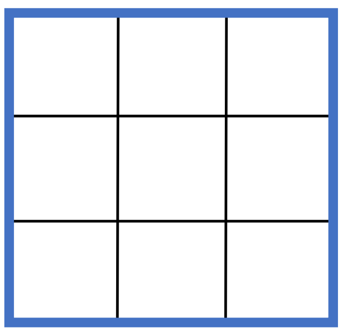
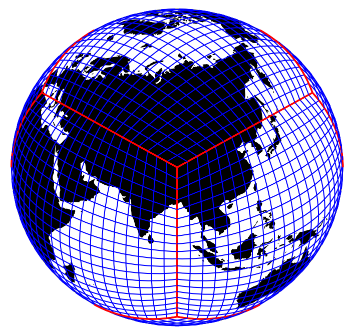
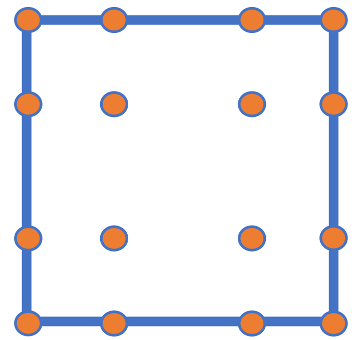
Climate Modeling with the Spectral-Element Method

P.H. Lauritzen[1], A.R. Herrington[2], M.A. Taylor[3], K.A.Reed[2]

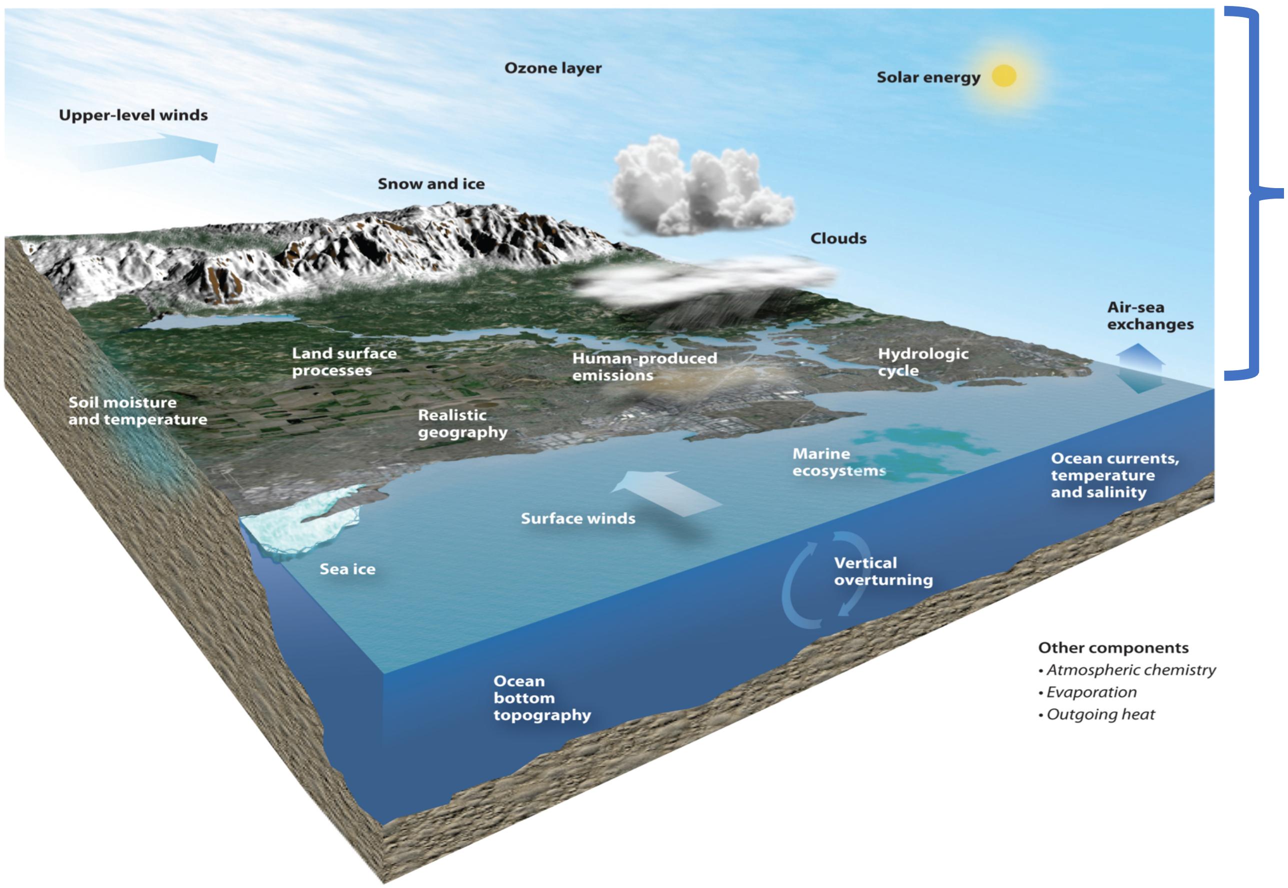
[1]National Center for Atmospheric Research, Boulder, Colorado
[2]Stony Brook University, Stony Brook, New York [3]Sandia National Laboratories, Albuquerque, New Mexico

Minisymposium on *Numerical Methods for Weather, Oceans and Climate*

2018 SIAM Conference on Mathematics of Planet Earth

September 13-14, 2018, Philadelphia

Setting the stage: NCAR's CESM (Community Earth System Model)



Community Atmosphere Model (CAM)

Climate model setup: dynamics, physics, physics-dynamics coupling

NCAR
NATIONAL CENTER FOR ATMOSPHERIC RESEARCH

Dynamical core module

$$\frac{\partial \vec{u}}{\partial t} + (\zeta + f)\,\hat{k}\times\vec{u} + \nabla\left(\frac{1}{2}\vec{u}^2 + \Phi\right) + \frac{1}{\rho}\nabla p = \nu\nabla^4\vec{u},$$

$$\frac{\partial T}{\partial t} + \vec{u}\cdot\nabla T - \frac{1}{c_p \rho}\omega = \nu\nabla^4 T, ,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial p_d}{\partial \eta}\right) + \nabla\cdot\left(\frac{\partial p_d}{\partial \eta}\vec{u}\right) = \nu\nabla^4\left(\frac{\partial p_d}{\partial \eta}\right),$$

$$\frac{\partial}{\partial t}\left(\frac{\partial p_d}{\partial \eta}m_i\right) + \nabla\cdot\left(\frac{\partial p_d}{\partial \eta}m_i\vec{u}\right) = \nu\nabla^4\left(m_i\right), \quad i = v, cl, ci, \ldots$$

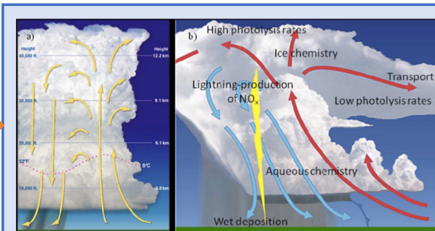
Approximates the solution to the adiabatic equations of motion (“resolved” scales):

- Momentum (u,v)
- Thermodynamic equation (T)
- Continuity equation for air (p)
- Continuity equation for
  - forms of water (water vapor, cloud liquid, cloud ice, rain, …)
  - quantities needed to represent aerosols
  - chemical species

Physics-dynamics coupling layer

Climate/weather models usually use low-order coupling (Euler forward time-stepping)

Physics (parameterization) module



Roughly speaking, processes that can not be resolved on model grid (hence physics is also referred to as sub-grid-scale processes):

Radiation
Boundary layer turbulence
Sub-grid-scale orographic drag
Shallow and deep convection
Microphysics
Aerosol processes
Vertical mixing
…

Climate model setup: dynamics, physics, physics-dynamics coupling

NCAR
NATIONAL CENTER FOR ATMOSPHERIC RESEARCH

Dynamical core module

$$\frac{\partial \vec{u}}{\partial t} + (\zeta + f)\,\hat{k}\times\vec{u} + \nabla\left(\frac{1}{2}\vec{u}^2 + \Phi\right) + \frac{1}{\rho}\nabla p = \nu\nabla^4\vec{u},$$

$$\frac{\partial T}{\partial t} + \vec{u}\cdot\nabla T - \frac{1}{c_p\rho}\omega = \nu\nabla^4 T,,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial p_d}{\partial \eta}\right) + \nabla\cdot\left(\frac{\partial p_d}{\partial \eta}\vec{u}\right) = \nu\nabla^4\left(\frac{\partial p_d}{\partial \eta}\right),$$

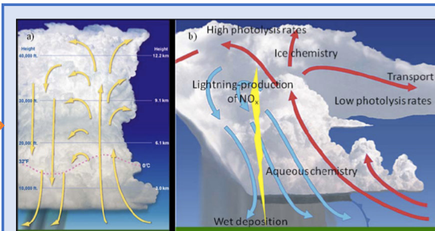
$$\frac{\partial}{\partial t}\left(\frac{\partial p_d}{\partial \eta}m_i\right) + \nabla\cdot\left(\frac{\partial p_d}{\partial \eta}m_i\vec{u}\right) = \nu\nabla^4\left(m_i\right), \quad i = v, cl, ci, \ldots$$

Approximates the solution to the adiabatic equations of motion ("resolved scales):

- Momentum (u,v)
- Thermodynamic equation (T)
- Continuity equation for air (p)
- Continuity equation for
  - forms of water (water vapor, cloud liquid, cloud ice, rain, …)
  - quantities needed to represent aeroso
  - chemical species

Physics-dynamics coupling layer

Physics (parameterization) module



Roughly speaking, processes that can not be resolved on model grid hence physics is also referred to as ub-grid-scale processes):

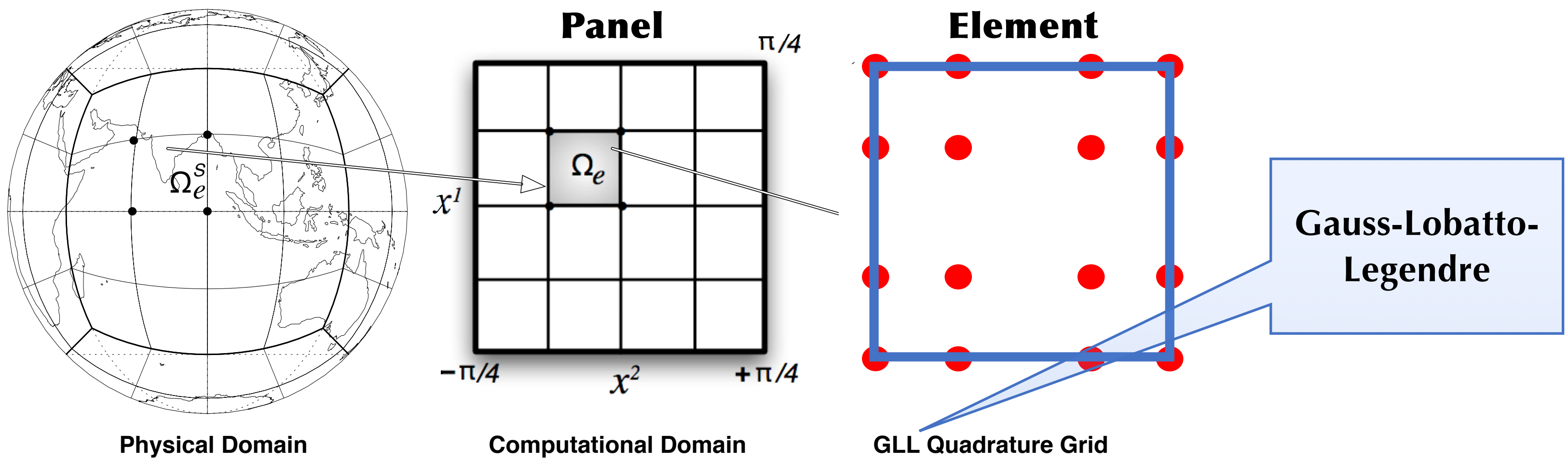
adiation
oundary layer turbulence
ub-grid-scale orographic drag
hallow and deep convection
licrophysics
erosol processes
'ertical mixing
…

Topic of this talk:

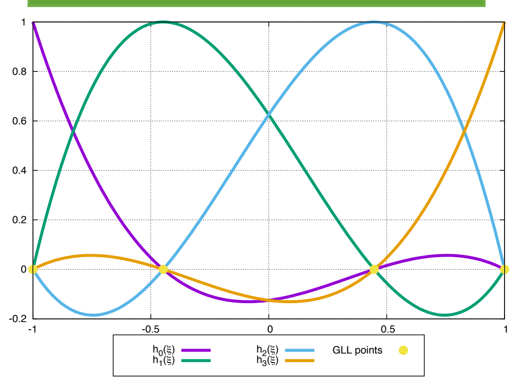
Rethinking physics-dynamics coupling with high-order element-based Galerkin method

Part I: why?
Part II: a solution

The spectral-element method: discretization grid



Nodal 1D polynomial basis functions

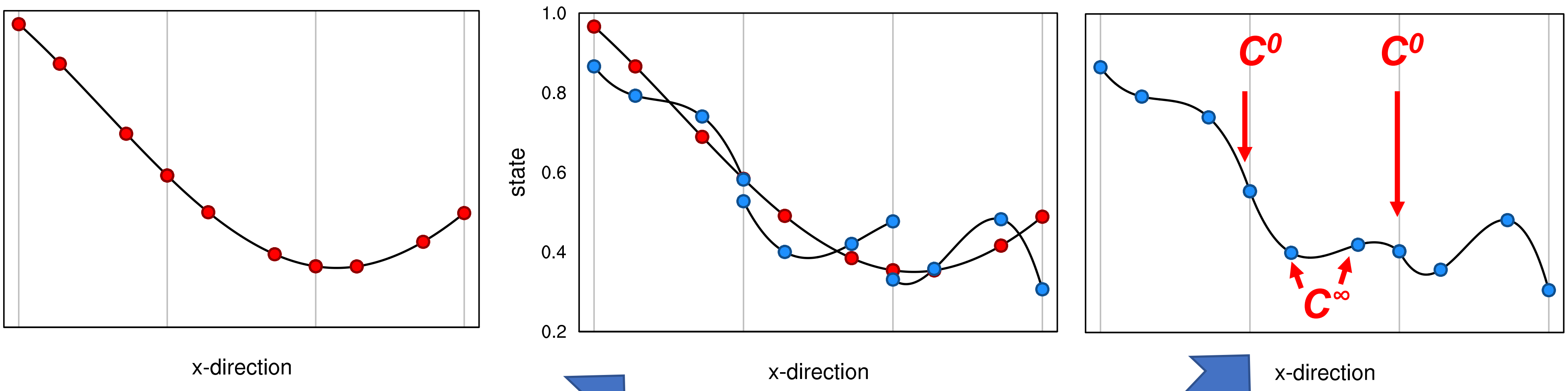


For any arbitrary variable f (e.g., T, u, v, p, ...) one can approximate f as a function of a tensor product of 1D basis functions on the 2D GLL grid:

$$f(x, y) = \sum_{i,j} f_{i,j} h_i(x_i) h_j(y_j),$$

where $f_{i,j}$ is grid point values of f.

The spectral-element method: advancing solution



Continuity equation for Δp:

$$\left\langle h_k, \frac{\partial \Delta p}{\partial t} \right\rangle = \langle h_k, -\nabla \cdot \Delta p \vec{v} \rangle + \langle h_k, \tau \nabla^4 \Delta p \rangle ,$$

where $\langle h_k, \cdot \rangle$ is inner product

$$\langle h_k, f \rangle = \sum_{i,j} w_{i,j} h_k(x_i, y_j) f(x_i, y_j) \sim \iint h_k f \, dA.$$

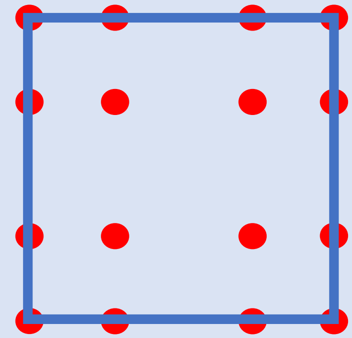
- Projection step

$$\Delta p^{n+1} = DSS\,(\Delta p^*)$$

  where *DSS* refers to *Direct Stiffness Summation* (also referred to as assembly or inverse mass matrix step).
- Choice of GLL quadrature based inner product and nodal basis functions gives a diagonal mass matrix (Maday and Patera, 1987).

The spectral-element method:

NCAR
NATIONAL CENTER FOR ATMOSPHERIC RESEARCH



Advantages of the spectral-element method:

- **Halo is small (MPI message sizes are small)**
- **Easily adaptable for variable-resolution meshes**
- **Discretization is mimetic (conserves mass and energy)**
- **High-order accuracy for smooth problems**

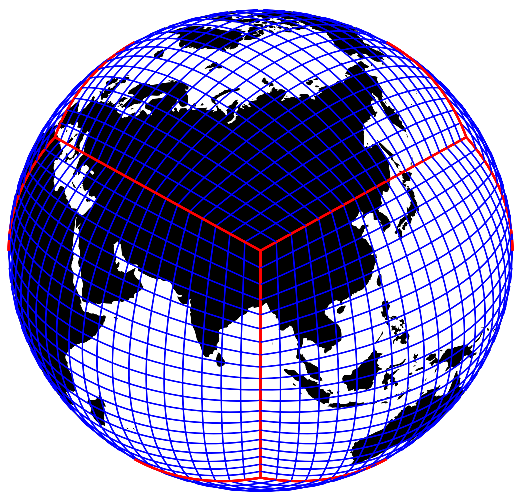
“Disadvantages”:

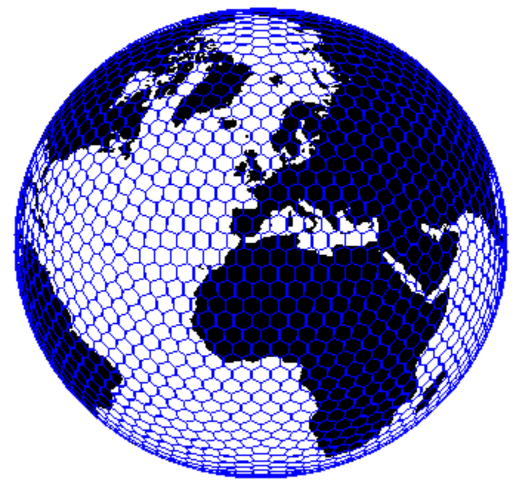
- **Need to rethink physics-dynamics coupling**

The physics dynamics coupling paradigm

NCAR
NATIONAL CENTER FOR ATMOSPHERIC RESEARCH

Assumptions inherent to the physical parameterizations require the state passed by the dynamical core represent a 'large-scale state', for example, in quasi-equilibrium-type convection schemes (Arakawa and Schubert 1974)

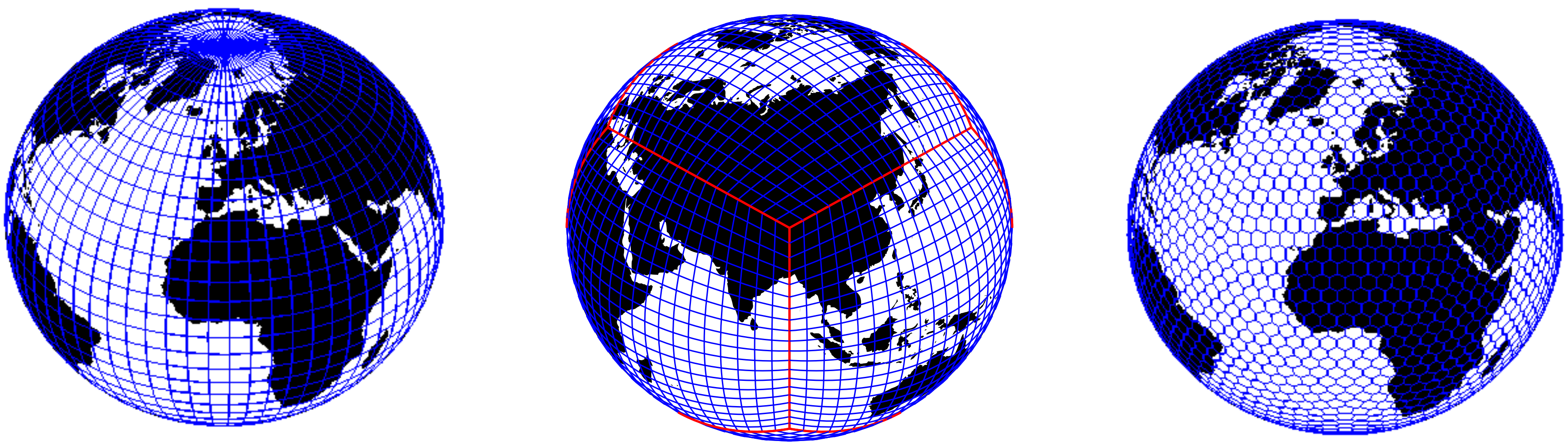




The physics dynamics coupling paradigm

Finite-volume methods : dynamical core state = average state over a control volume
Finite-difference methods : point value representative for dynamical core state - in the vicinity of point value one can usually associate a volume with the grid-point that is representative of state.

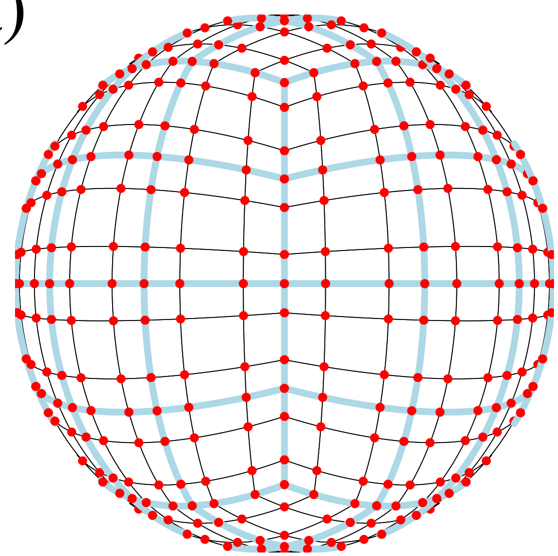
For the regular latitude-longitude, cubed-sphere and icosahedral grids the distance between the grid-points is gradually varying for finite-volume/finite-difference discretizations!

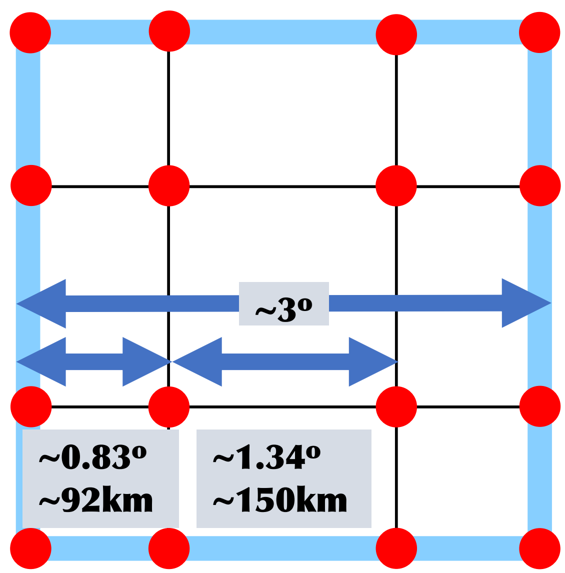


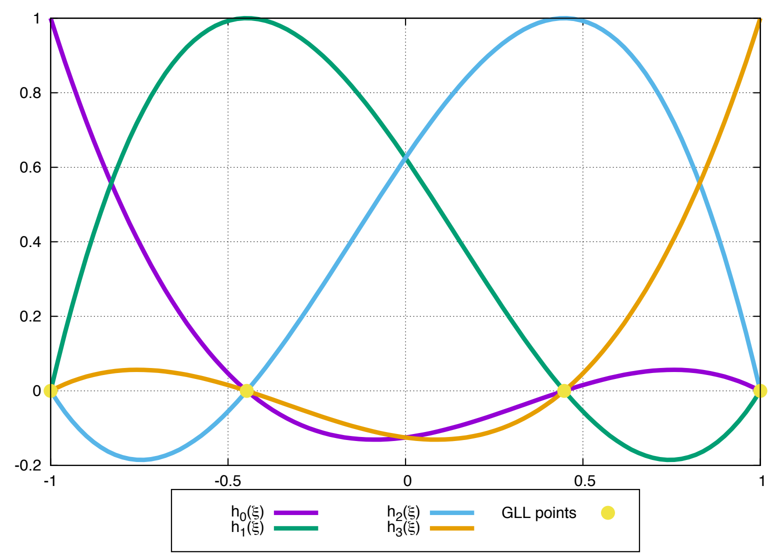
The physics dynamics coupling paradigm

- A unique aspect of the GLL quadrature rules is that the nodes within an element are located at the roots of the basis functions, which may be irregularly spaced
- Resolved scales of motion are NOT determined by the distance between GLL nodes, but rather the degree of the polynomial basis in each element.
- The nodes may be viewed as irregularly spaced samples of an underlying spectrally truncated state.

(a)

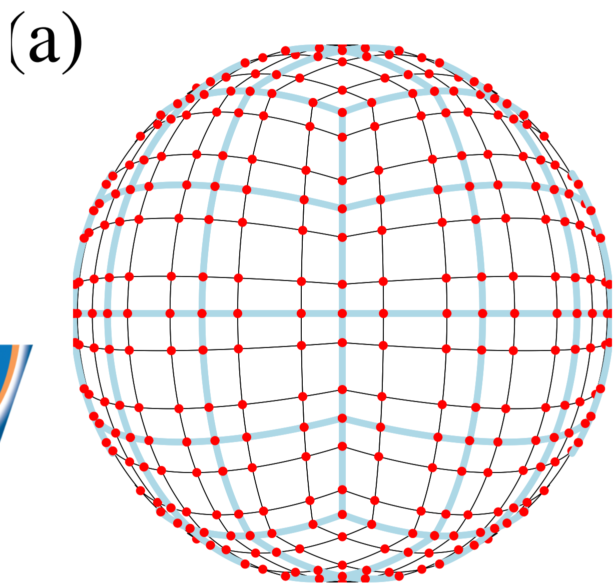


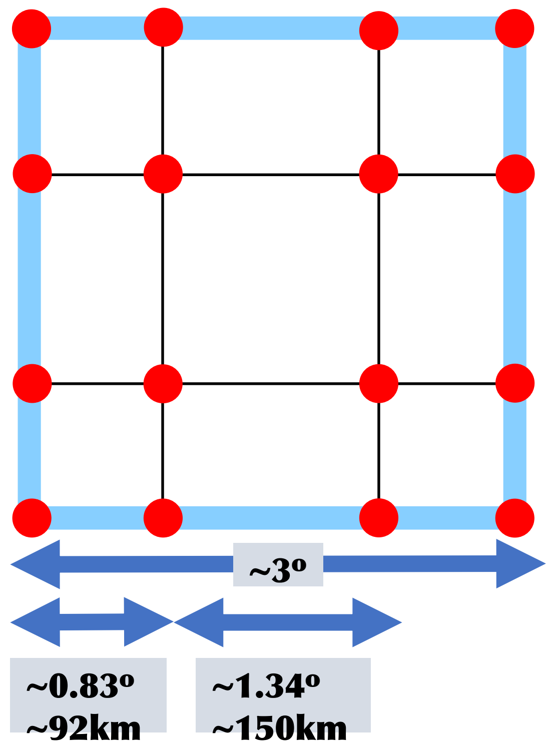


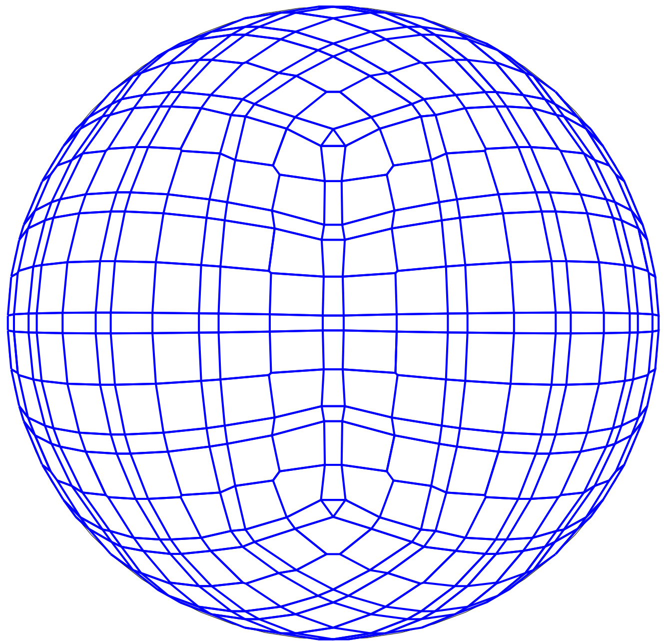


If we apply the conventional physics-dynamics coupling paradigm to higher-order Galerkin methods …

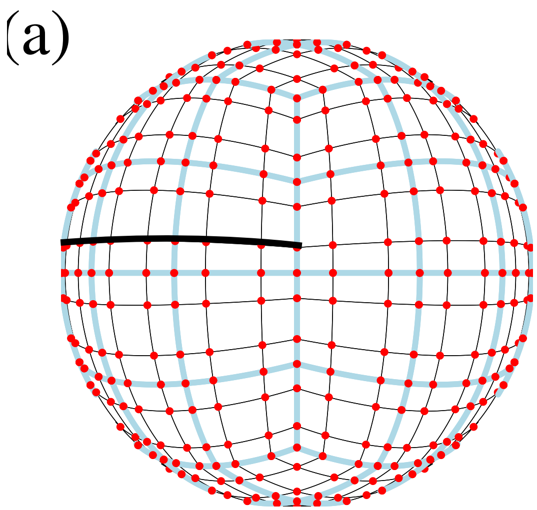
then state passed to physics is the state at the quadrature node values

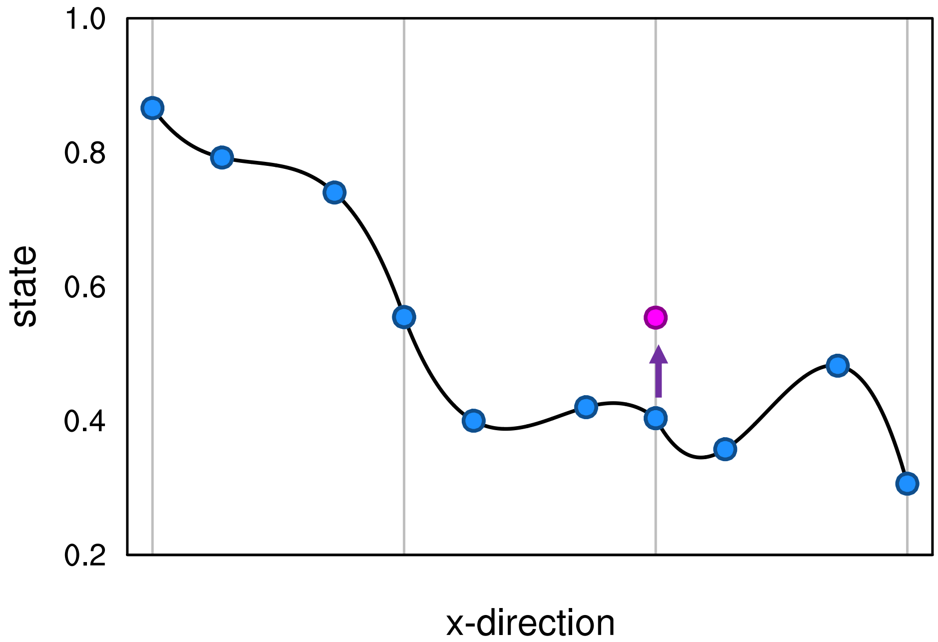






If we apply convention the conventional physics-dynamics coupling paradigm to higher-order Galerkin methods …

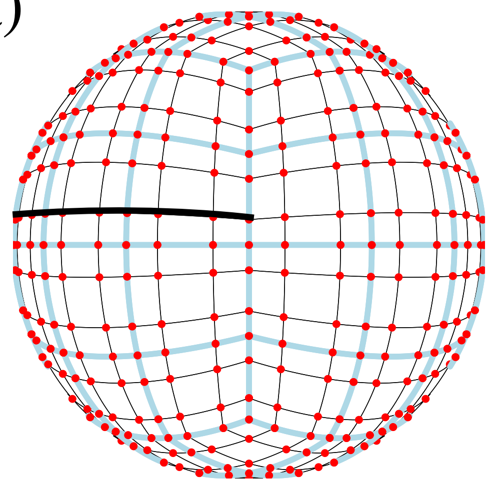


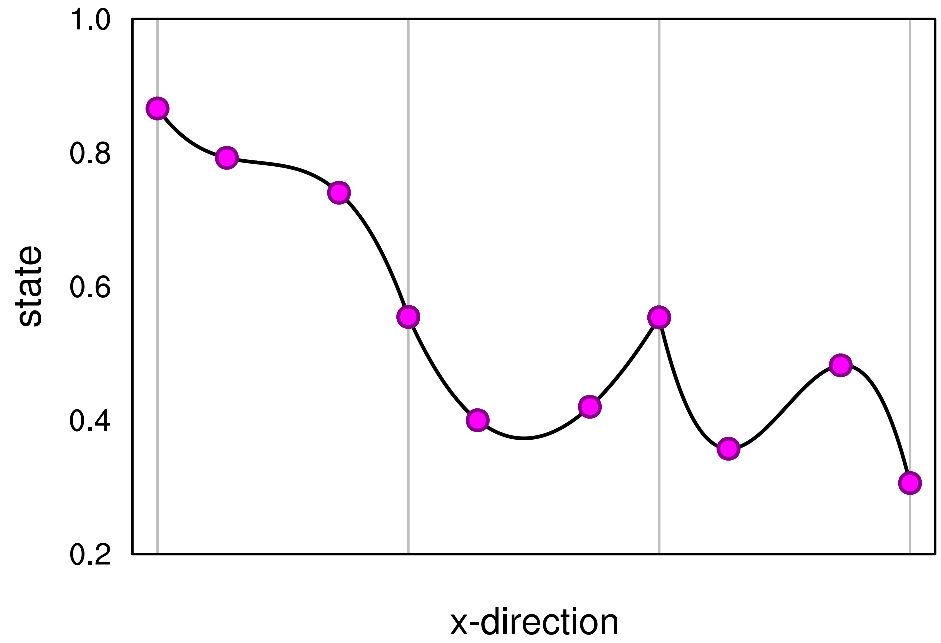


The physics forms a cloud on a boundary node

If we apply convention the conventional physics-dynamics coupling paradigm to higher-order Galerkin methods …

(a)



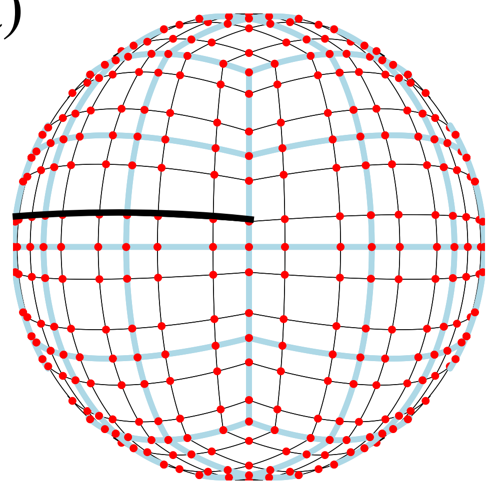


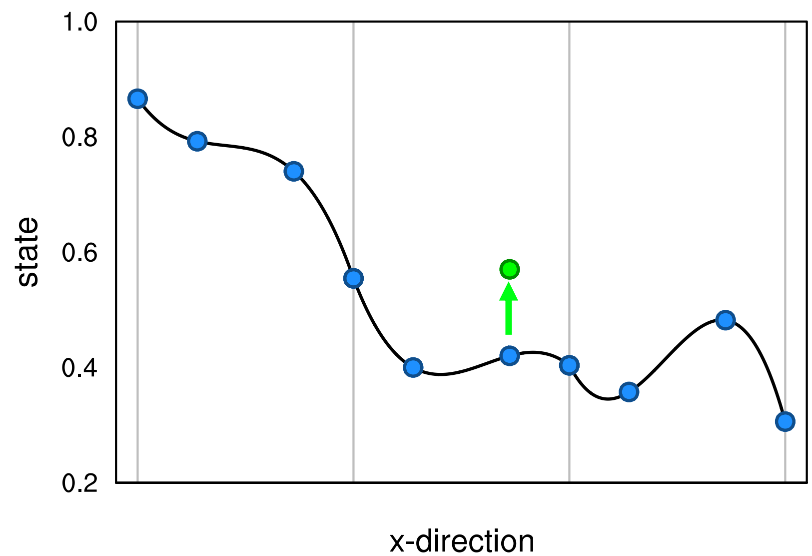
Note … non-local effect by changing one node value

The physics forms a cloud on a boundary node

If we apply convention the conventional physics-dynamics coupling paradigm to higher-order Galerkin methods …

(a)



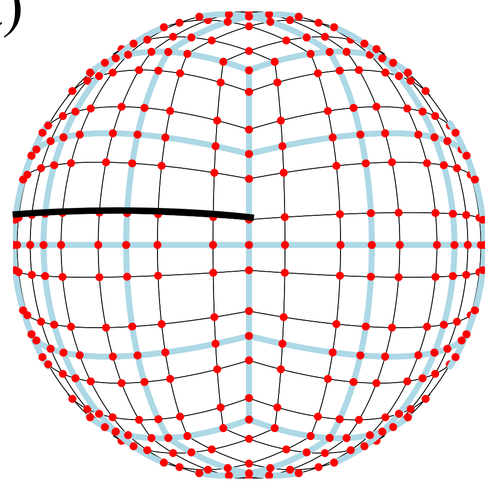


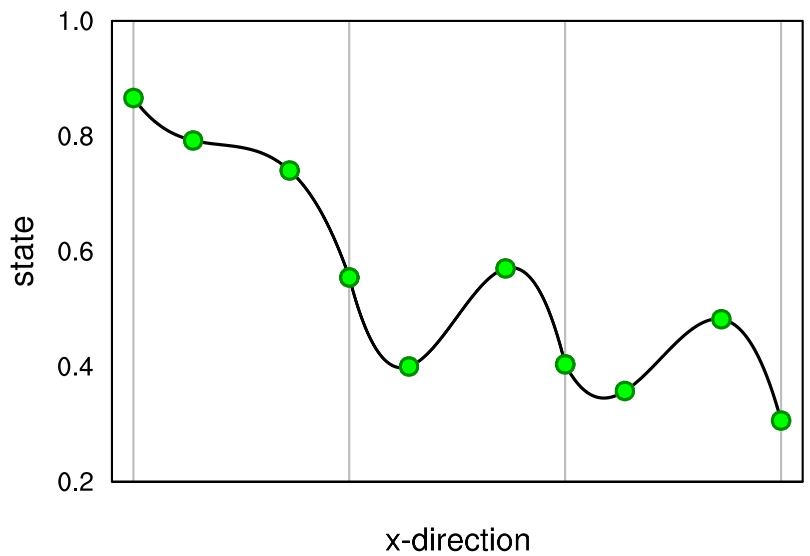
Note … non-local effect by changing one node value

Lets say the cloud instead forms at an interior node…

If we apply convention the conventional physics-dynamics coupling paradigm to higher-order Galerkin methods …

(a)

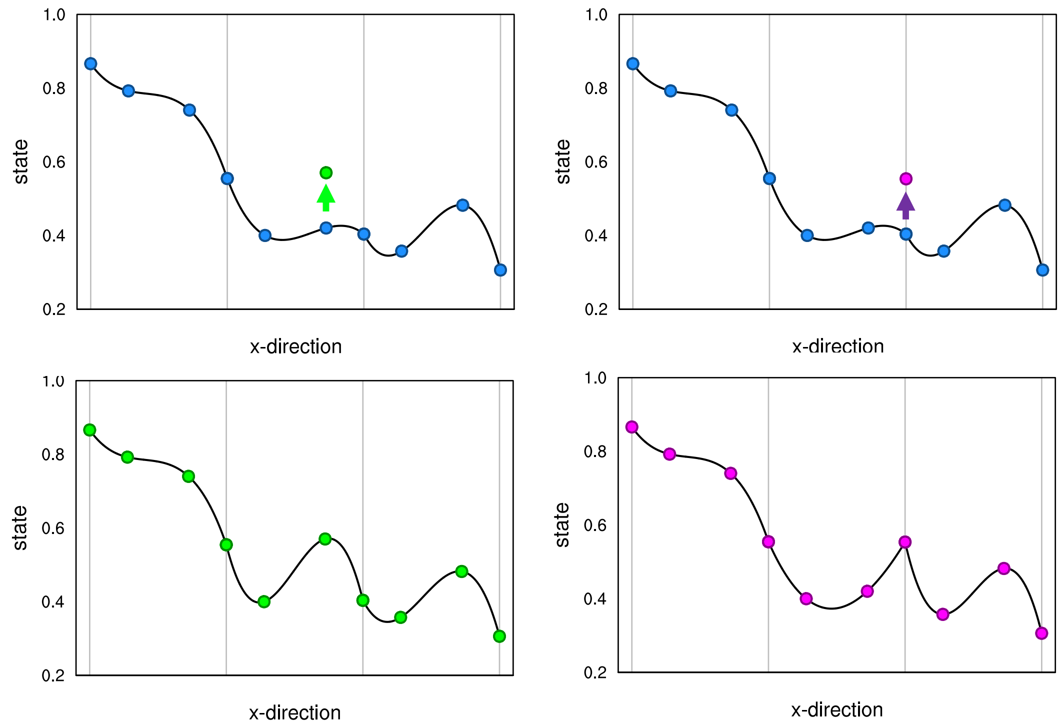




Note … non-local effect by changing one node value

Lets say the cloud instead forms at an interior node…

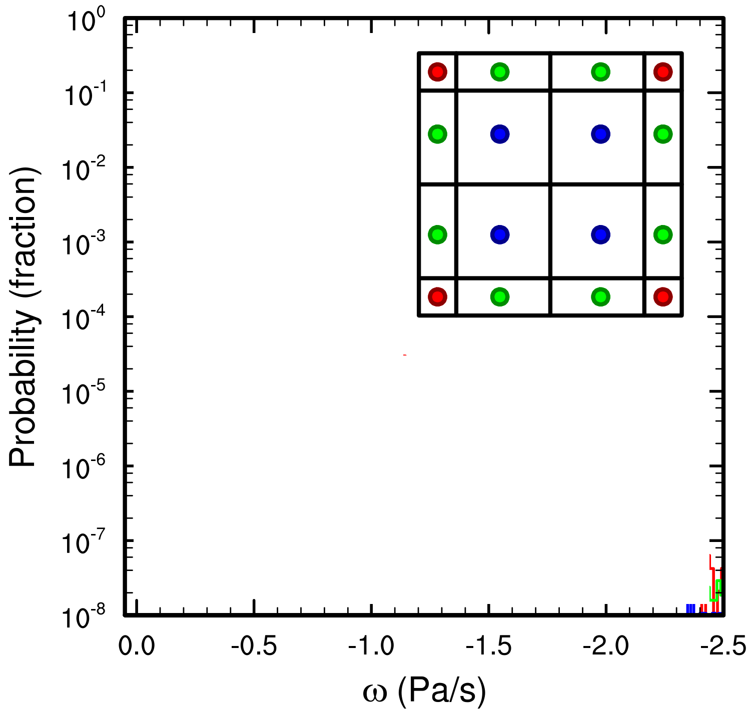
If we apply convention the conventional physics-dynamics coupling paradigm to higher-order Galerkin methods …



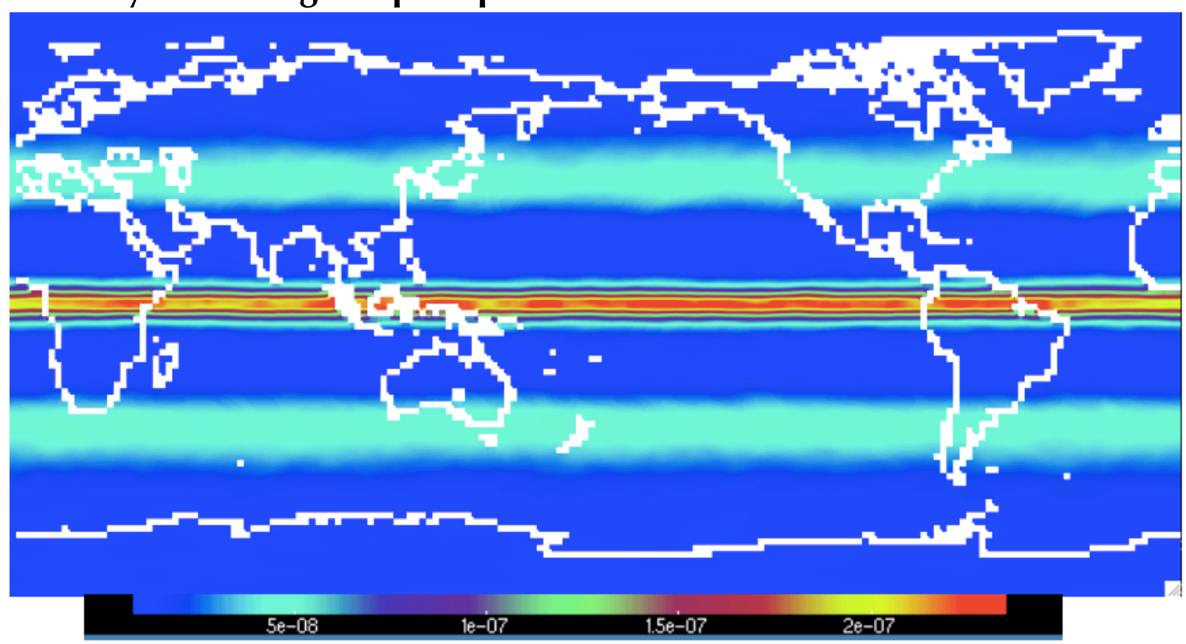
The irregular physical distance between nodes seems to have less bearing on the solution, compared with whether one is, or is not, on an element boundary

For an Aqua-planet simulation the climatology (of any variable) should be zonal:

… so the climatology at any quadrature node should be the same!

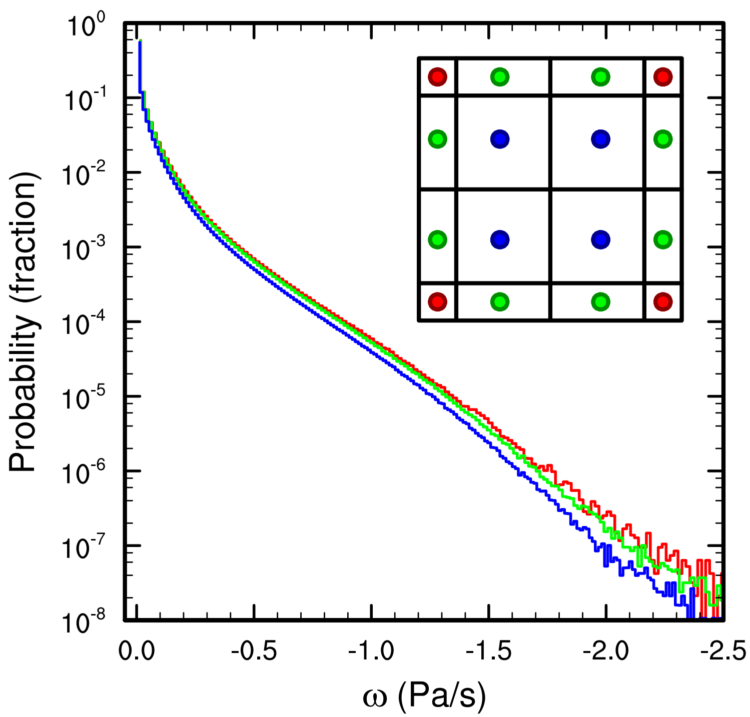


Multi-year average of precipitation rate

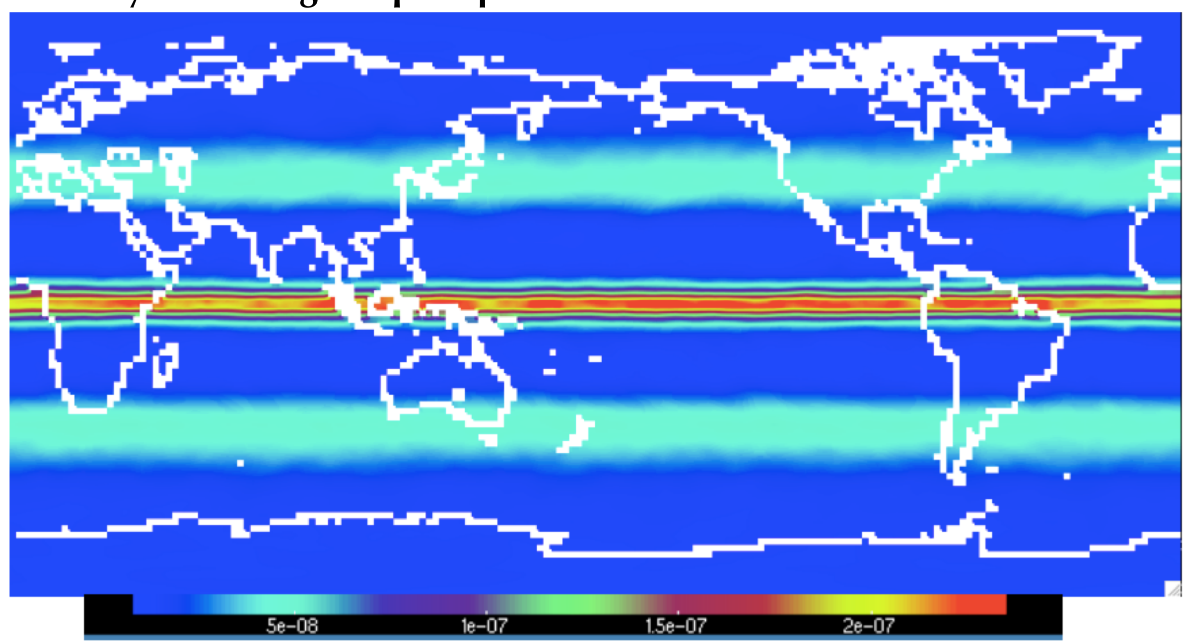


For an Aqua-planet simulation the climatology (of any variable) should be zonal:

… so the climatology at any quadrature node should be the same!



Multi-year average of precipitation rate



For an Aqua-planet simulation the climatology (of any variable) should be zonal:

... so the climatology at any quadrature node should be the same!

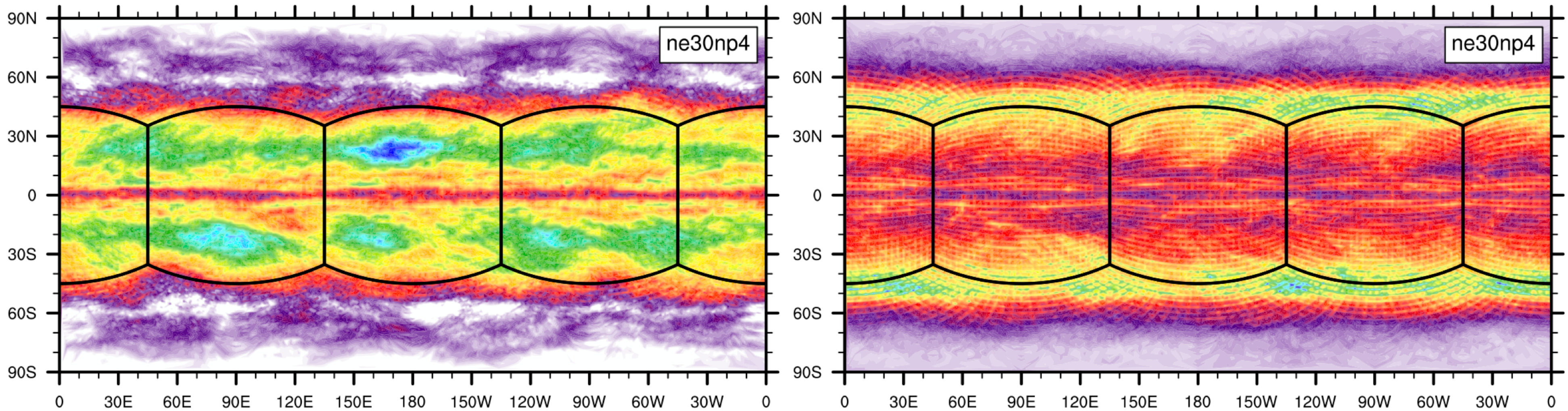
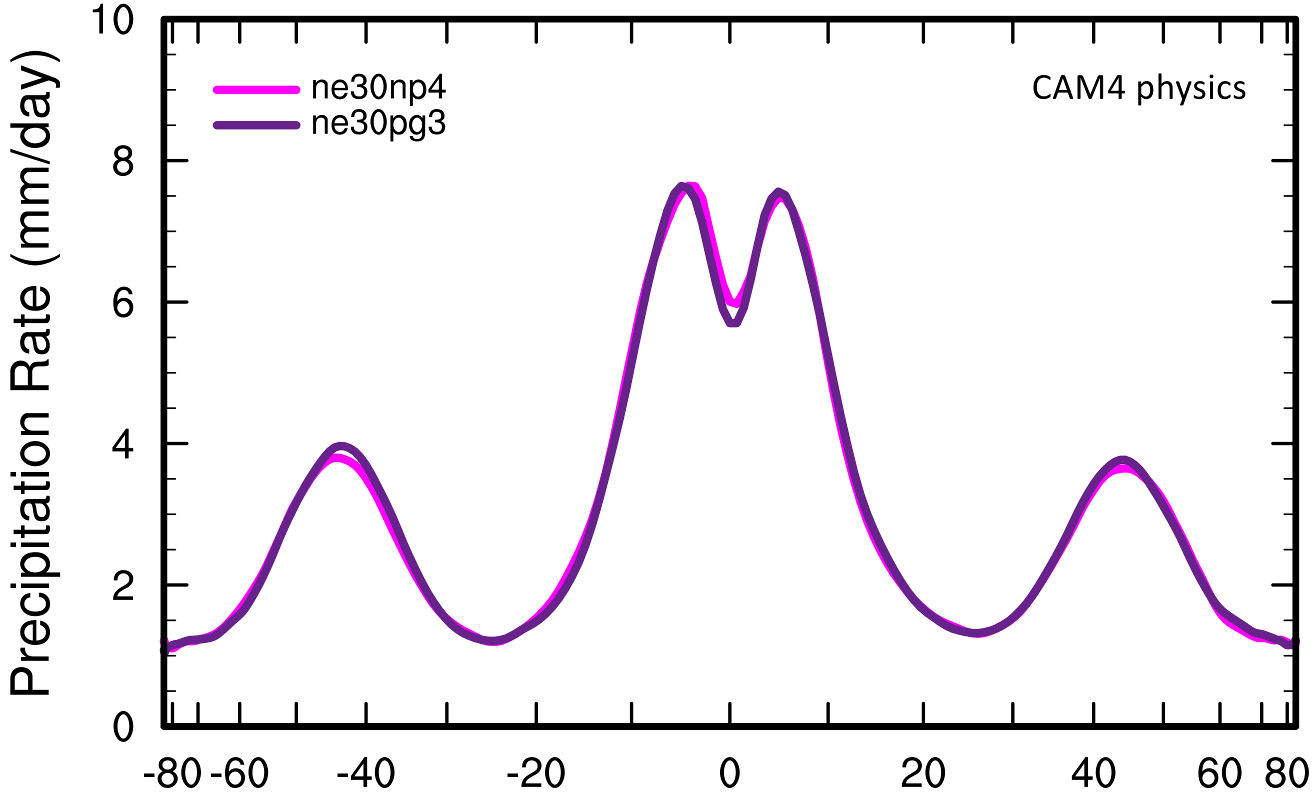


Figure: (left) Mean and (right) variance of low level temperature tendency (using CAM4 physics)

That said, the zonal means look very similar …



Held-Suarez simulation with real-world topography

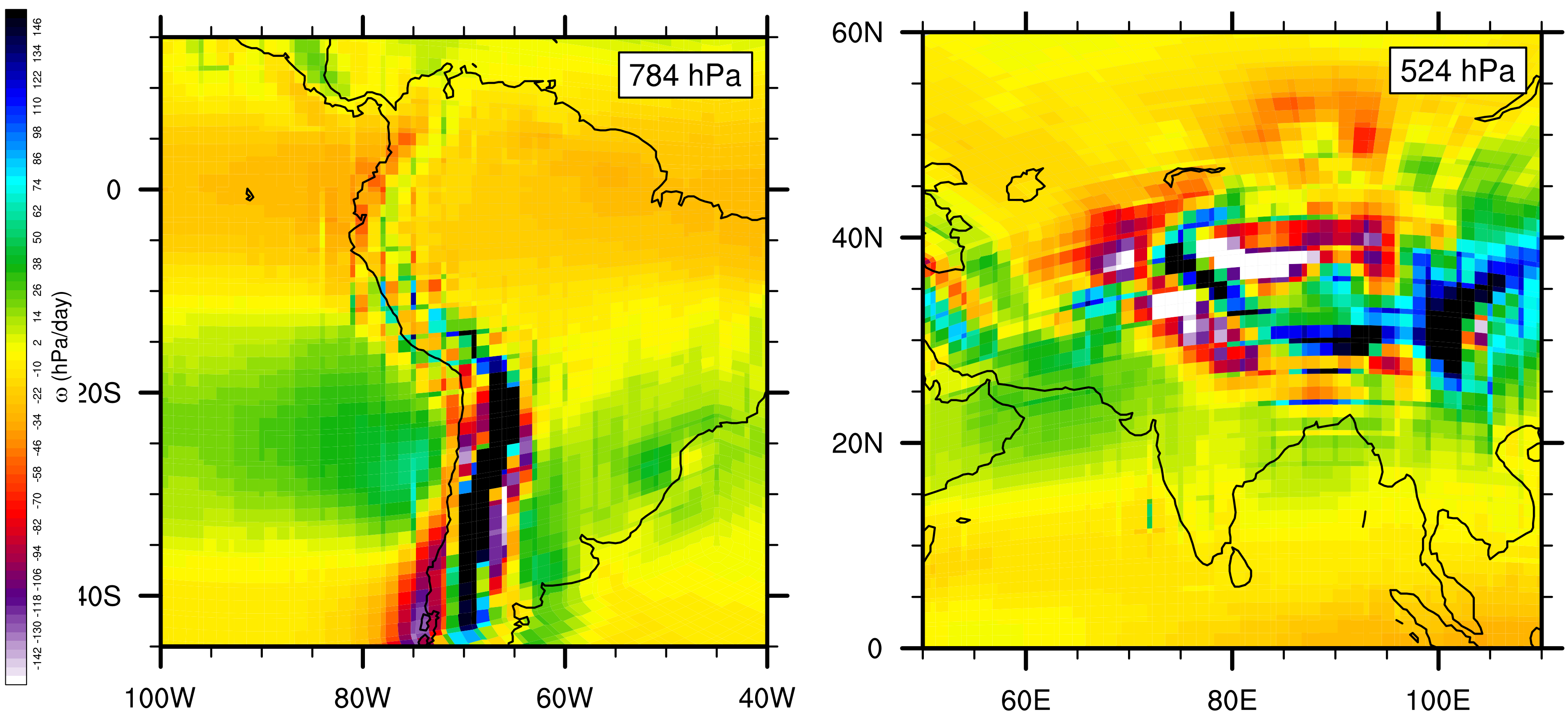


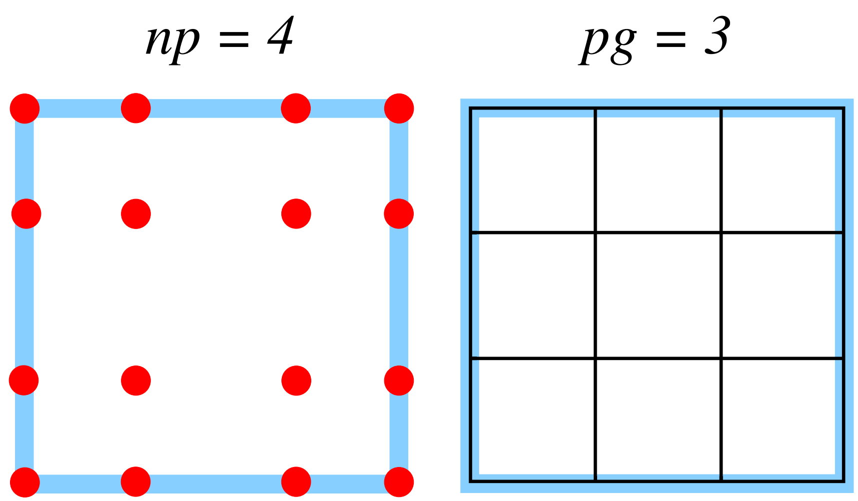
Figure: Mean OMEGA for CAM-SE at two model levels in the middle troposphere, in a Held-Suarez configuration outfitted with real world topography. The data are contoured according to a ‘cell fill’ approach.

NCAR
NATIONAL CENTER FOR ATMOSPHERIC RESEARCH

-> using the conventional physics-dynamics coupling paradigm leads to spurious dependencies on location within element

Solution: Quasi-equal area physics grid

Introducing an ~equal area physics grid



Herrington et al. (MWR, revised)

Mapping u,v, T, and omega from dynamics grid (GLL) to finite-volume grid:

Important properties for mapping operators

1. conservation of scalar quantities such as mass (and dry thermal energy),

2. for tracers; shape-preservation (monotonicity), i.e. the mapping method must not introduce new extrema in the interpolated field, in particular, negatives,

3. consistency, i.e. the mapping preserves a constant,

4. linear correlation preservation.

Other properties that may be important, but not pursued here, includes total energy conservation and axial angular momentum.

Herrington et al. (MWR, revised)

Mapping u,v, T and tracer tendencies from finite-volume grid to dynamics grid (GLL)

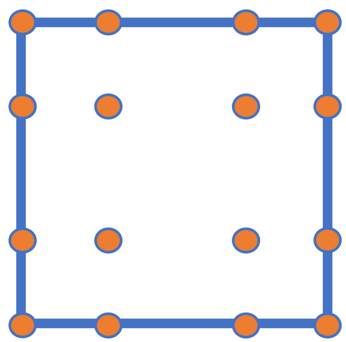
Important properties for mapping operators

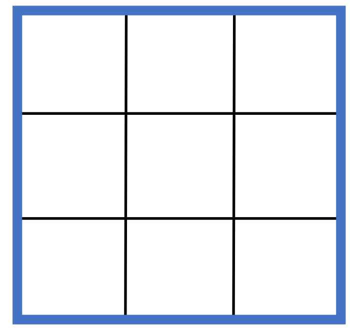
1. for tracers; mass tendency is conserved,
2. for tracers; in each tracer grid cell the mass tendency from physics must not exceed tracer mass available in tracer grid cell (i.e. physics tendency will not drive tracer mixing ratio negative on the GLL grid),
3. linear correlation preservation (at least for tracers),
4. consistency, i.e. the mapping preserves a constant tendency.

Other properties that may be important, but not pursued here, includes total energy conservation (incl. components of total energy) and axial angular momentum conservation.

Herrington et al. (MWR, revised)

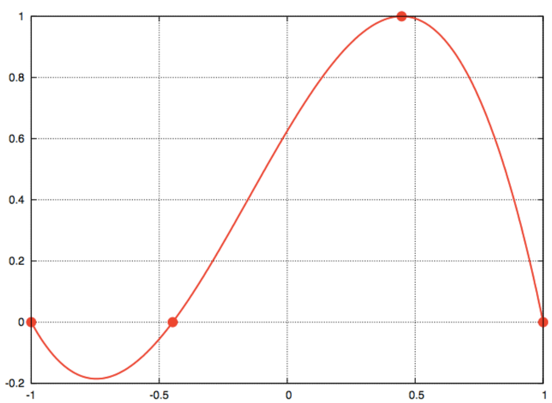
To my knowledge there is no reversible map using the SE Lagrange basis
(let alone shape-preserving and mass conservative)

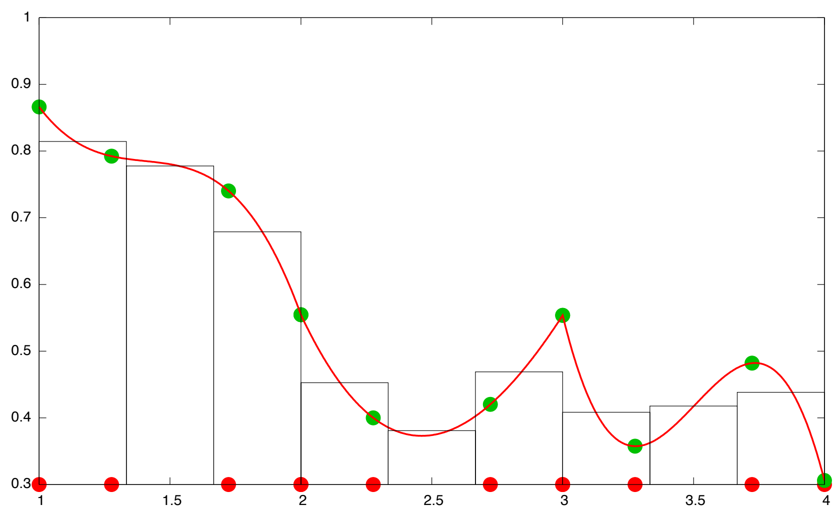




Herrington et al. (MWR, revising)

To my knowledge there is no reversible map using the SE Lagrange basis
(let alone shape-preserving and mass conservative)



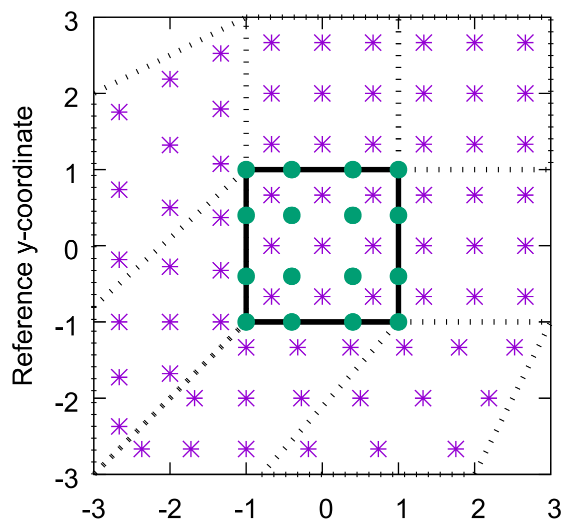


Integrate SE basis functions over finite-volume control volumes (lower right plot):

- conserves scalar mass
- not shape-preserving (see left plot)

Herrington et al. (MWR, revised)

To my knowledge there is no reversible map using the SE Lagrange basis
(let alone shape-preserving and mass conservative)



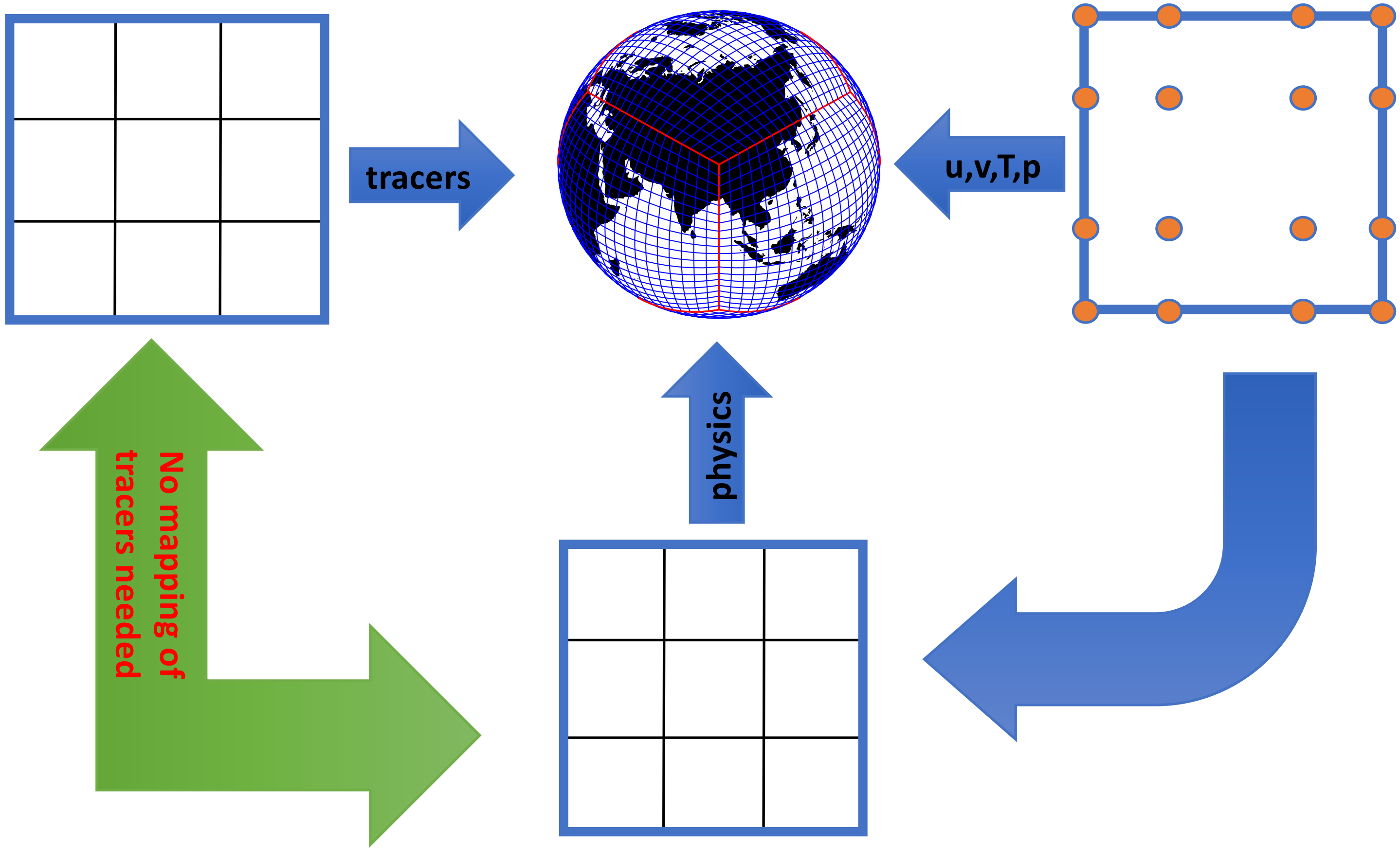
Cubic tensor-product interpolation in central angle coordinates
(high-order interpolation was found to be important!)

- Preserves a constant
- Not scalar mass conserving

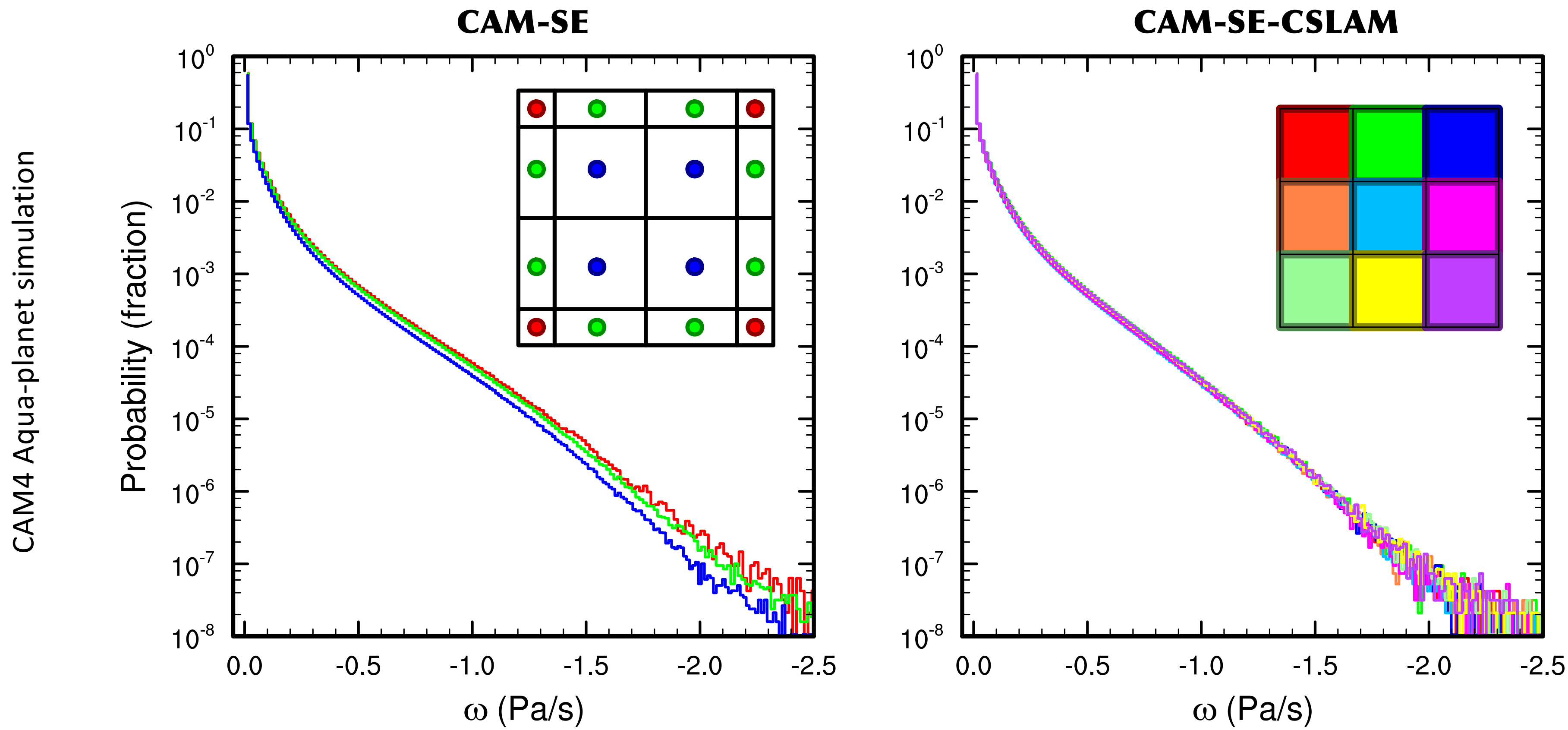
Herrington et al. (MWR, revised)

To achieve scalar mass-conservation in physics-dynamics coupling use CSLAM for transport:
conservation, consistency & shape-preservation in tracer physics-dynamics coupling

CSLAM = Conservative Semi-Lagrangian Multi-tracer transport scheme
(Lauritzen et al., 2010, 2016)
Consistent coupling of CSLAM with the SE method is described in detail in Lauritzen et al. (2016)



Results – CAM4 Aqua-planets



State the physics 'see' is now independent of location within element!

Herrington et al. (MWR, revised)

Results – CAM4 Aqua-planets

NCAR
NATIONAL CENTER FOR ATMOSPHERIC RESEARCH

CAM-SE

CAM-SE-CSLAM

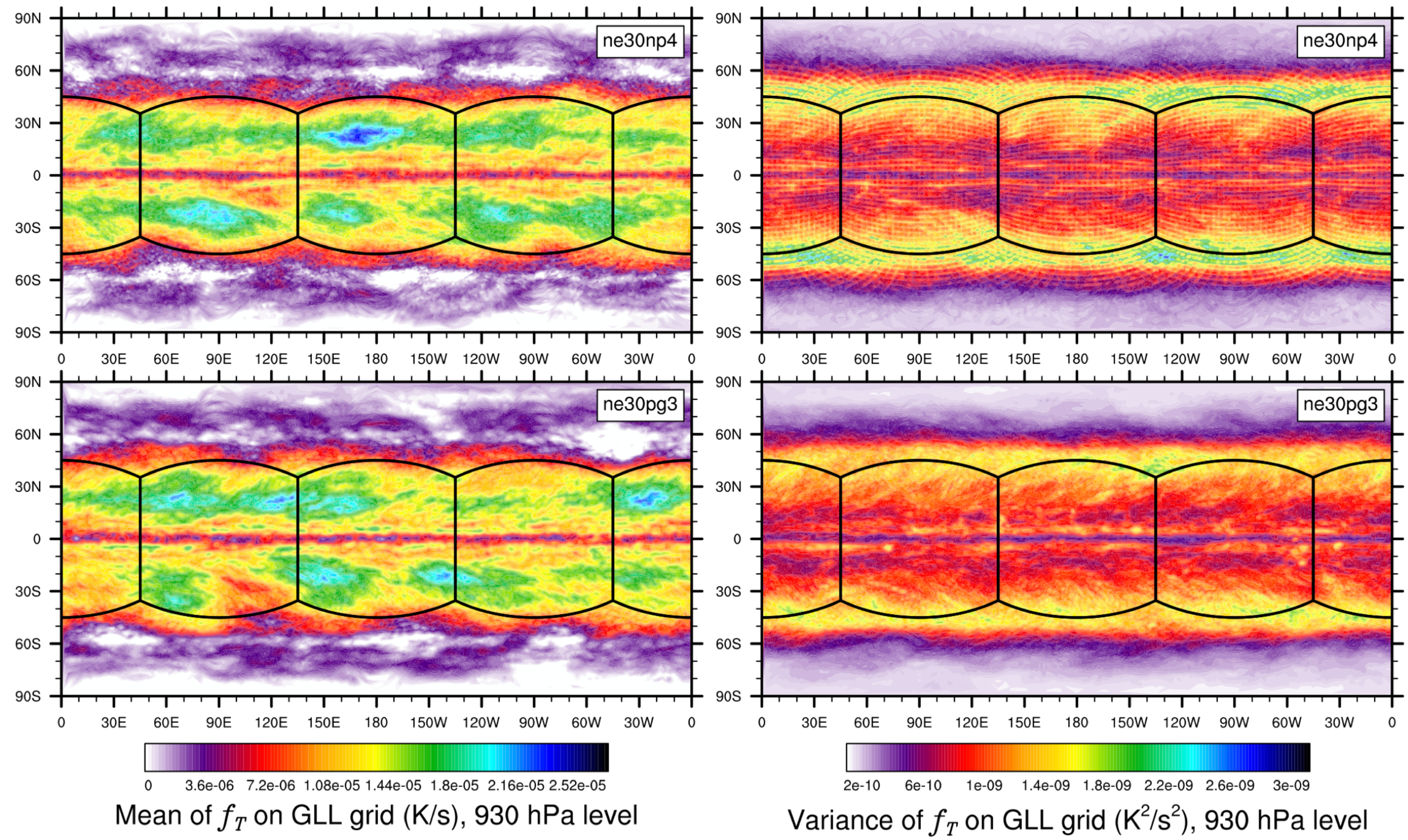


Figure: (left column) Mean and (right column) variance of low level temperature tendency

Herrington et al. (MWR, revised)

Held-Suarez simulation with real-world topography

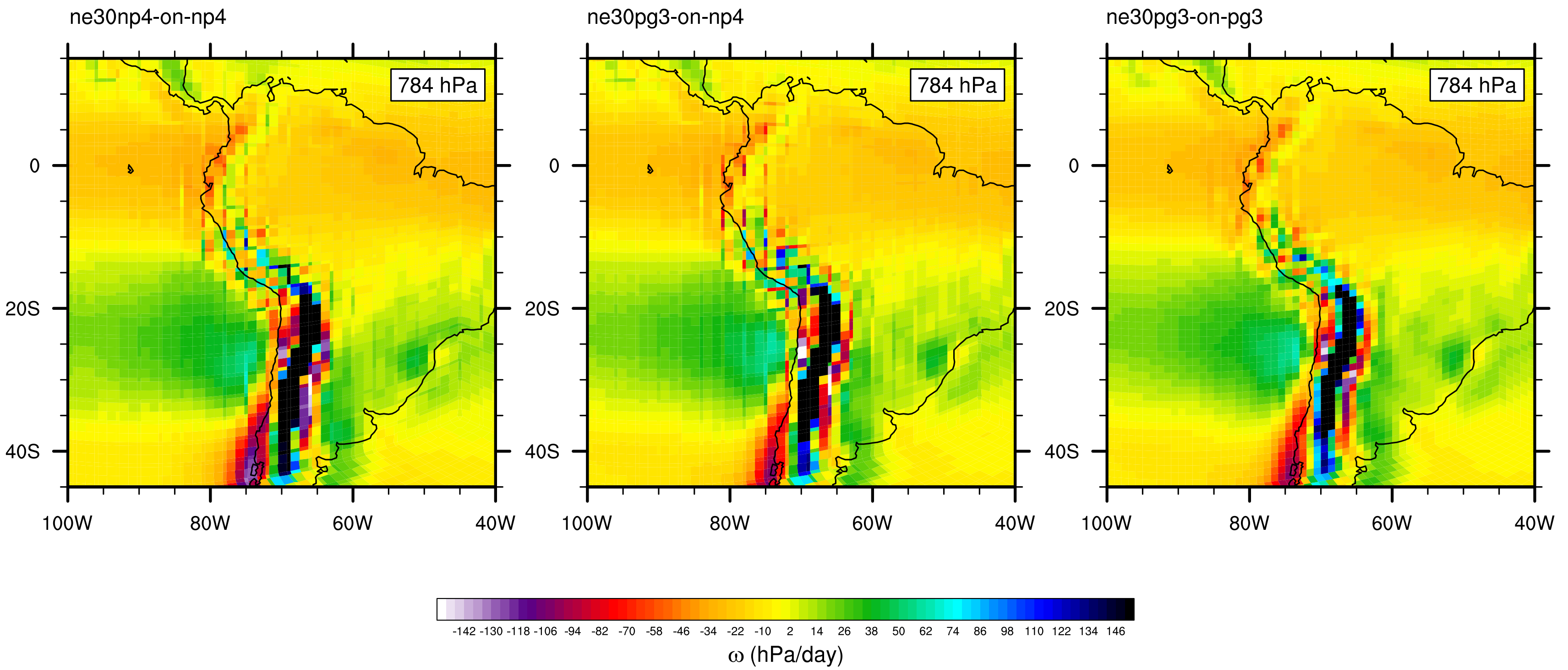


Figure: Mean OMEGA for CAM-SE (left), CAM-SE-CSLAM but on GLL grid and CAM-SE-CSLAM grid. The data are contoured according to a 'cell fill' approach.

Herrington et al. (MWR, revised)

Held-Suarez simulation with real-world topography

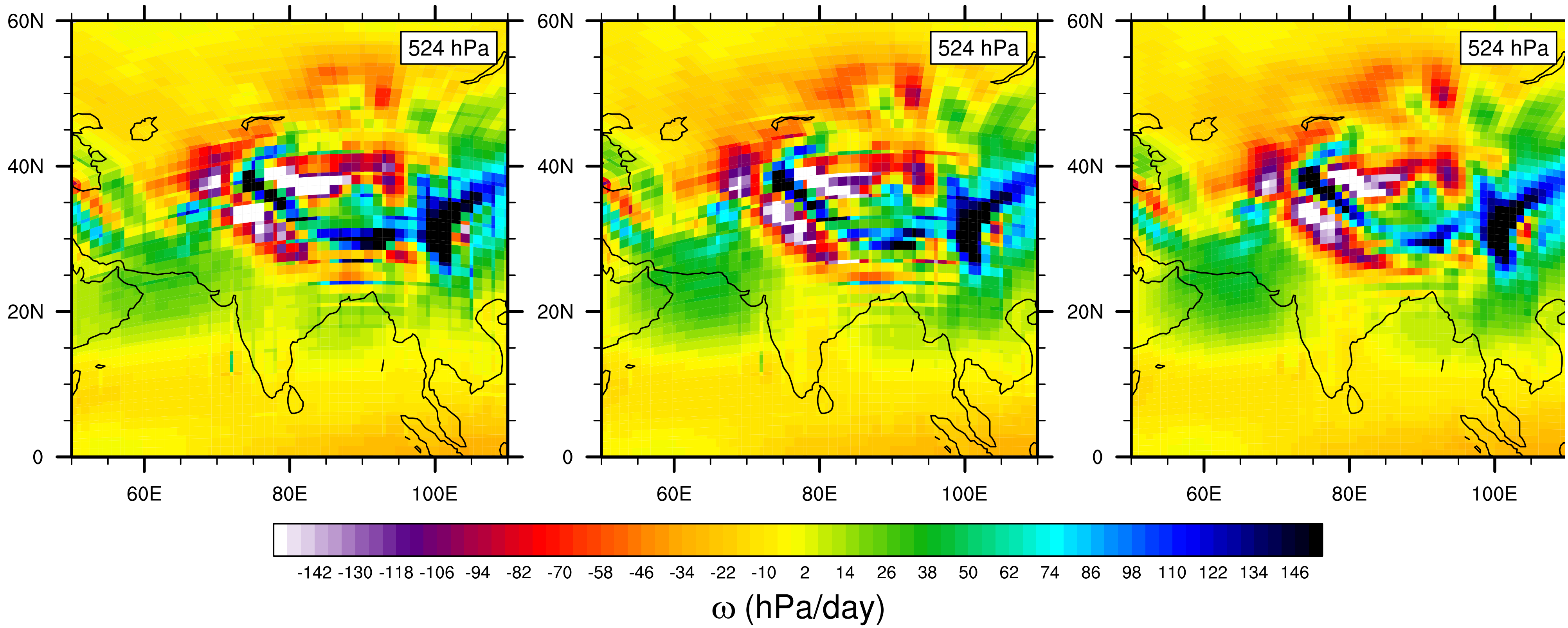


Figure: Mean OMEGA for CAM-SE (left), CAM-SE-CSLAM but on GLL grid and CAM-SE-CSLAM grid.
The data are contoured according to a 'cell fill' approach.

Herrington et al. (MWR, revising)

Results – CAM6 AMIP simulations

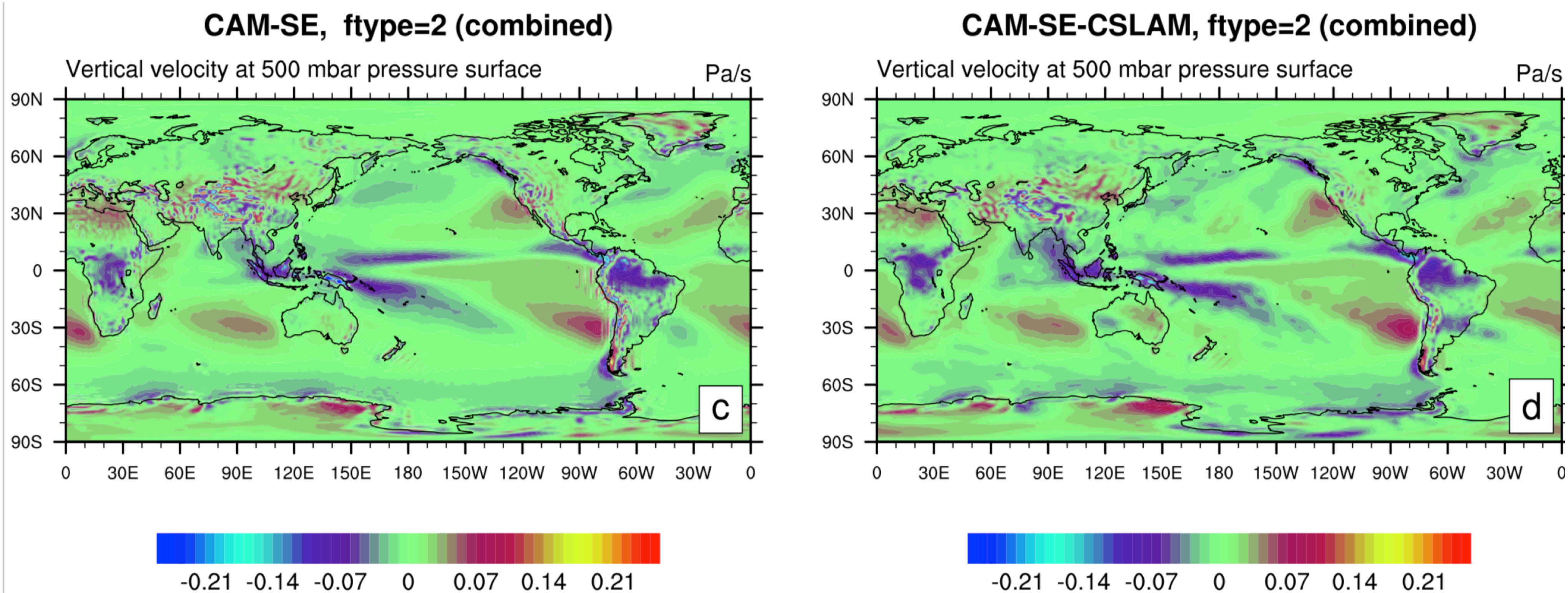


Figure: Multi-year mean vertical pressure velocity in `real-world' (AMIP) simulation.

Results – CAM6 AMIP simulations

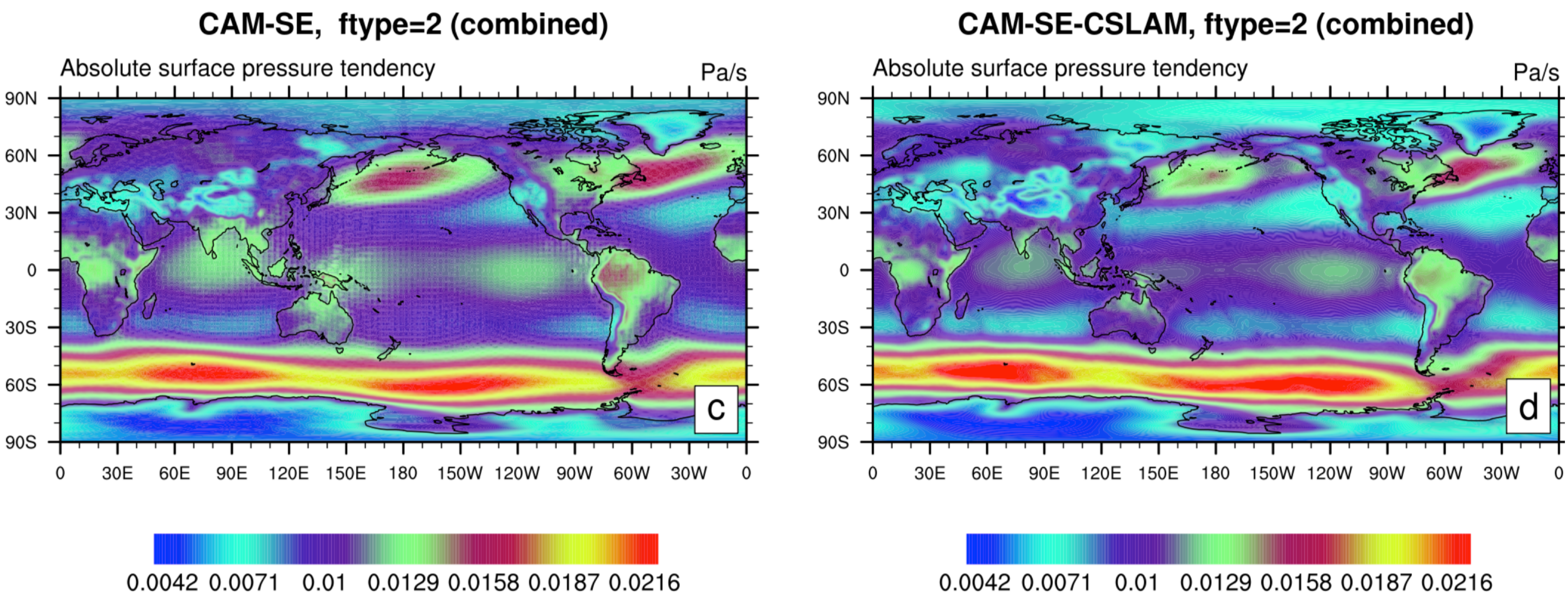
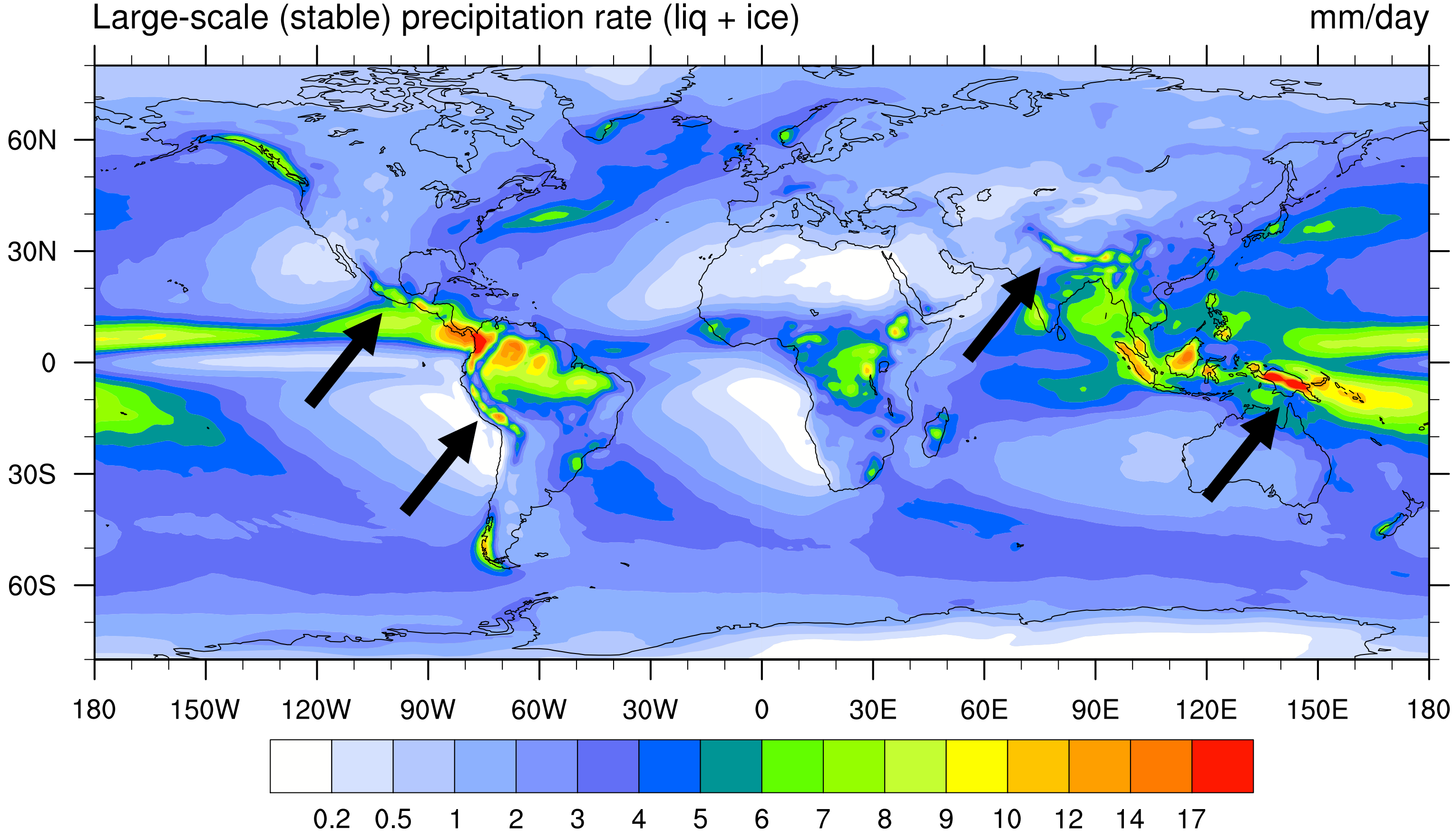


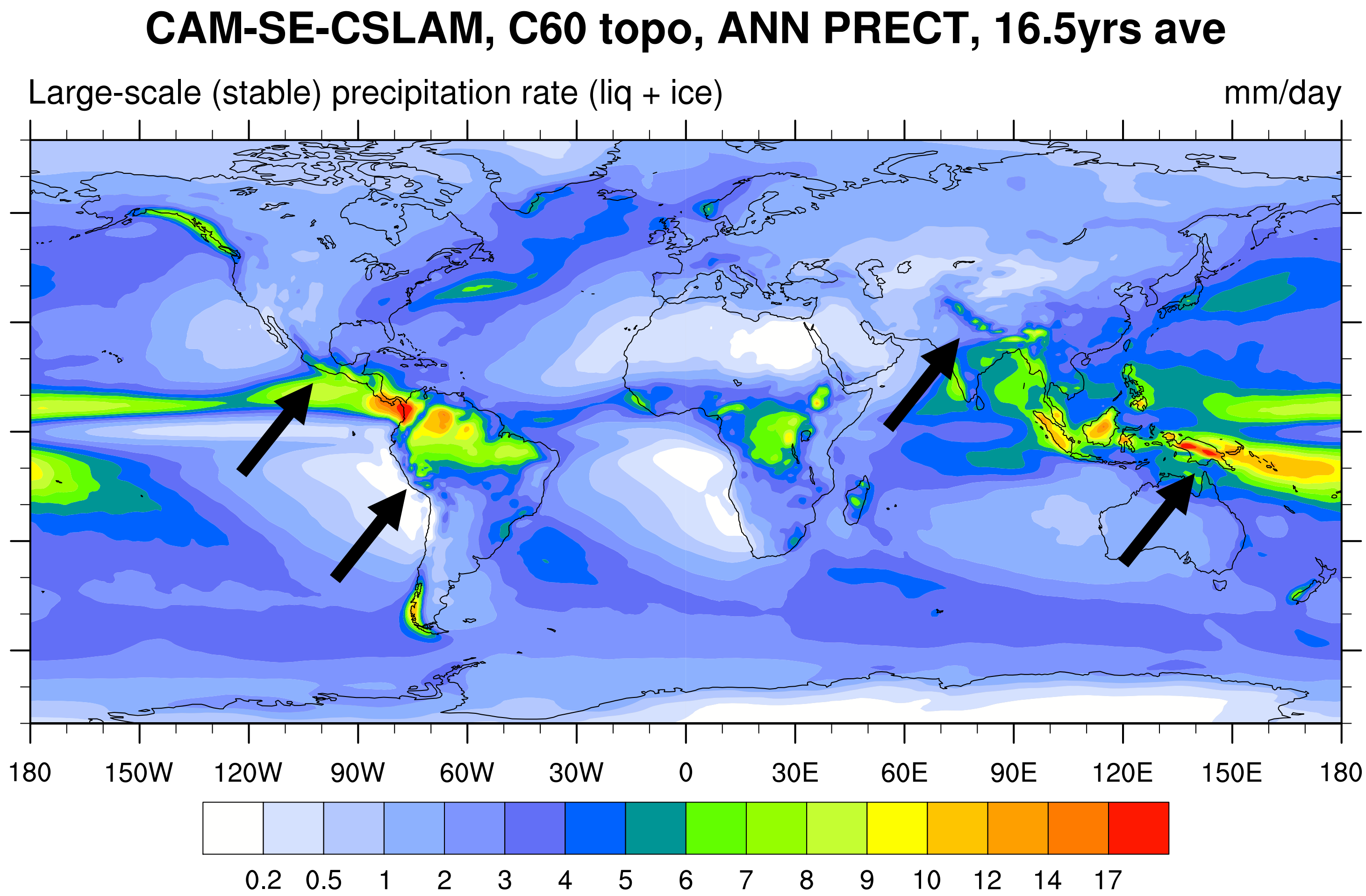
Figure: Multi-year mean absolute surface pressure tendency.

AMIP simulation

CAM-SE, C60 topo, ANN PRECT, 16.5yrs ave



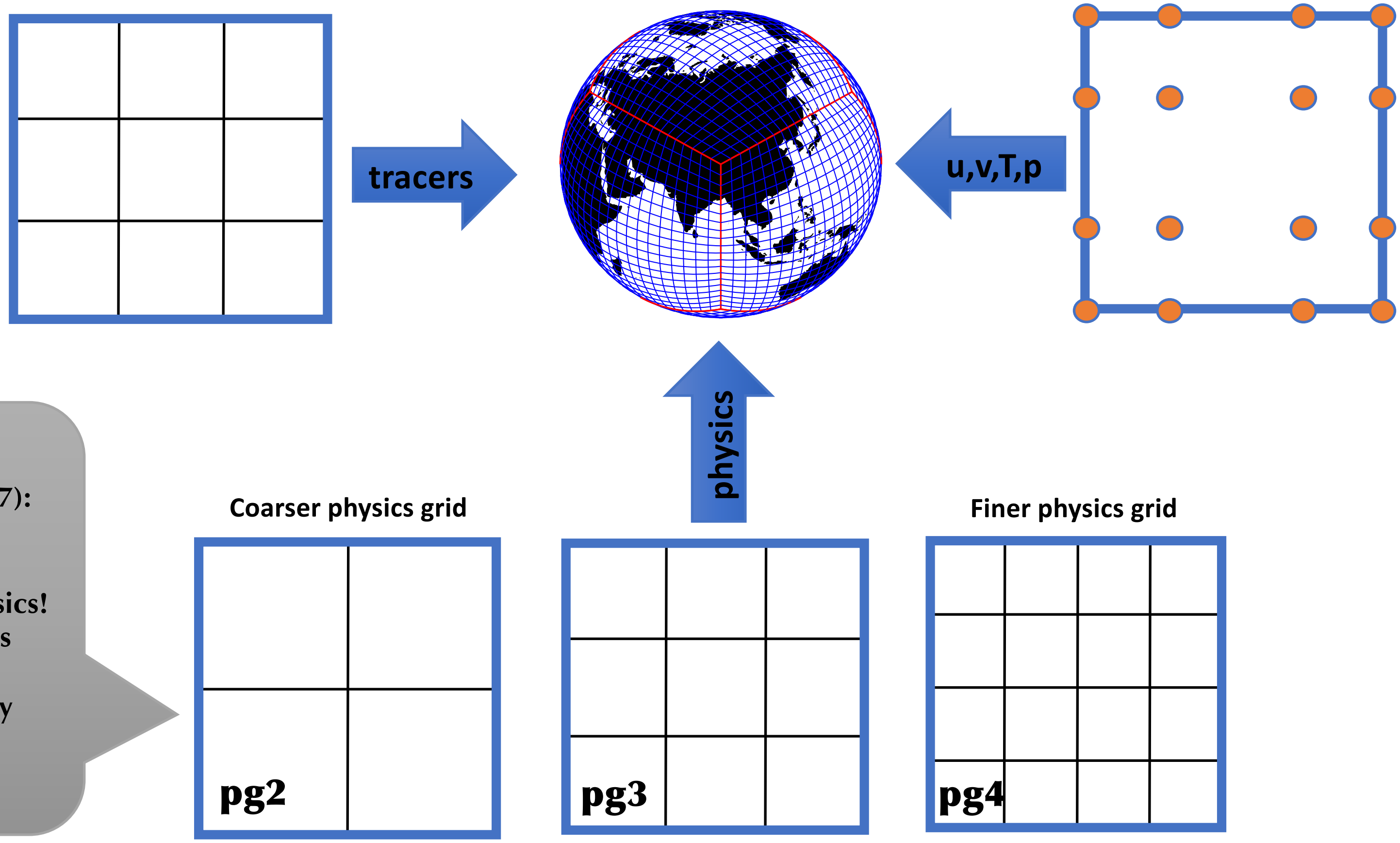
CAM6 release physics



AMIP simulation

CAM6 release physics

CAM-SE-CSLAM: varying physics grid resolution



- Lander and Hoskins (1997): only pass "believable" scales to physics!
- 4 physics cells instead of 9 => effectively speed-up of physics

NCAR
NATIONAL CENTER FOR ATMOSPHERIC RESEARCH

pel@ucar.edu
http://www.cgd.ucar.edu/cms/pel/index.html