia.org/wiki/Digital_elevation_model)



**Global climate/weather model**



# Power spectra for surface elevation





# From DEM to weather/climate model

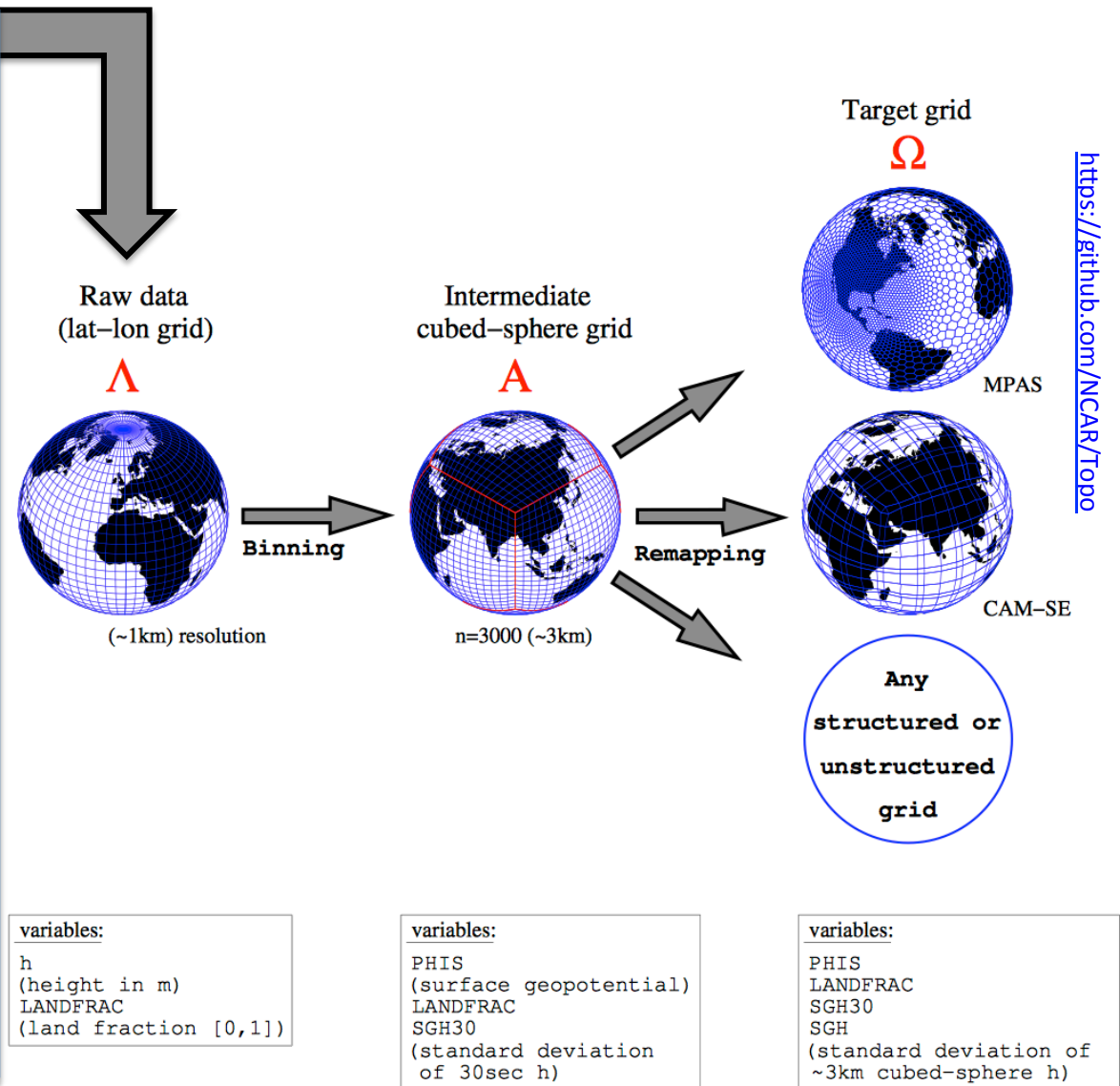
## Global DEM's

- **GTOPO30 (from 1996)**
- **GMTED2010**
- **Several commercially DEMs (especially for regional applications)**

## Input Data Sources

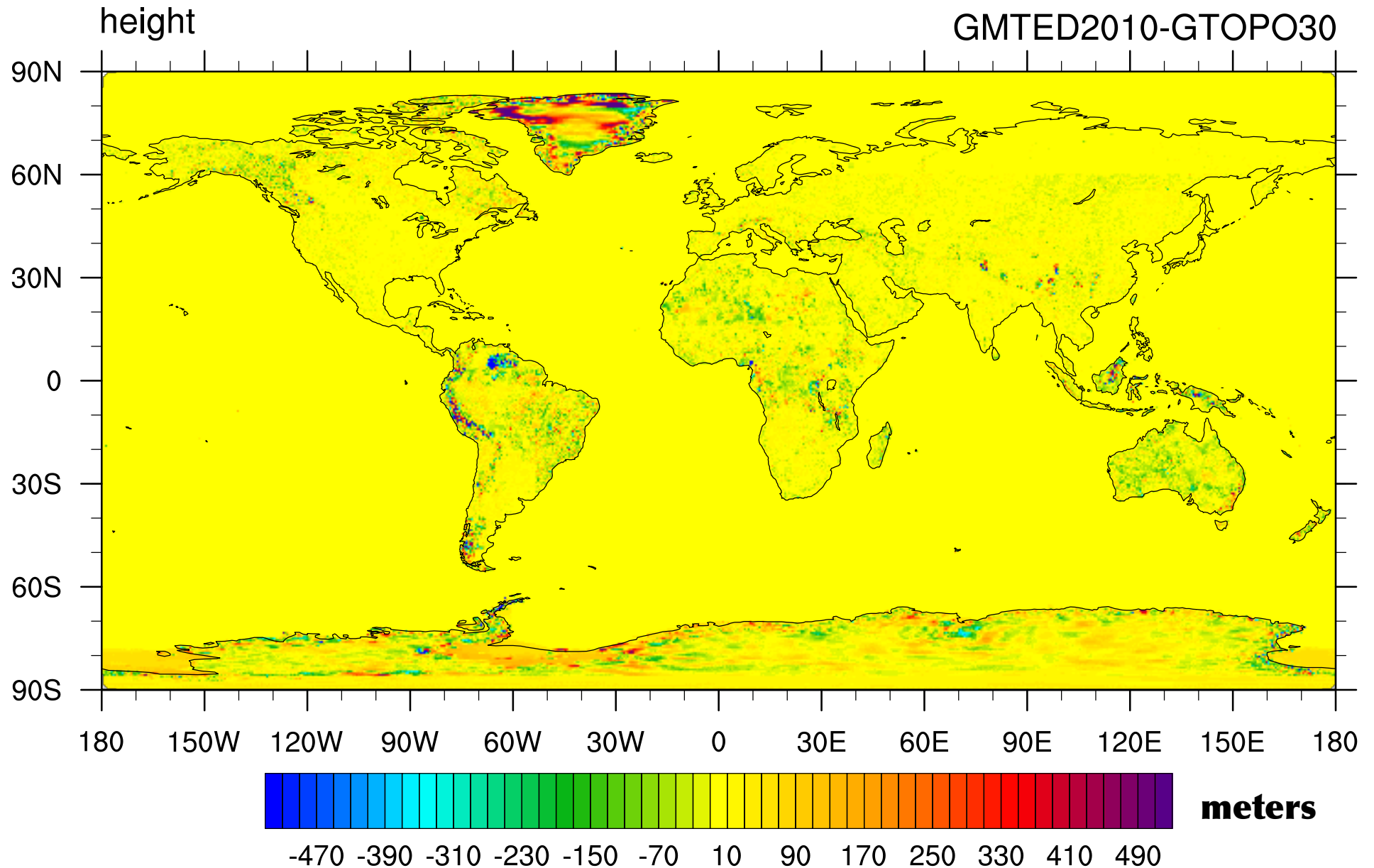
GMTED2010 is based on data derived from 11 raster-based elevation sources. The primary source dataset for GMTED2010 is NGA's SRTM Digital Terrain Elevation Data (DTED<sup>®</sup>2, <http://www2.jpl.nasa.gov/srtm/>) (void-filled) 1-arc-second data. For the geographic areas outside the SRTM coverage area and to fill in remaining holes in the SRTM data, the following sources were used: (1) non-SRTM DTED<sup>®</sup>, (2) Canadian Digital Elevation Data (CDED) at two resolutions, (3) Satellite Pour l'Observation de la Terre (SPOT 5) Reference3D, (4) National Elevation Dataset (NED) for the continental United States and Alaska, (5) GEODATA 9 second digital elevation model (DEM) for Australia, (6) an Antarctica satellite radar and laser altimeter DEM, and (7) a Greenland satellite radar altimeter DEM. Each is described below.

<http://pubs.usgs.gov/of/2011/1073/pdf/of2011-1073.pdf>

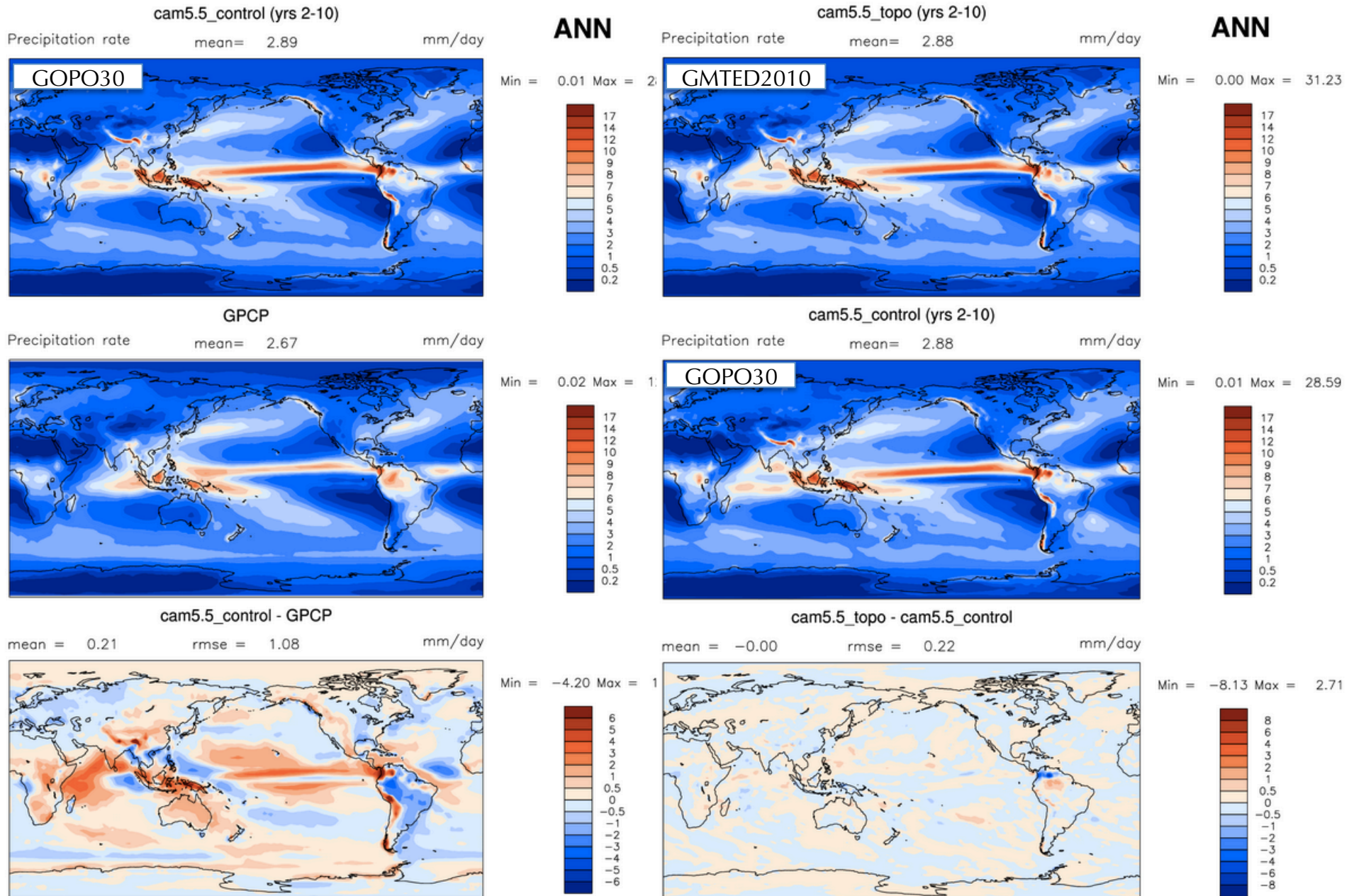


# Raw data differences

(on 3km cubed-sphere grid)



# Raw data differences: effect on precipitation rates





# From DEM to weather/climate model

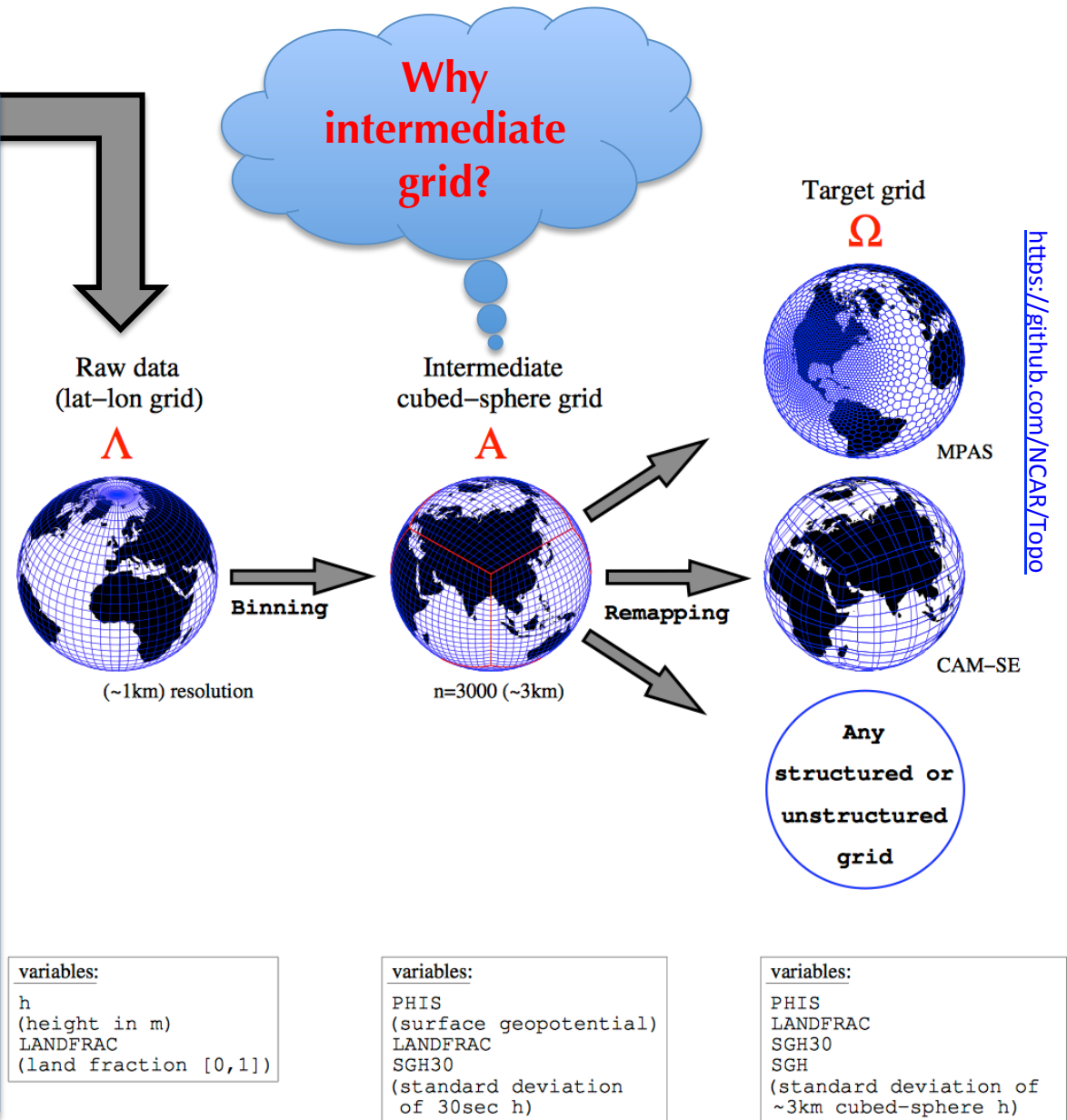
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# Linear theory for topographic waves (Holton, 2004) - infinite periodic mountain range

When do gravity waves propagate in the vertical? (infinite mountain range)

Stability ( $N$ ) constant, topography scale (expressed in terms of wave number:  $k = 2\pi/L$ ), and mean wind speed ( $U$ ) determine if gravity waves propagate in the vertical or not:

$$\frac{Uk}{N} > 1 \Rightarrow \text{no vertical propagation} \quad \frac{Uk}{N} < 1 \Rightarrow \text{vertical propagation} \quad (1)$$

$$N = 0.01\text{s}^{-1}, U = 15\text{ms}^{-1} \Rightarrow \frac{Uk}{N} = 1 \text{ for } L \approx 9 \text{ km}$$

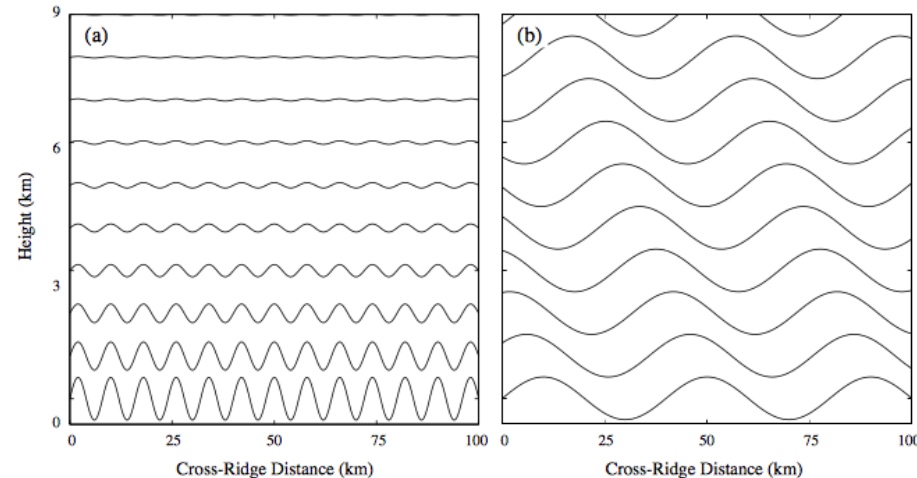


Figure 1: Streamlines in steady airflow over an infinite series of sinusoidal ridges when  $N = .01 \text{ s}^{-1}$ ,  $U = 15 \text{ ms}^{-1}$ , and the wavelength of the topography is (a) 8 km (case  $Uk > N$ ) or (b) 40 km (case  $Uk < N$ ). The flow is from left to right. The lowest streamline coincides with the topography.

Figure courtesy of Dale Durran (University of Washington)



# From DEM to weather/climate model

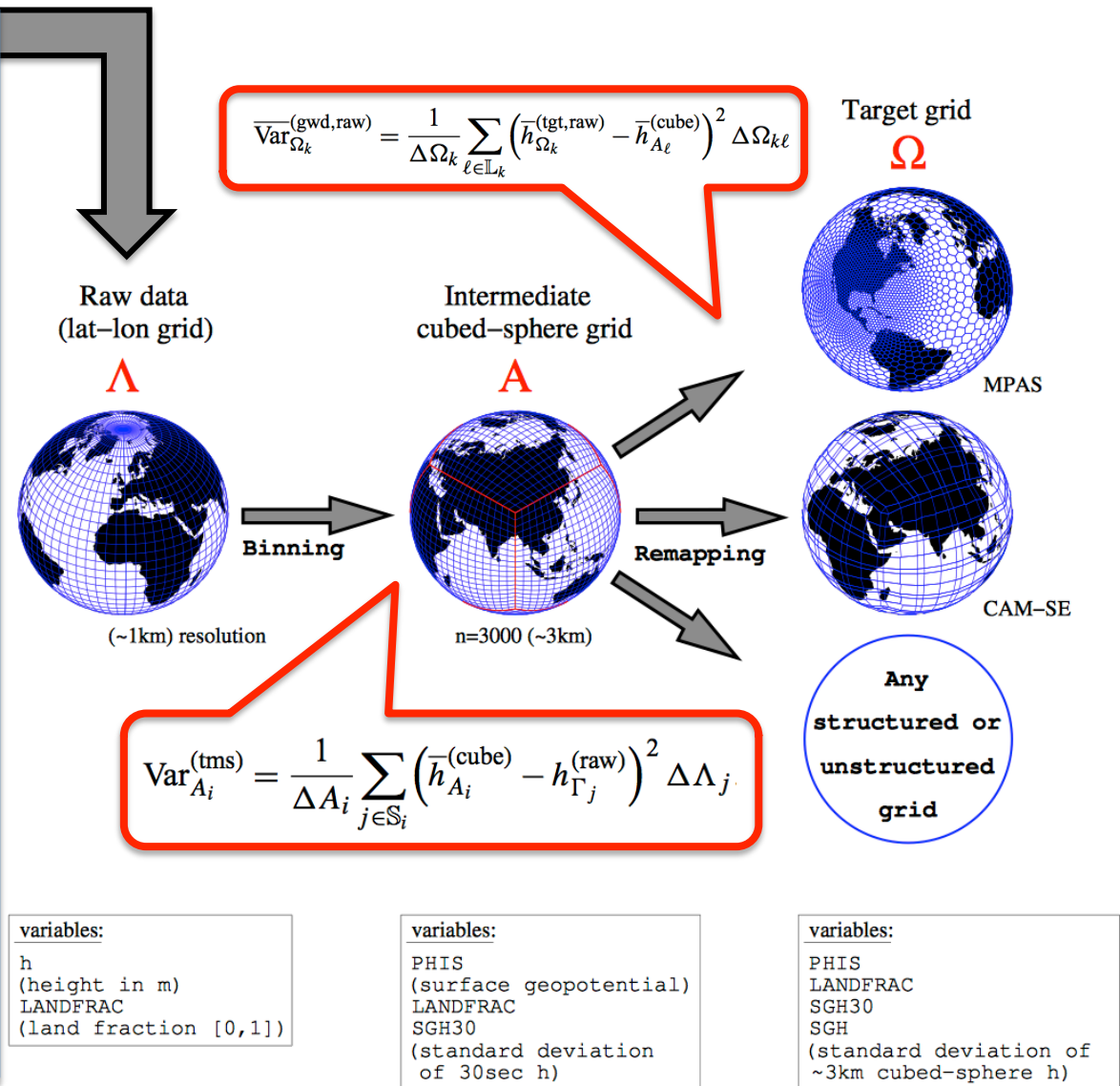
## Global DEM's

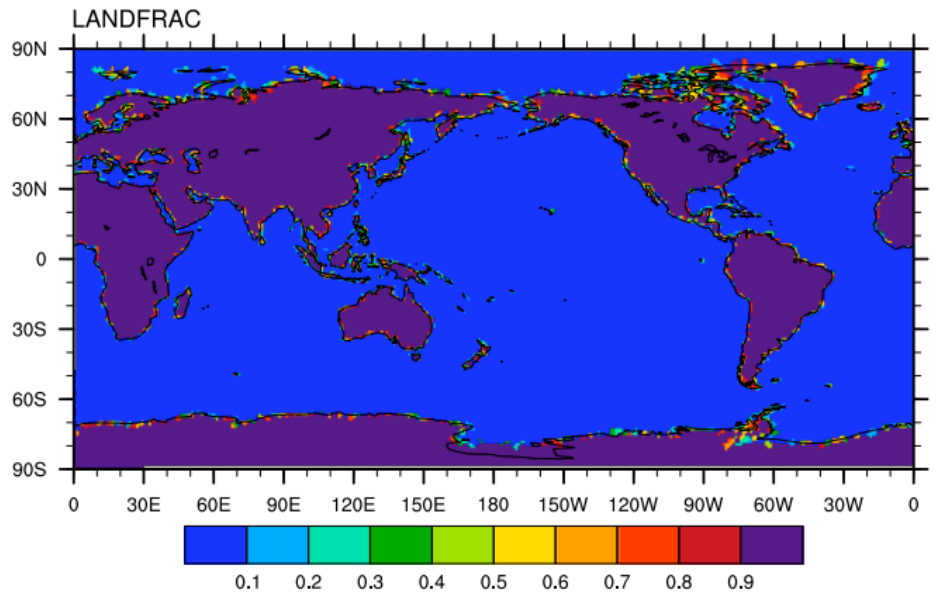
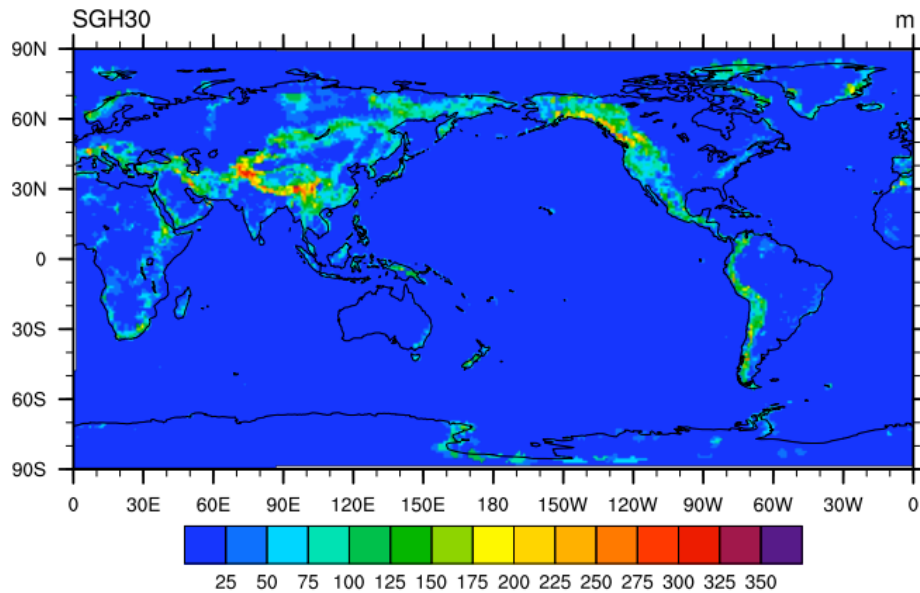
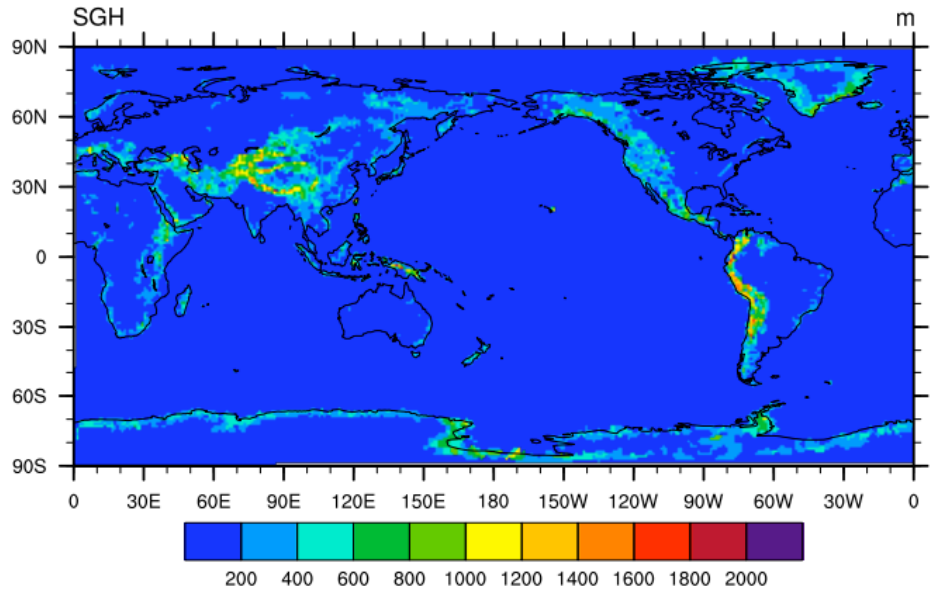
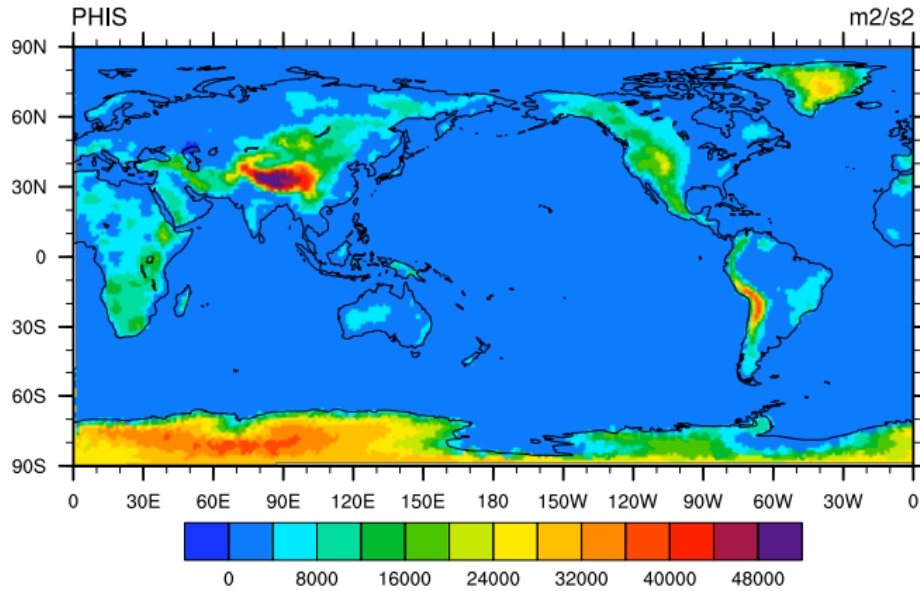
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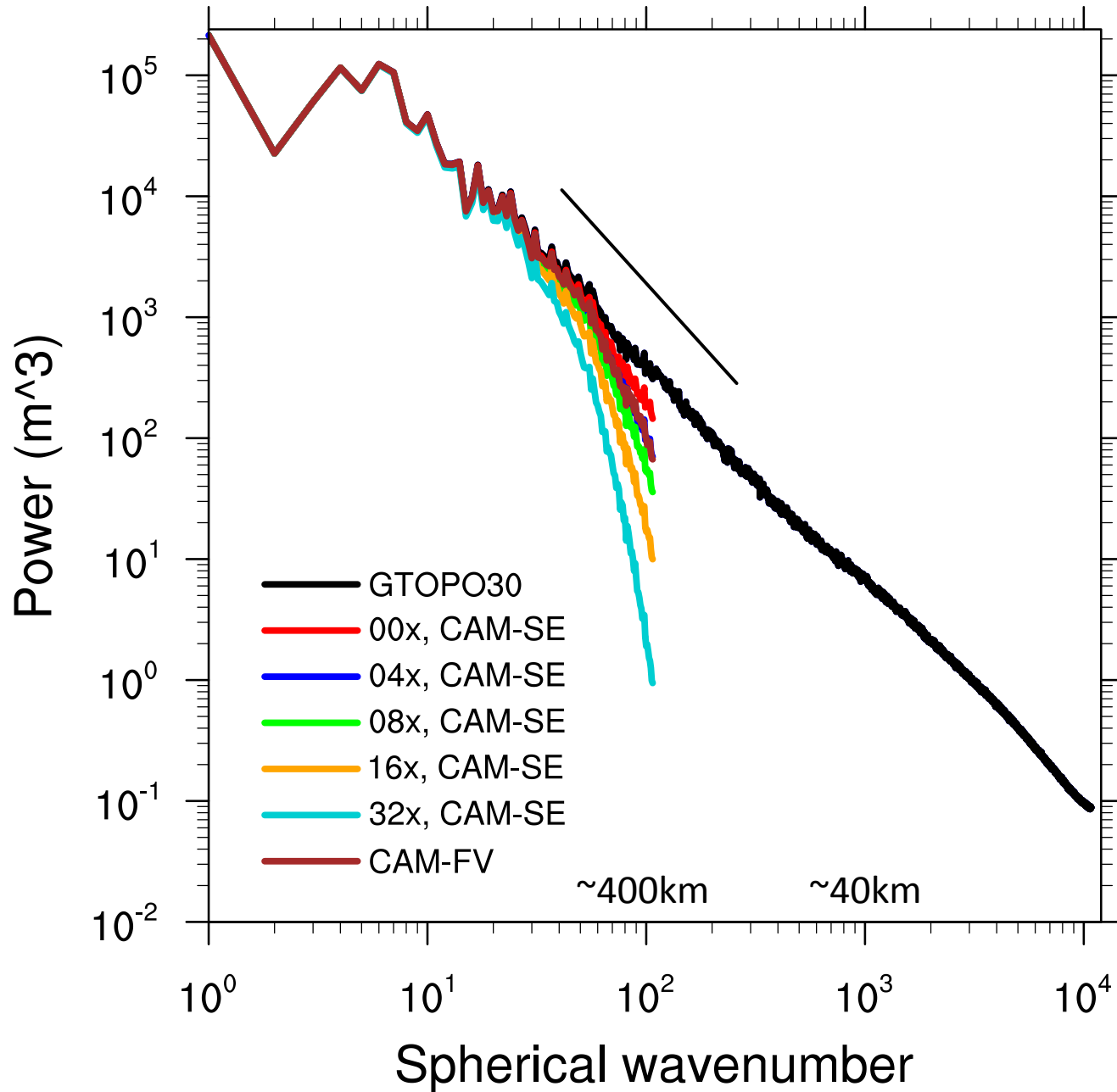
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# Power spectra for surface elevation



## No smoothing of PHIS

Although the binning process smoothes PHIS significantly it is not sufficient

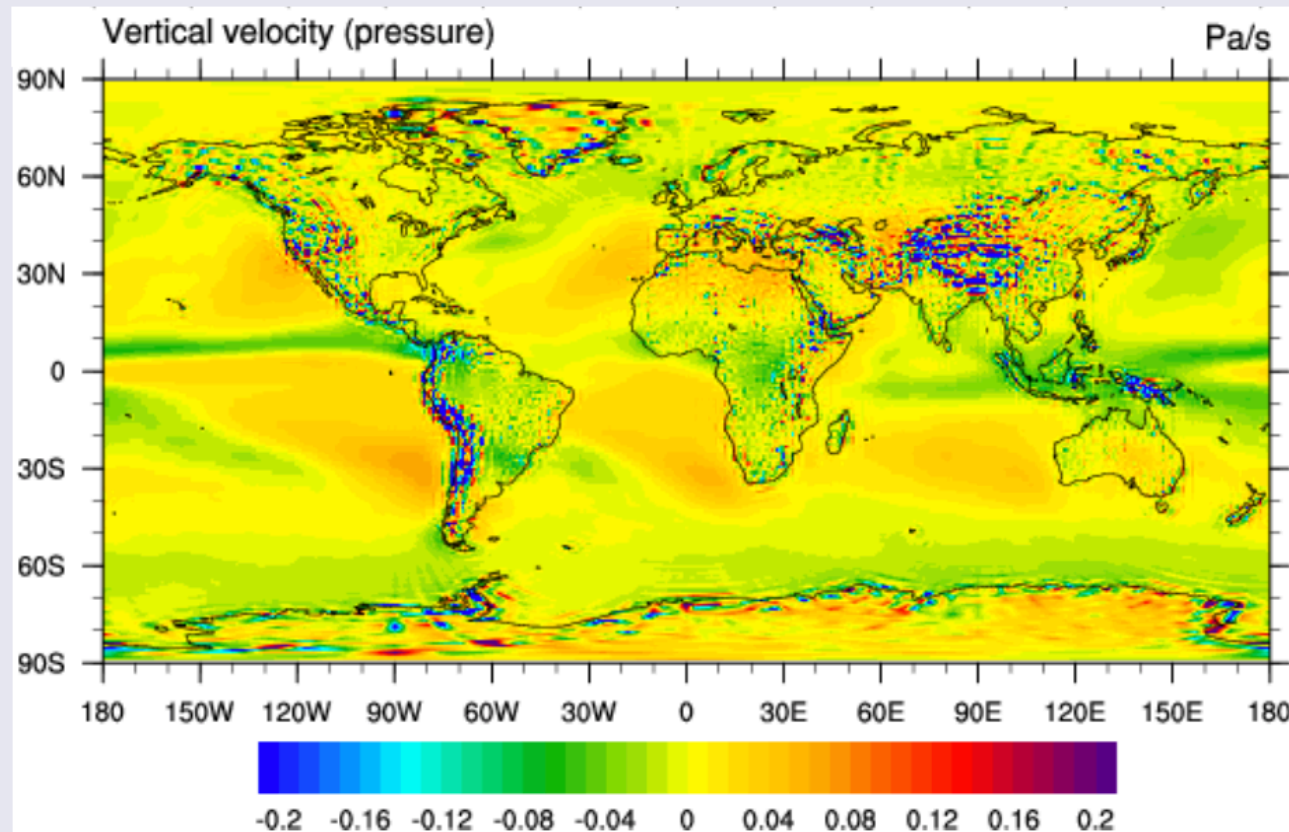


Figure courtesy of Mark Taylor (Sandia)

Figure: 5 year averaged 500hPa vertical pressure velocity  $\omega = \frac{dp}{dt}$  for CAM-SE using (almost) unsmoothed PHIS. Excessive generation of gravity waves contaminates the solution



## Some smoothing of PHIS is necessary

Smoothing of PHIS is necessary for many dynamical cores - how and how much smoothing that is needed is a 'grey' area of climate modeling and it is usually highly dependent on the internal diffusion properties of the numerical method used by the dynamical core as well as the strength of external diffusion operators (e.g., hyperdiffusion operators  $\nabla^4$ ).

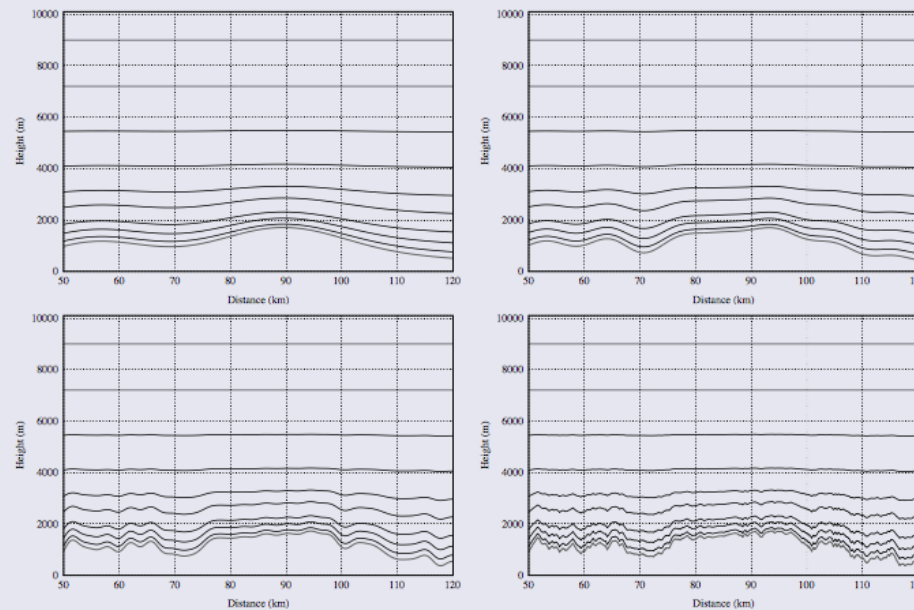
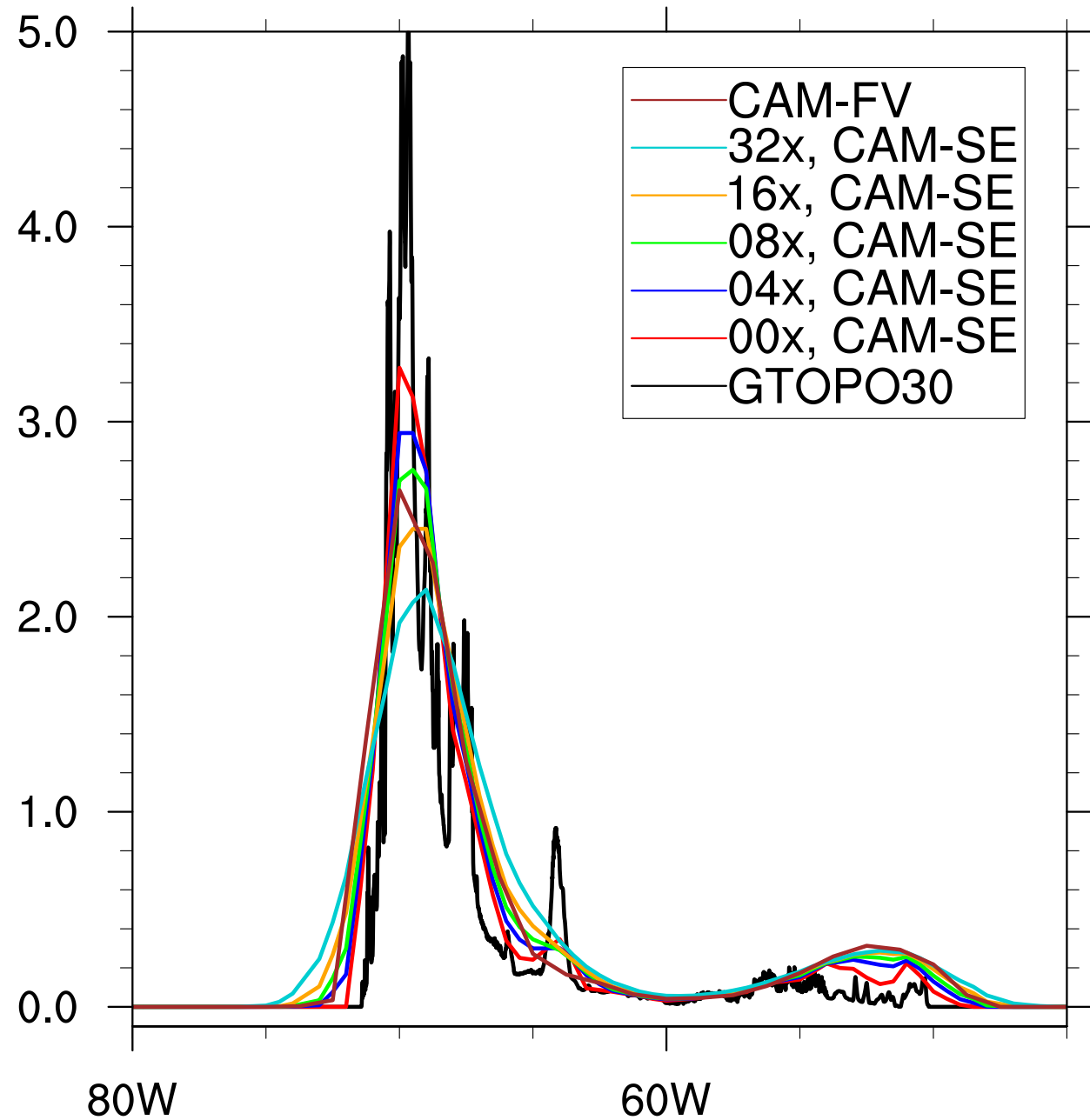


FIGURE 3.3. Hypothetic vertical coordinate based on Carpathian profiles (smoothing to the resolution of 32 km, 10 km, 3.3m and raw data, see Fig. 3.2). A section from 50 km to 120 km of the profile in Fig. 3.1 is shown.

Figure from Rontu (2007)

## Height [km] along latitude $-30^{\circ}$





# Divergence damping

fourth-order divergence damping (with  $q = 2$ ) can be expressed as

$$\frac{\partial u}{\partial t} = \dots - \frac{1}{a \cos \phi} \frac{\partial}{\partial \lambda} (\nu_4 \nabla^2 D) \quad (13.67)$$

$$\frac{\partial v}{\partial t} = \dots - \frac{1}{a} \frac{\partial}{\partial \phi} (\nu_4 \nabla^2 D) \quad (13.68)$$

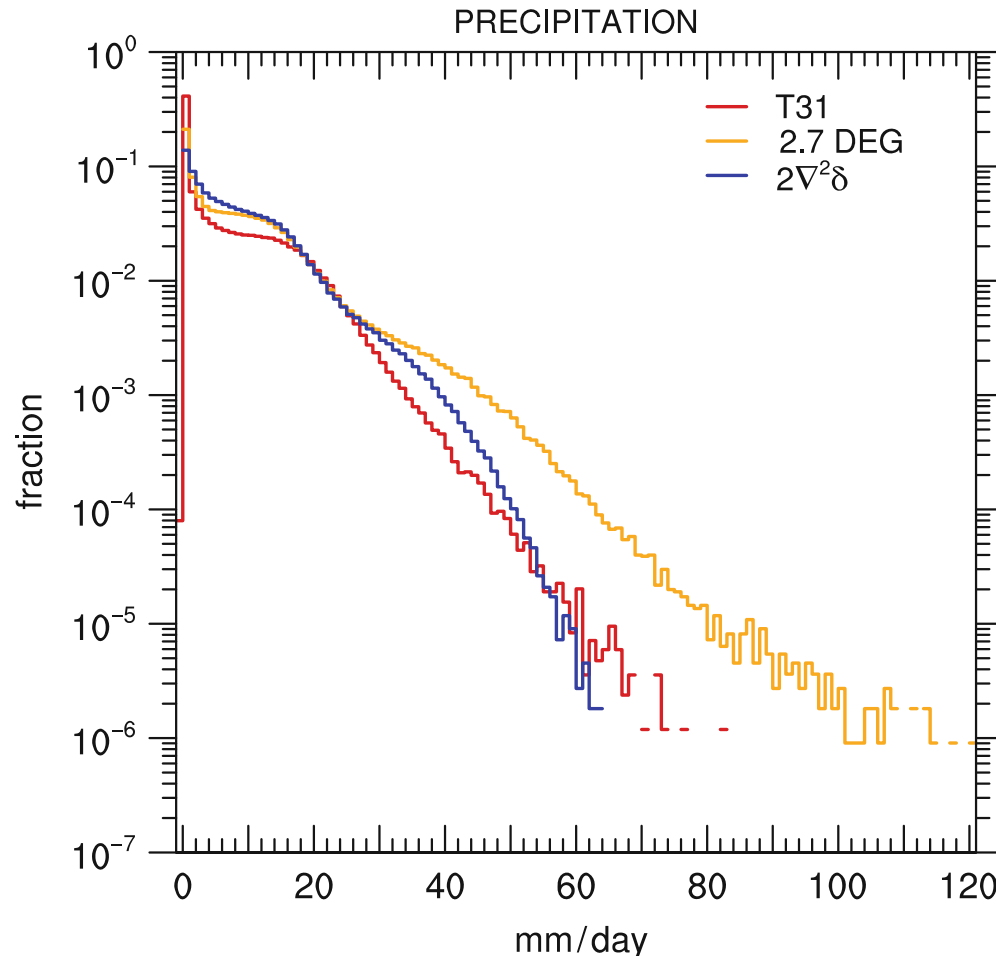
where  $\nu_4$  is the fourth-order damping coefficient. This leads to the evolution equation for the divergence

$$\frac{\partial D}{\partial t} = \dots - \nu_4 \nabla^4 D \quad (13.69)$$

in case of a horizontally constant coefficient. Fourth-order damping is an option in NCAR's model CAM 5 (Neale et al. 2010) which utilizes the FV dynamical core on a latitude–longitude grid.

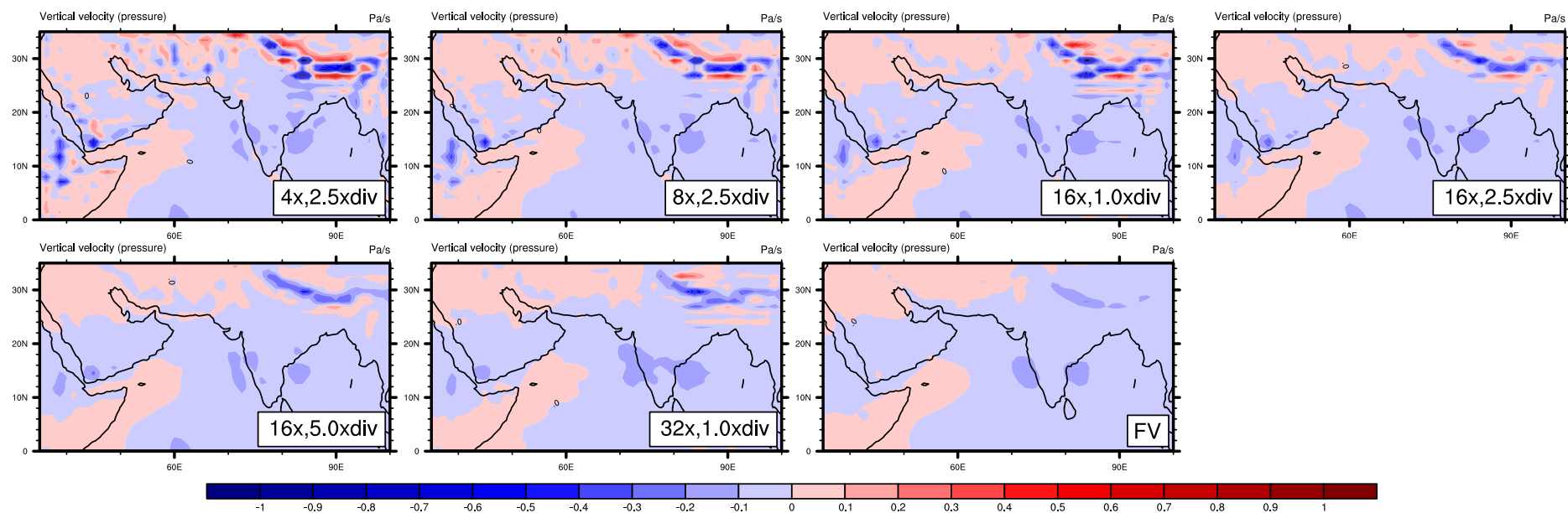


# Divergence damping and extreme precip

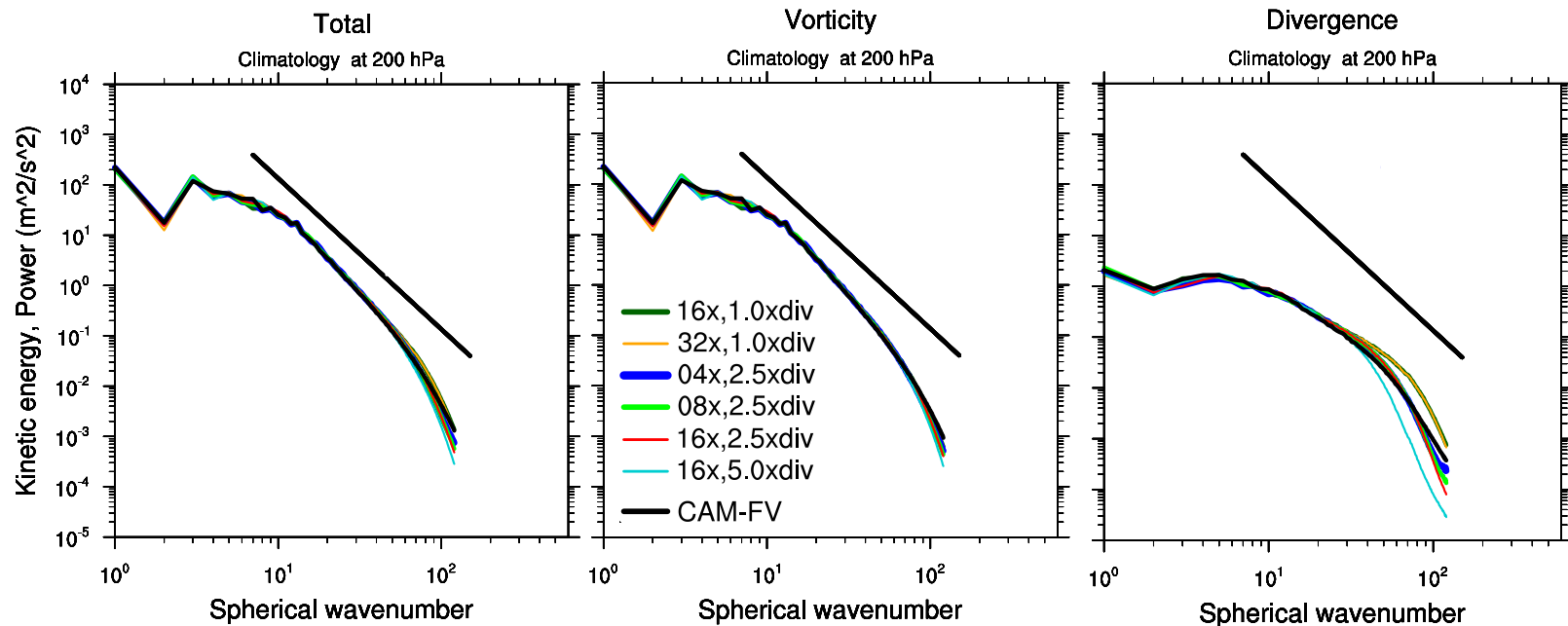


**Fig. 13.8** Fraction of the time the tropical precipitation is in  $1 \text{ mm day}^{-1}$  bins ranging from 0 to  $120 \text{ mm day}^{-1}$ , calculated from 6-h averages for all grid points between  $\pm 10^\circ$ . This frequency distribution is an annual average. The aqua-planet simulations are (*blue, yellow*) CAM FV at the coarse  $lat \times lon$  resolution  $2.7^\circ \times 3.3^\circ$  L26 and (*red*) CAM EUL at the resolution T31L26 (with time step  $\Delta t = 1,800 \text{ s}$ ). *Yellow FV curve*: standard second-order divergence damping (13.70). *Blue curve*: FV simulation with a doubled coefficient. The figure is courtesy of Peter H. Lauritzen,

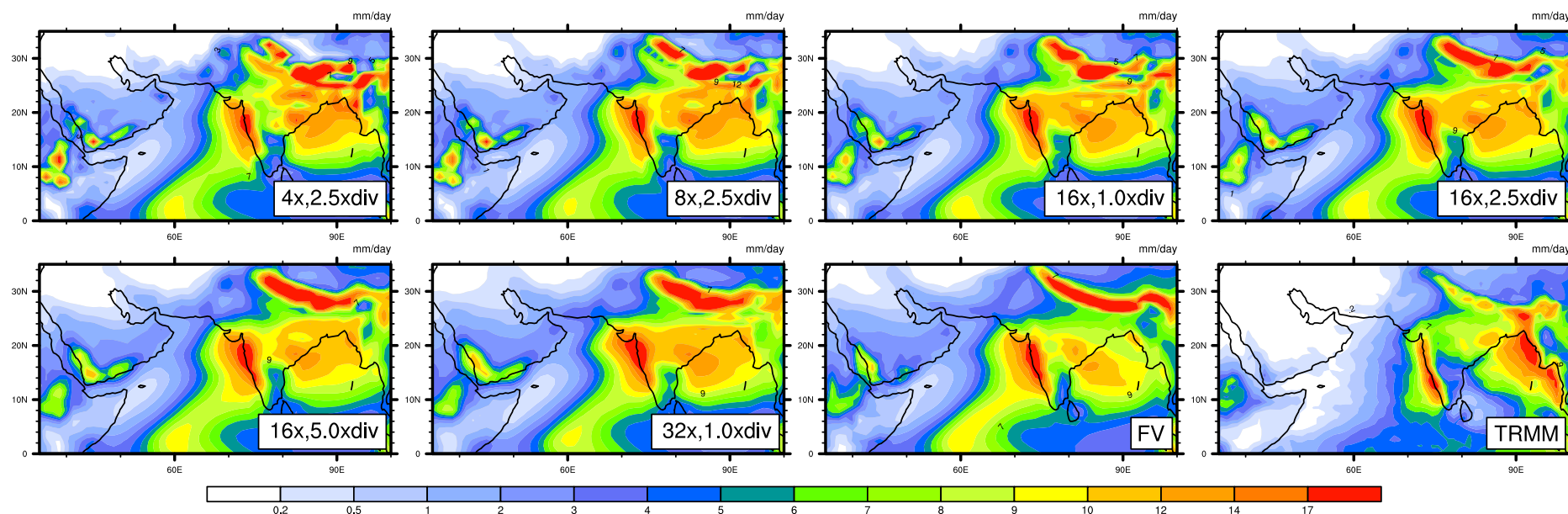
OMEGA, JJA, model level 16 (approximately 323 hPa)



**Figure 7.** Diagnostics for 30 year AMIP simulations with CAM5.2. Upper and lower group of plots are model level 16 vertical velocity and total precipitation rate differences, respectively. Except for the lower right-most plot on the lower group of plots, the diagnostics are for CAM-SE with different amounts of smoothing of  $\Phi_s$  and different levels of divergence damping. The amount of smoothing follows the same notation as Fig. 2 (right) and 1.0 x div, 2.5 x div, 5.0 x div refers to increasing divergence damping by a factor 1.0,  $2.5^2$ , and  $5.0^2$ , respectively. The second right-most plot on each group of plots (labeled FV) show results for CAM-FV. Lower right plot in the second group of plots show TRMM observations, respectively.



**Figure 8.** (left) Total kinetic energy spectrum for the velocity field at 200 hPa as a function of spherical wave number  $K$  for CAM-FV and different configurations of CAM-SE. The labeling for the CAM-SE configurations is the same as in Fig. 7. The solid-straight black line indicates the  $K^{-3}$  reference slope. The middle and right plots show the kinetic energy partitioned into divergent and rotational modes, respectively. The spectra have been computed using daily instantaneous wind and surface pressure data for a 2 month period.





# **Part 3: Vertical coordinates and topography**

# Vertical coordinate: hybrid sigma ( $\sigma = p/p_s$ )-pressure ( $p$ ) coordinate

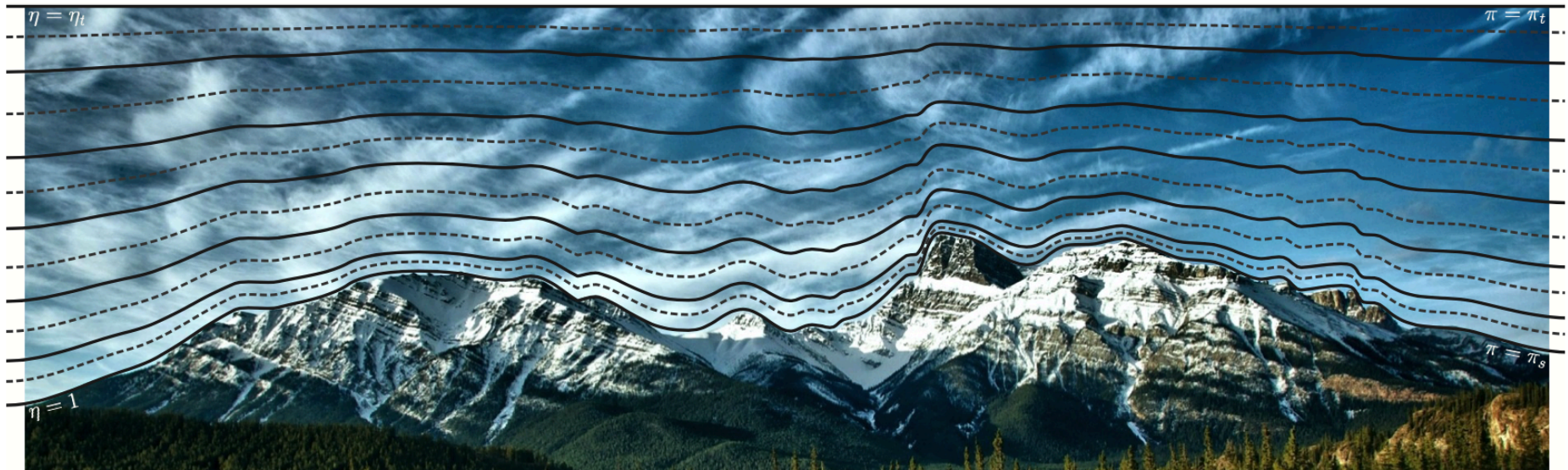


Figure courtesy of David Hall (CU Boulder).

*Sigma layers at the bottom (following terrain) with isobaric (pressure) layers aloft.*

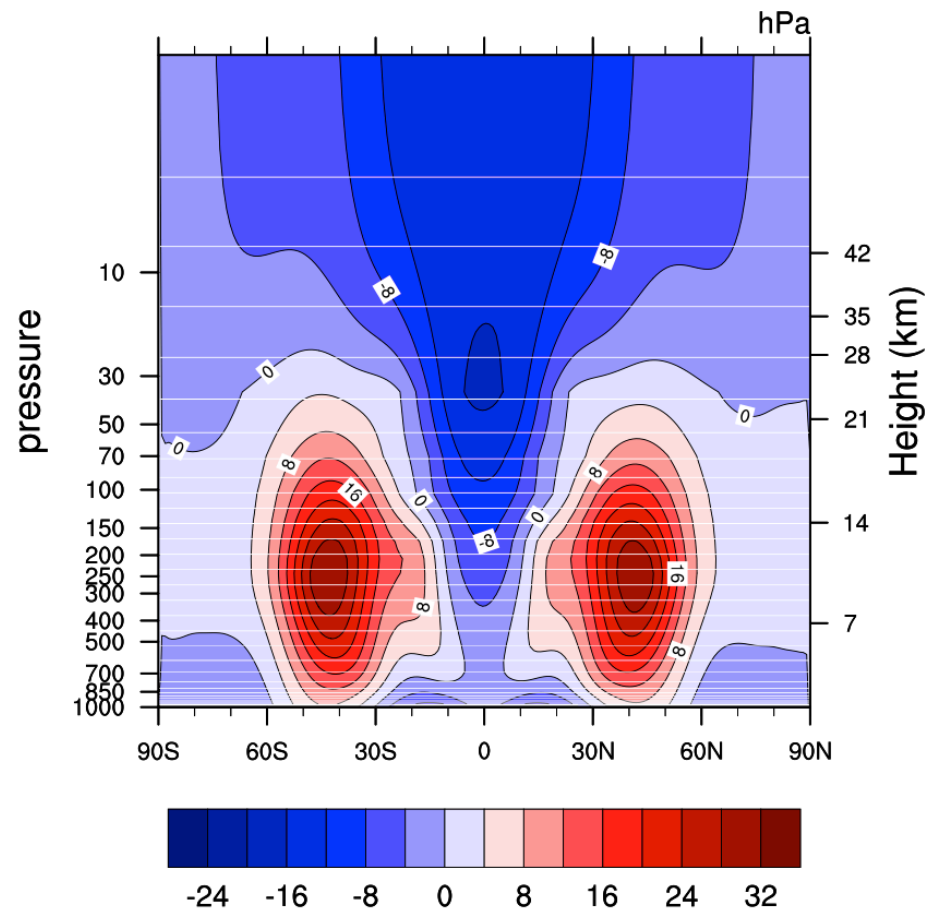
Pressure at model level interfaces

$$p_{k+1/2} = A_{k+1/2} p_0 + B_{k+1/2} p_s,$$

where  $p_s$  is surface pressure,  $p_0$  is the model top pressure, and  $A_{k+1/2} (\in [0 : 1])$  and  $B_{k+1/2} (\in [1 : 0])$  hybrid coefficients (in model code: *hyai* and *hybi*). Similarly for model level mid-points.

Note: vertical index is 1 at model top and *klev* at surface.

Vertical coordinate: hybrid sigma ( $\sigma = p/p_s$ )-pressure ( $p$ ) coordinate



Time & zonally averaged zonal wind (Held-Suarez forcing); overlaid CAM5 levels ( $klev = 30$ ).

## Aside: hybrid sigma ( $\sigma = p/p_s$ )-pressure ( $p$ ) coordinate

While terrain-following coordinates simplify the bottom boundary condition, they may introduce errors:

- Pressure gradient force (PDF) errors:  $\frac{1}{\rho} \nabla_z p = \frac{1}{\rho} \nabla_\eta p + \frac{1}{\rho} \frac{dp}{dz} \nabla_\eta z$ , (Kasahara, 1974) where  $\rho$  is density,  $p$  pressure and  $z$  height.
- Errors in modeling flow along constant  $z$ -surfaces near the surface

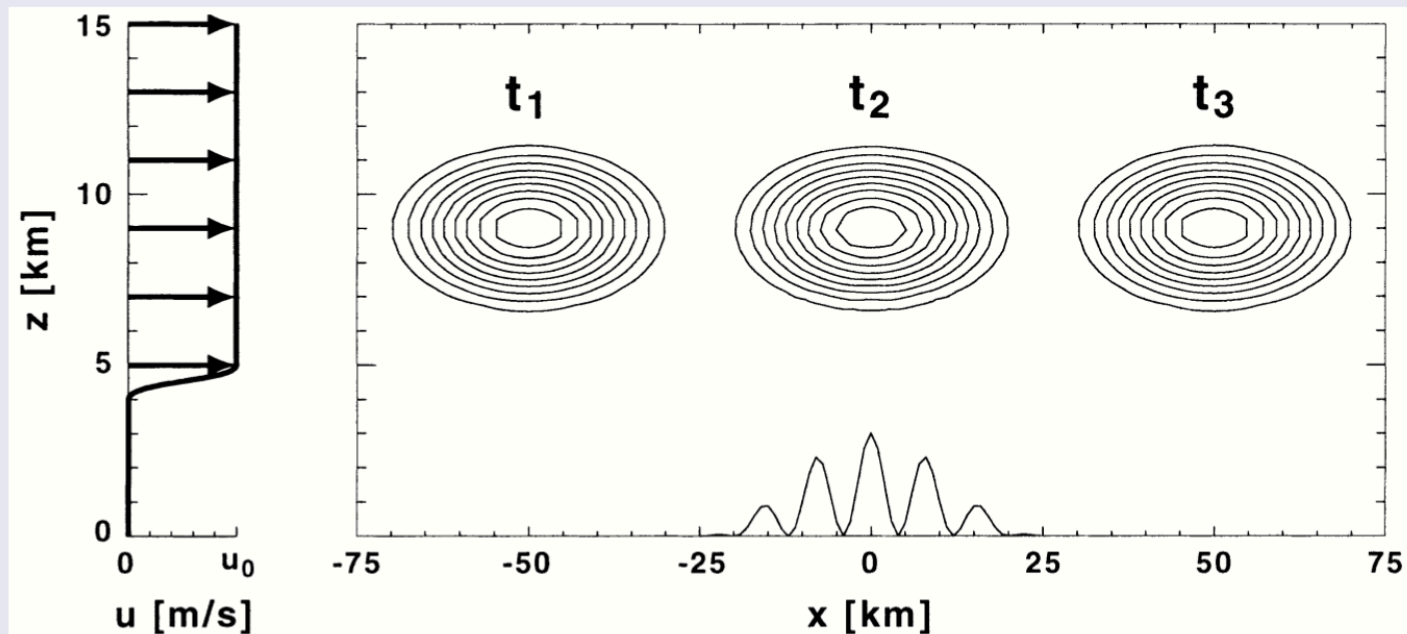
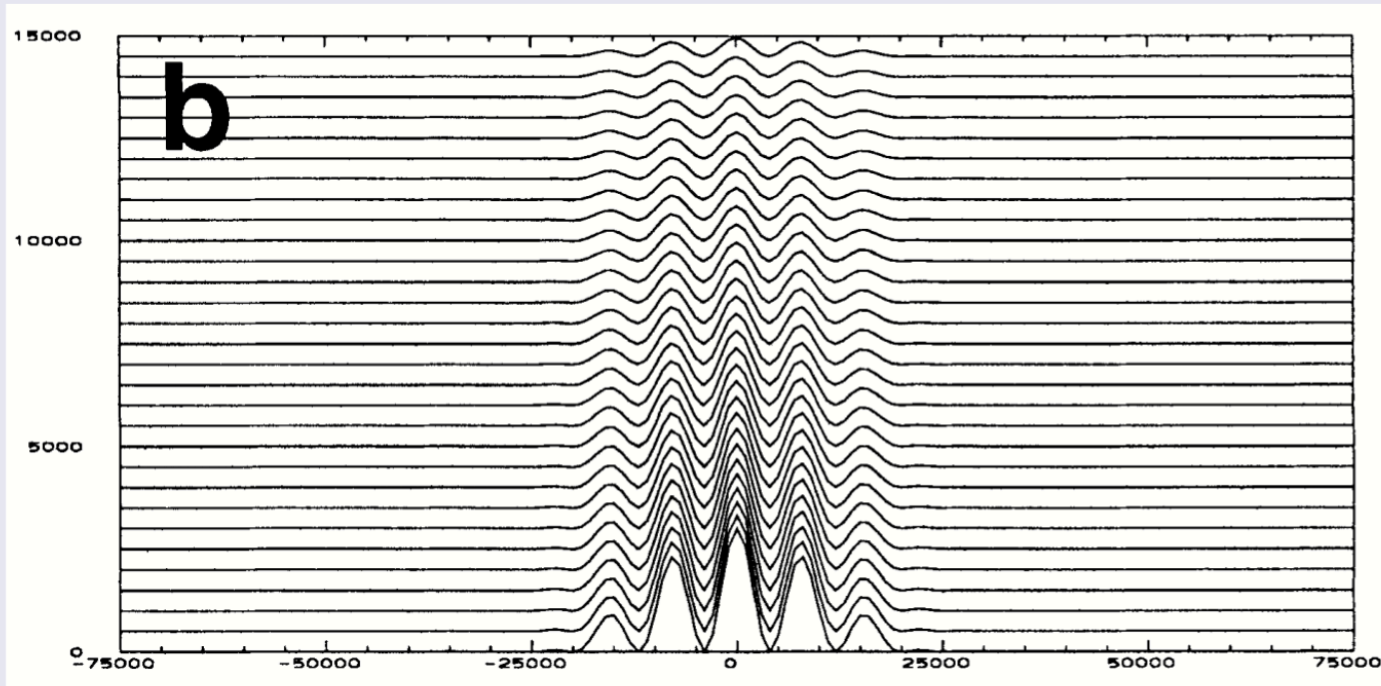


FIG. 4. Vertical cross section of the idealized two-dimensional advection test. The topography is located entirely within a stagnant pool of air, while there is a uniform horizontal velocity aloft. The analytical solution of the advected anomaly is shown at three instances.

## Aside: hybrid sigma ( $\sigma = p/p_s$ )-pressure ( $p$ ) coordinate

While terrain-following coordinates simplify the bottom boundary condition, they may introduce errors:

- Errors in modeling flow along constant  $z$ -surfaces near the surface



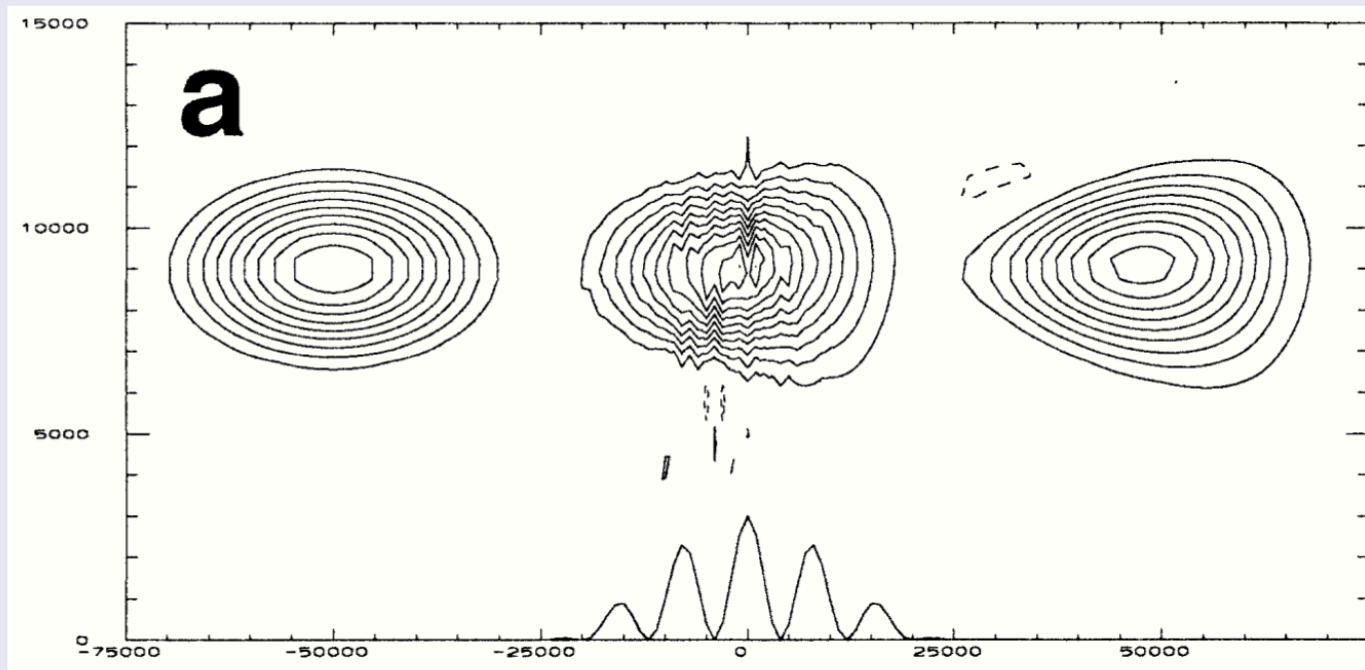
Schär et al. (2002)



## Aside: hybrid sigma ( $\sigma = p/p_s$ )-pressure ( $p$ ) coordinate

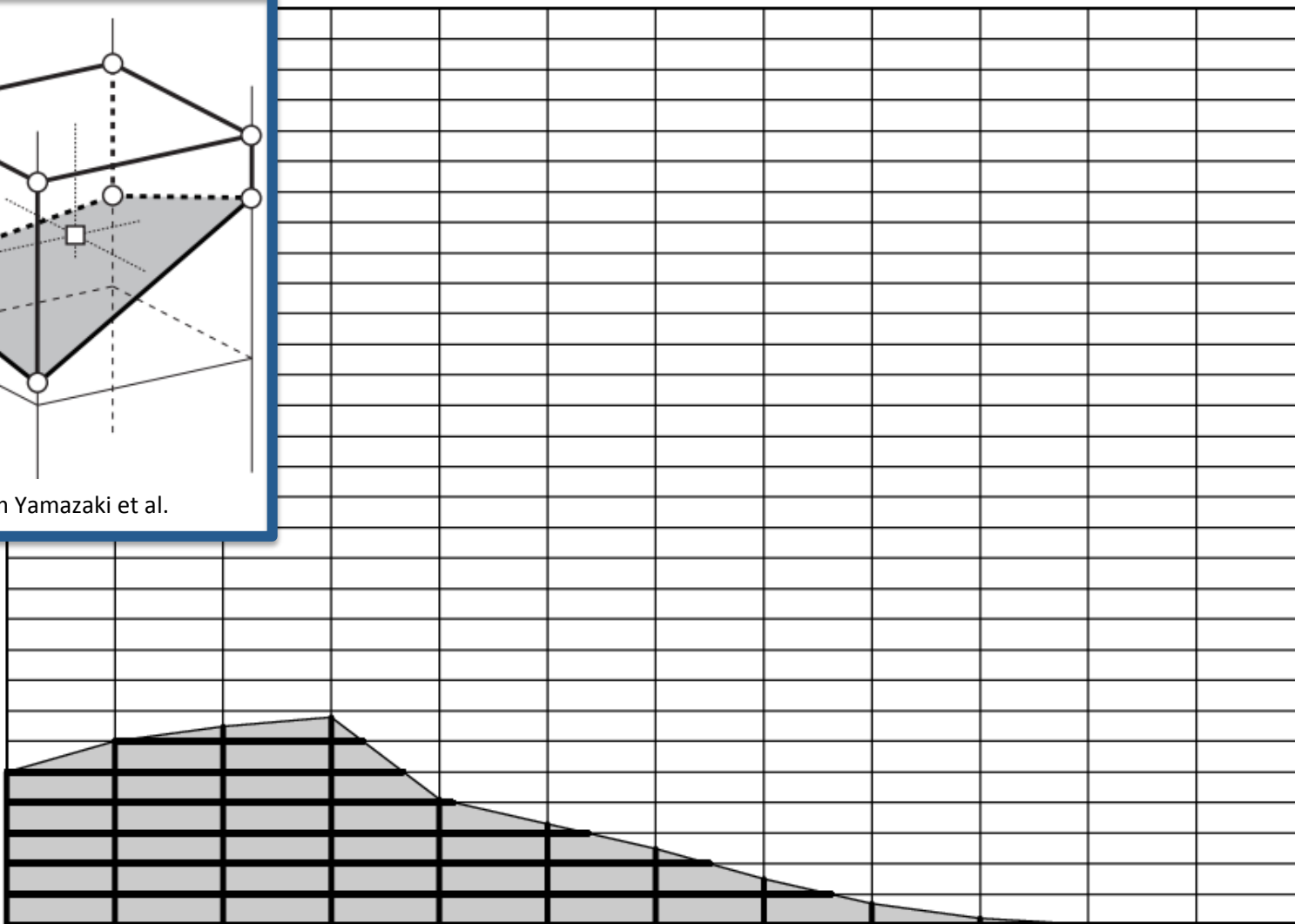
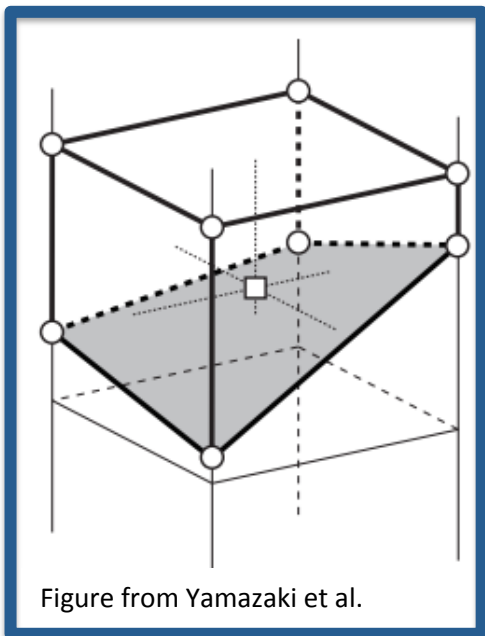
While terrain-following coordinates simplify the bottom boundary condition, they may introduce errors:

- Errors in modeling flow along constant  $z$ -surfaces near the surface



Schär et al. (2002)

# Cut cell method



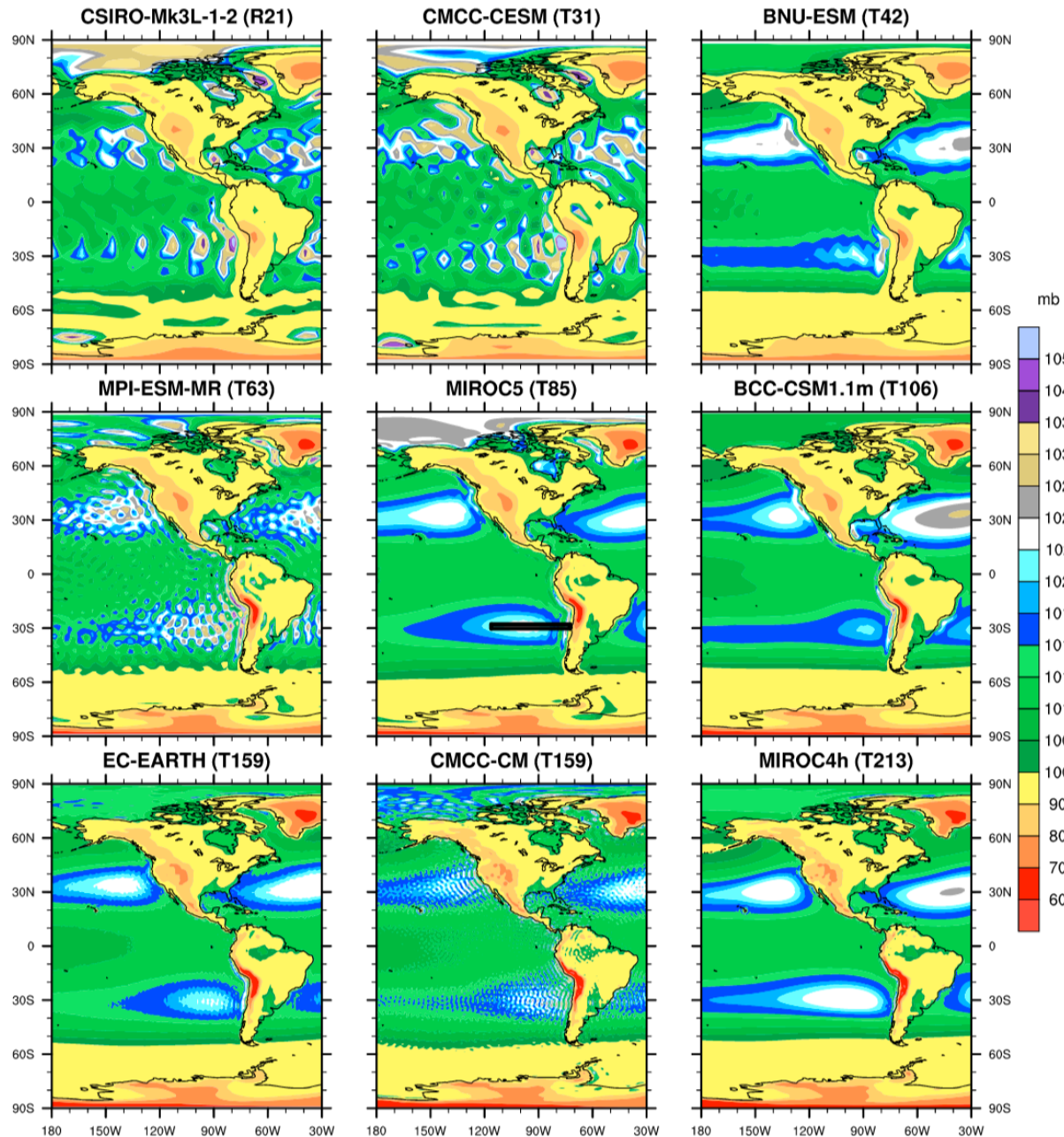
From R. Walko

# Final remarks

**Representation of topography in models remains a challenge!**

- **Resolved and un-resolved topography**
- **Numerical accuracy of fluid-flow solver near topography**

# Quantitative characterization of spurious numerical oscillations in 48 CMIP5 models



**Figure 1.** Surface pressure (mb) for nine spectral models (spectral resolution increasing from top left to bottom right) shows the large range in wavelength and amplitude of the spurious numerical oscillations (aka Gibbs oscillations in spectral models). The middle plot displays the location of the transect (horizontal black line) used to quantify numerical oscillations in subsequent figures.

Lauritzen · Jablonowski  
Taylor · Nair (Eds.)



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LECTURE NOTES IN COMPUTATIONAL  
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80

This book surveys recent developments in numerical techniques for global atmospheric models. It is based upon a collection of lectures prepared by leading experts in the field. The chapters reveal the multitude of steps that determine the global atmospheric model design. They encompass the choice of the equation set, computational grids on the sphere, horizontal and vertical discretizations, time integration methods, filtering and diffusion mechanisms, conservation properties, tracer transport, and considerations for designing models for massively parallel computers. A reader interested in applied numerical methods but also the many facets of atmospheric modeling should find this book of particular relevance.



Numerical Techniques for Global Atmospheric Models

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