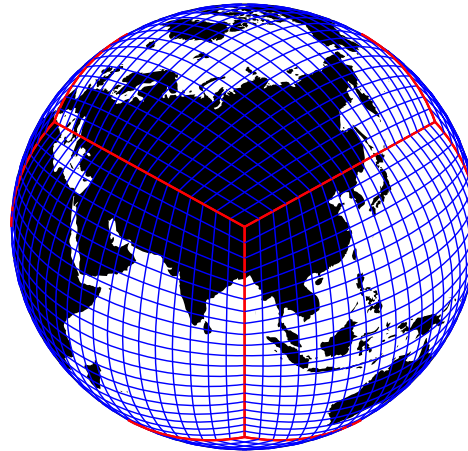


The challenge of Energy budget closure in Earth system models



Peter Hjort Lauritzen

National Center for Atmospheric Research, Boulder, Colorado

“The Earth’s Energy Imbalance and its implications”, 13 – 16 November 2018, Toulouse, France

Research question

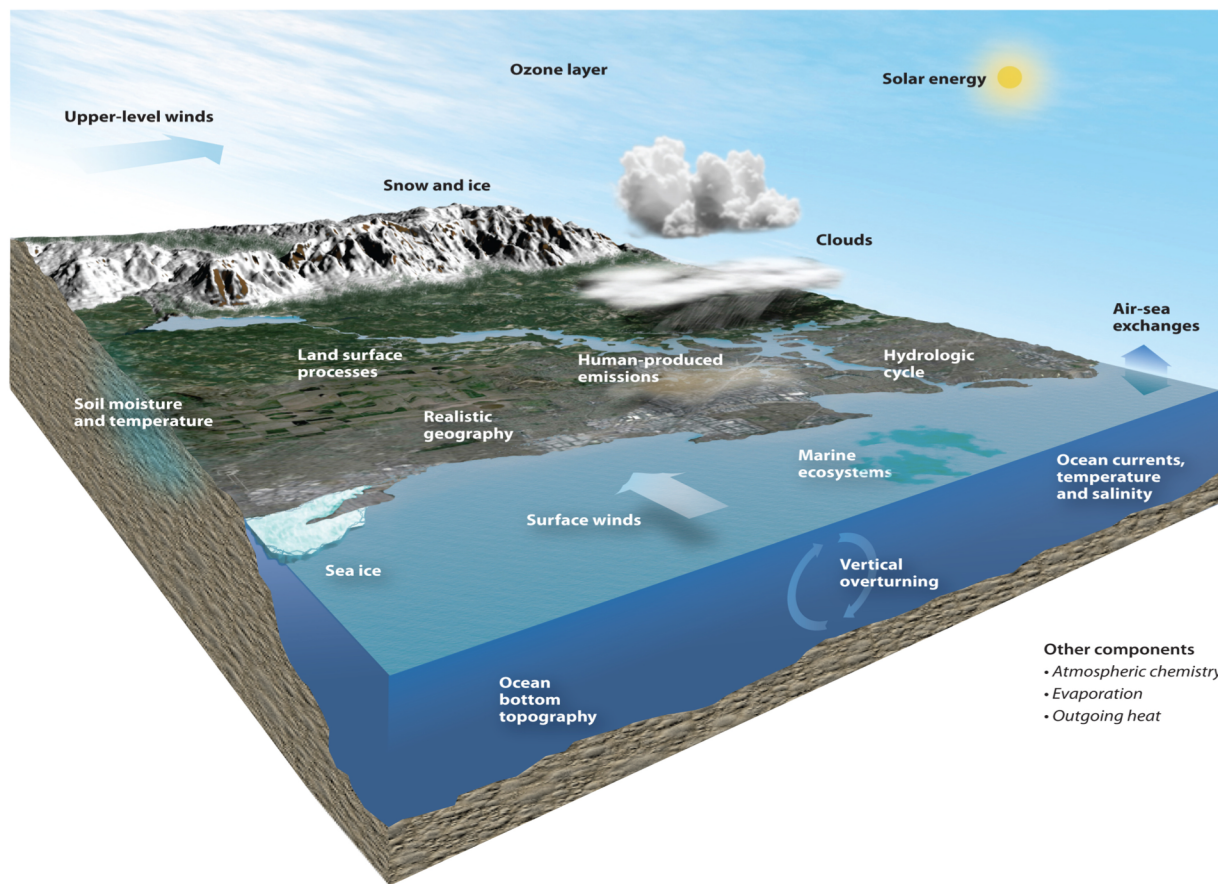
How large are the spurious total energy sources/sinks in an atmosphere model and where are they coming from?

WARNING: Total energy in Earth system models is ...



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Setting the stage: NCAR's CESM (Community Earth System Model)



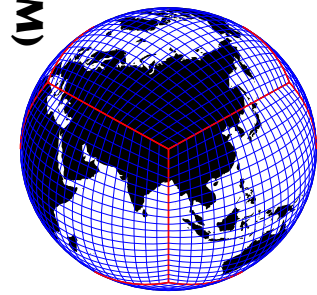
Community Atmosphere Model (CAM)

- Other components
- Atmospheric chemistry
 - Evaporation
 - Outgoing heat

CAM-FV: 1 degree
CMIP6 model



CAM-SE (Spectral Elements):
¼ degree CMIP6 model with
mesh-refinement capability



Climate model setup: dynamics, physics, physics-dynamics coupling

Dynamical core module

$$\frac{\partial \vec{u}}{\partial t} + (\zeta + f) \hat{k} \times \vec{u} + \nabla \cdot \left(\frac{1}{2} \vec{u}^2 + \Phi \right) + \frac{1}{\rho} \nabla p = \nu \nabla^4 \vec{u},$$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T - \frac{1}{c_p \rho} \omega = \nu \nabla^4 T,$$

$$\frac{\partial}{\partial t} \left(\frac{\partial p_d}{\partial \eta} \right) + \nabla \cdot \left(\frac{\partial p_d}{\partial \eta} \vec{u} \right) = \nu \nabla^4 \left(\frac{\partial p_d}{\partial \eta} \right),$$

$$\frac{\partial}{\partial t} \left(\frac{\partial p_d}{\partial \eta} m_i \right) + \nabla \cdot \left(\frac{\partial p_d}{\partial \eta} m_i \vec{u} \right) = \nu \nabla^4 (m_i), \quad i = v, cl, ci, \dots$$

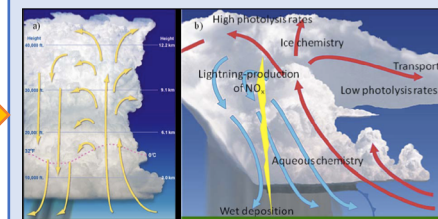
Approximates the solution to the adiabatic equations of motion (“resolved” scales):

- Momentum (u, v)
- Thermodynamic equation (T)
- Continuity equation for air (p)
- Continuity equation for
 - forms of water (water vapor, cloud liquid, cloud ice, rain, ...)
 - quantities needed to represent aerosols
 - chemical species

Physics-dynamics coupling layer

Climate/weather models usually use low-order coupling (Euler forward time-stepping)

Physics (parameterization) module



Roughly speaking, processes that can not be resolved on model grid (hence physics is also referred to as sub-grid-scale processes):

Radiation

Boundary layer turbulence

Sub-grid-scale orographic drag

Shallow and deep convection

Microphysics

Aerosol processes

Vertical mixing

...

Climate model setup: dynamics, physics, physics-dynamics coupling

Dynamical core module

$$\frac{\partial \vec{u}}{\partial t} + (\zeta + f) \hat{k} \times \vec{u} + \nabla \cdot \left(\frac{1}{2} \vec{u}^2 + \Phi \right) + \frac{1}{\rho} \nabla p = \nu \nabla^4 \vec{u},$$

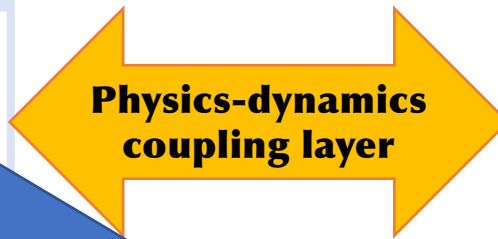
$$\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T - \frac{1}{c_p \rho} \omega = \nu \nabla^4 T,$$

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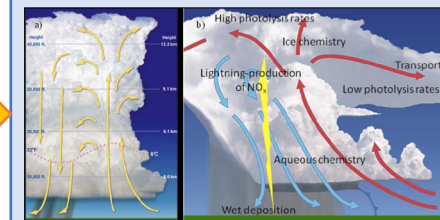
$$\frac{\partial}{\partial t} \left(\frac{\partial p_d}{\partial \eta} m_i \right) + \nabla \cdot \left(\frac{\partial p_d}{\partial \eta} m_i \vec{u} \right) = \nu \nabla^4 (m_i), \quad i =$$

Approximates the solution to the adiabatic equations of motion (at grid scales):

- Momentum (u, v)
- Thermodynamic equation
- Continuity equation for air
- Continuity equation for
 - forms of water (water vapor, cloud liquid, cloud ice, rain)
 - quantities needed to represent
 - chemical species



Physics (parameterization) module



processes that can be resolved on a model grid (these are also referred to as resolved processes):

sub-grid scale processes:

- Turbulence
- Orographic drag
- Convection

Frictional heating

If a hyperviscosity term or some other diffusion is added to the momentum equations, then one can diagnose the local energy dissipation from such damping and add a corresponding heating to balance it.

Climate model setup: dynamics, physics, physics-dynamics coupling

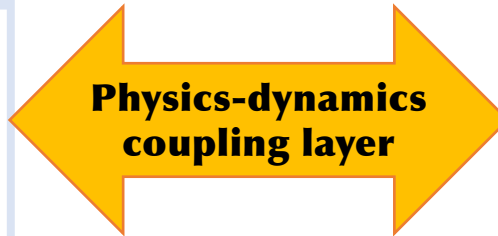
Dynamical core module

$$\frac{\partial \vec{u}}{\partial t} + (\zeta + f) \hat{k} \times \vec{u} + \nabla \cdot \left(\frac{1}{2} \vec{u}^2 + \Phi \right) + \frac{1}{\rho} \nabla p = \nu \nabla^4 \vec{u},$$

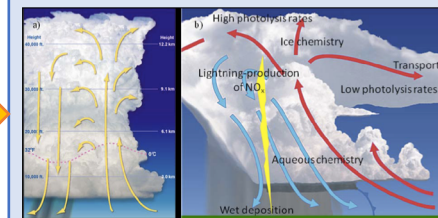
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Physics (parameterization) module



Mimetic discretizations

There are examples of numerical discretizations of the adiabatic frictionless equations motion that are designed so that total energy is conserved (in the absence of time-truncation and filtering errors)

models
order
forward

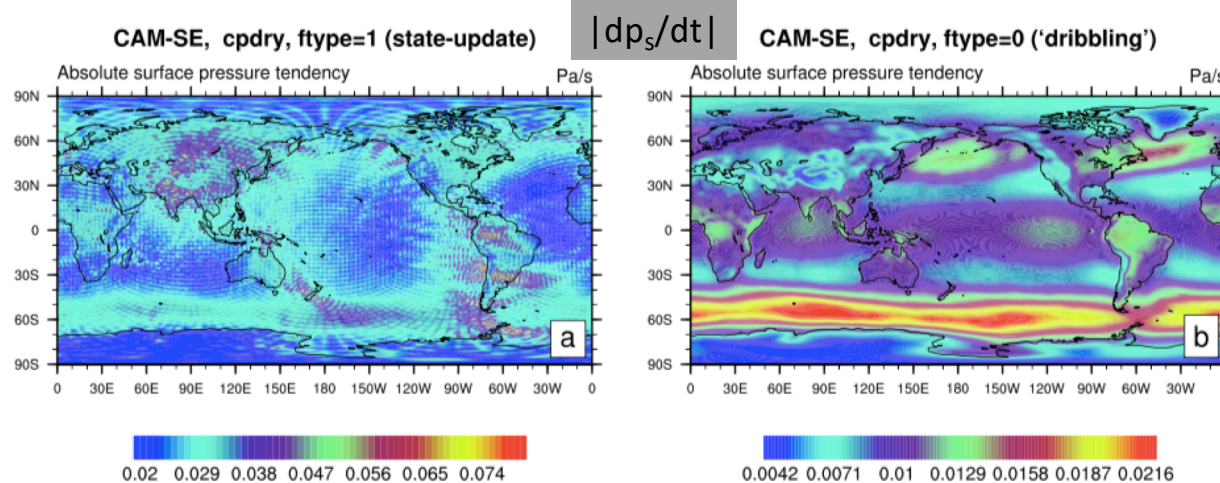
Roughly speaking, processes that can not be resolved on model grid (hence physics is also referred to as sub-grid-scale processes):

- Radiation
- Boundary layer turbulence
- Sub-grid-scale orographic drag
- Shallow and deep convection
- Microphysics
- Aerosol processes
- Vertical mixing

...

Physics-dynamics coupling in CAM

**Advance dynamical core
(30 minutes)**



**Compute physics
tendencies based
on dynamics
updated state**

**Dynamical core is responsible for
updating dynamics state with
physics tendencies:**

- **State-up-dating (ftype=1)**
- **“Dribbling” (ftype=0)**

Total energy (TE) equation

- dry atmosphere (height coordinates)

In the following it is assumed that the model top and bottom are coordinate surfaces and that there is no flux of mass through the model top and bottom. In a dry atmosphere the TE equation integrated over the entire sphere is given by

$$\frac{d}{dt} \int_{z=z_s}^{z=z_{top}} \iint_S E_v \rho^{(d)} dA dz = \int_{z=z_s}^{z=z_{top}} \iint_S F_{net} \rho^{(d)} dA dz,$$

[e.g., Kasahara, 1974] where F_{net} is net flux calculated by the parameterizations (e.g., heating and momentum forcing), d/dt the total/material derivative, z_s is the height of the surface, S the sphere, $\rho^{(d)}$ the density of dry air, E_v is the TE and dA is an infinitesimal area on the sphere. E_v can be split into kinetic energy $K = \frac{1}{2} \mathbf{v}^2$ (\mathbf{v} is the wind vector), internal energy $c_v^{(d)} T$, where $c_v^{(d)}$ is the heat capacity of dry air at constant volume, and potential energy $\Phi = gz$

$$E_v = K + c_v^{(d)} T + \Phi.$$

Calculated by
parameterizations (e.g.,
heating, momentum forcing)

Total energy (TE) equation

- dry atmosphere (mass coordinates)

If the vertical integral is performed in a mass-based vertical coordinate, e.g., pressure, then the integrated TE equation for a dry atmosphere can be written as

$$\frac{d}{dt} \int_{p=p_s}^{p=p_{top}} \iint_S E_p \rho^{(d)} dA dp + \frac{d}{dt} \iint_S \Phi_s p_s dA = \int_{p=p_s}^{p=p_{top}} \iint_S F_{net} \rho dA dp,$$

[e.g., Kasahara, 1974] where

$$E_p = K + c_p^{(d)} T.$$

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[e.g., Kasahara, 1974] where

$$E_p = K + c_p^{(d)} T.$$

In a moist atmosphere, however, there are several definitions of TE used in the literature related to what heat capacity is used for water vapor and whether or not condensates are accounted for in the energy equation. To explain the details of that we focus on the energy equation for CAM-SE.

CAM-SE = Community Atmosphere Model – Spectral Element dynamical core (Lauritzen et al., 2018)

Total energy (TE) equation

- moist atmosphere (mass coordinates)

Define the dry mixing ratios for the water variables (vapor 'wv', cloud liquid 'cl', cloud ice 'ci', rain 'rn' and snow 'sw')

$$m^{(\ell)} \equiv \frac{\rho^{(\ell)}}{\rho^{(d)}}, \text{ where } \ell = \text{'wv', 'cl', 'ci', 'rn', 'sw'},$$

where $\rho^{(d)}$ is the mass of dry air per unit volume of moist air and $\rho^{(\ell)}$ is the mass of the water substance of type ℓ per unit volume of moist air.

The density of a unit volume of moist air is related to the dry air density through

$$\rho = \rho^{(d)} \left(\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} \right).$$

$$\mathcal{L}_{all} = \{ 'd', 'wv', 'cl', 'ci', 'rn', 'sw' \}$$

SI unit for density: kg/m³

Total energy (TE) equation

- moist atmosphere (mass coordinates)

Dry versus specific ('wet') mixing ratios

$$m^{(\ell)} \equiv \frac{\rho^{(\ell)}}{\rho^{(d)}}$$

$$q^{(\ell)} \equiv \frac{\rho^{(\ell)}}{\rho}$$

Similarly for dry versus moist mass vertical coordinates

SI unit for density: kg/m³

$$\mathcal{L}_{all} = \{ 'd', 'wv', 'cl', 'ci', 'rn', 'sw' \}$$

Total energy (TE) equation

- moist atmosphere (and per unit area)

$$\frac{d\widehat{E}}{dt} = \widehat{F}_{net},$$

where

$$\frac{d\widehat{E}}{dt} = \frac{d}{dt} \left\{ \frac{1}{\Delta S} \int_{\eta=0}^{\eta=1} \iint_S \left(\frac{1}{g} \frac{\partial M^{(d)}}{\partial \eta^{(d)}} \right) \sum_{\ell \in \mathcal{L}_{all}} \left[m^{(\ell)} (K + c_p^{(\ell)} T + \Phi_s) \right] dA d\eta^{(d)} \right\},$$

and

$$\widehat{F}_{net} = \frac{1}{\Delta S} \int_{\eta=0}^{\eta=1} \iint_S \left(\frac{1}{g} \frac{\partial M^{(d)}}{\partial \eta^{(d)}} \right) \sum_{\ell \in \mathcal{L}_{all}} \left[m^{(\ell)} \right] F_{net} dA d\eta^{(d)}.$$

where ΔS is the surface area of the sphere, Φ_s is the surface geopotential and $\widehat{(\cdot)}$ refers to the global average.

Total energy (TE) equation

- moist atmosphere (and per unit area)

$$\frac{d\hat{E}}{dt} = \hat{F}_{net},$$

Dynamical core (although it redistributes energy locally) should conserve total energy globally:

$$\frac{d\hat{E}}{dt} = 0$$

Total energy (TE) equation

- moist atmosphere (and per unit area)

$$\frac{d\hat{E}}{dt} = \hat{F}_{net},$$

Conserving total energy to within $\sim 0.01 \text{ W/m}^2$ is considered “good enough” for coupled climate modeling (Boville, 2000; Williamson et al., 2015)

$$\frac{d\hat{E}}{dt} \leq 0.01 \text{ W/m}^2$$

Total energy (TE) equation

- moist atmosphere (and per unit area)

$$\frac{d\hat{E}}{dt} = \hat{F}_{net},$$

Column physics: TE change in column should be balanced by fluxes in/out of the top and bottom

$$\frac{d\hat{E}}{dt} = \frac{1}{\Delta S} \iint_S (p_{top} F_{net} - p_s F_{net}) dA.$$

Potential spurious sources/sinks of total energy in an atmosphere model:



- **Parameterization errors:** Individual parameterizations may not have a closed energy budget. CAM parameterizations are required to have a closed energy budget under the assumption that pressure remains constant during the computation of the subgrid-scale parameterization tendencies. In other words, the TE change in the column is exactly balanced by the net sources/sinks given by the fluxes through the column.
- **Pressure work:** That said, if parameterizations update specific humidity then the surface pressure changes (e.g., moisture entering or leaving the column). In that case the pressure changes which, in turn, changes TE. This is referred to as pressure work [section 3.1.8 in Neale et al., 2012].
- **Continuous TE formula discrepancy:** If the continuous equations of motion for the dynamical core conserve a TE different from the one used in the parameterizations then an energy inconsistency is present in the system as a whole. In CAM this mismatch arose from the evolutionary nature of the model development and not by deliberate design; and should be eliminated in the future.
- **Dynamical core errors:** Energy conservation errors in the dynamical core, not related to physics-dynamics coupling errors, can arise in multiple parts of the algorithms used to solve the equations of motion.
- **Physics-dynamics coupling (PDC):** Assume that physics computes a tendency. Usually the tendency (forcing) is passed to the dynamical core which is responsible for adding the tendencies to the state.

Potential spurious sources/sinks of total energy in an atmosphere model:

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CAM-SE dynamical core

$$\hat{E} = \frac{1}{\Delta S} \int_{\eta=0}^{\eta=1} \iint_S \left(\frac{1}{g} \frac{\partial M^{(d)}}{\partial \eta^{(d)}} \right) \sum_{\ell \in \mathcal{L}_{all}} [m^{(\ell)} (K + c_p^{(\ell)} T + \Phi_s)] dA d\eta^{(d)}$$

CAM physics

$$\hat{E}_{phys} = \frac{1}{\Delta S} \int_{\eta=0}^{\eta=1} \iint_S \left(\frac{1}{g} \frac{\partial M^{(d)}}{\partial \eta^{(d)}} \right) (1 + m^{(wv)}) [(K + c_p^{(d)} T + \Phi_s)] dA d\eta^{(d)}$$

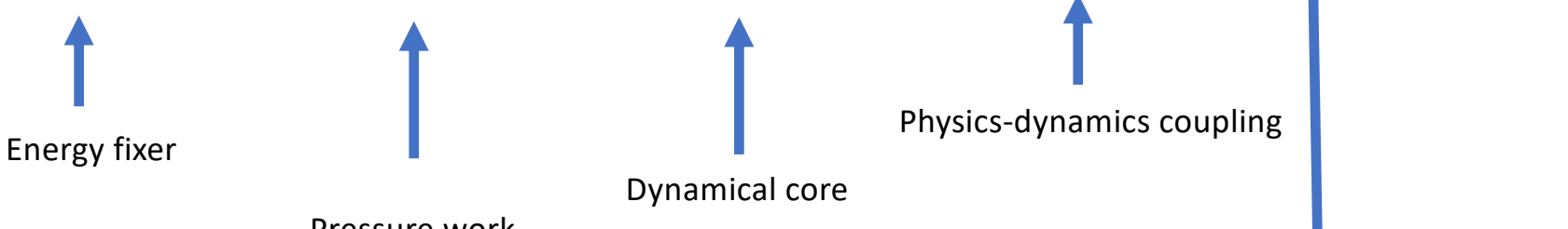
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Fixing spurious sources/sinks of total energy in an atmosphere model:

- **Compensating Energy fixers:** To avoid TE conservation errors which could accumulate and ultimately lead to a climate drift, it is customary to apply an arbitrary energy fixer to restore TE conservation. Since the spatial distribution of many energy errors, in general, is not known, global fixers are used. In CAM a uniform increment is added to the temperature field to compensate for TE imbalance from all processes, i.e. dynamical core, physics-dynamics coupling, TE formula discrepancy, energy change due to pressure work, and possibly parameterization errors if present.

$$-\partial \widehat{E}_{phys}^{(efix)} = \partial \widehat{E}_{phys}^{(pwork)} + \partial \widehat{E}_{dyn}^{(adiab)} + \partial \widehat{E}^{(pdc)} + \partial \widehat{E}^{(discr)}$$


Energy fixer

Pressure work

Dynamical core

Physics-dynamics coupling

Continuous TE formula discrepancy

```
do nt=1,ntotal
```

```
PARAMETERIZATIONS:
```

```
Last dynamics state received from dynamics
```

```
output 'pBF'
```

```
Energy fixer
```

```
output 'pBP'
```

```
Physics updates the state and state saved for energy fixer
```

```
output 'pAP'
```

```
Pressure work (dry mass correction)
```

```
output 'pAM'
```

```
Physics tendency (forcing) passed to dynamics
```

```
DYNAMICAL CORE
```

```
output 'dED'
```

```
do ns=1,nsplit
```

```
output 'dAF'
```

```
START PHYSICS-DYNAMICS COUPLING
```

```
Update dynamics state with (1/nsplit) of physics tendency (ftype=2)
```

```
if (ns=1) Update dynamics state with entire physics tendency (ftype=1)
```

```
DONE PHYSICS-DYNAMICS COUPLING
```

```
output 'dBD'
```

```
do nr=1,rsplit
```

```
Advance the adiabatic frictionless equations of motion  
in floating Lagrangian layer
```

```
do ns=1,hypervis_subcycle
```

```
output 'dBH'
```

```
Apply hyperviscosity operators
```

```
output 'dCH'
```

```
Add frictional heating to temperature
```

```
output 'dAH'
```

```
end do (ns=1,hypervis_subcycle)
```

```
end do (nr=1,rsplit)
```

```
output 'dAD'
```

```
Vertical remapping from floating Lagrangian levels to Eulerian levels
```

```
output 'dAR'
```

```
end do (ns=1,nsplit)
```

```
Dynamics state saved for next model time step and passed to physics
```

```
output 'dBF'
```

```
end do (nt=1,ntotal)
```

Diagnosing TE errors:

Implemented using CAM history infrastructure by computing column integrals of energy at various places in CAM and outputting the 2D energy fields. CAM history internally handles accumulation and averaging in time at each horizontal grid point.

(Lauritzen and Williamson., 2019)

Spurious sources/sinks of total energy in atmosphere model:

- **Parameterization errors:** Individual CAM parameterizations are required so that pressure remains constant during integration. In other words, the TE change in fluxes through the column.
- **Pressure work:** That said, if parameterizations (e.g., moisture entering or leaving) are not consistent. This is referred to as pressure work.
- **Continuous TE formula discrepancy:** A TE different from the one used in the model as a whole. In CAM this mismatch is a design; and should be eliminated.
- **Dynamical core errors:** Energy errors, can arise in multiple parts of the model.
- **Physics-dynamics coupling (PDC) errors:** Energy errors passed to the dynamical core when

Budget closed in CAM 😊

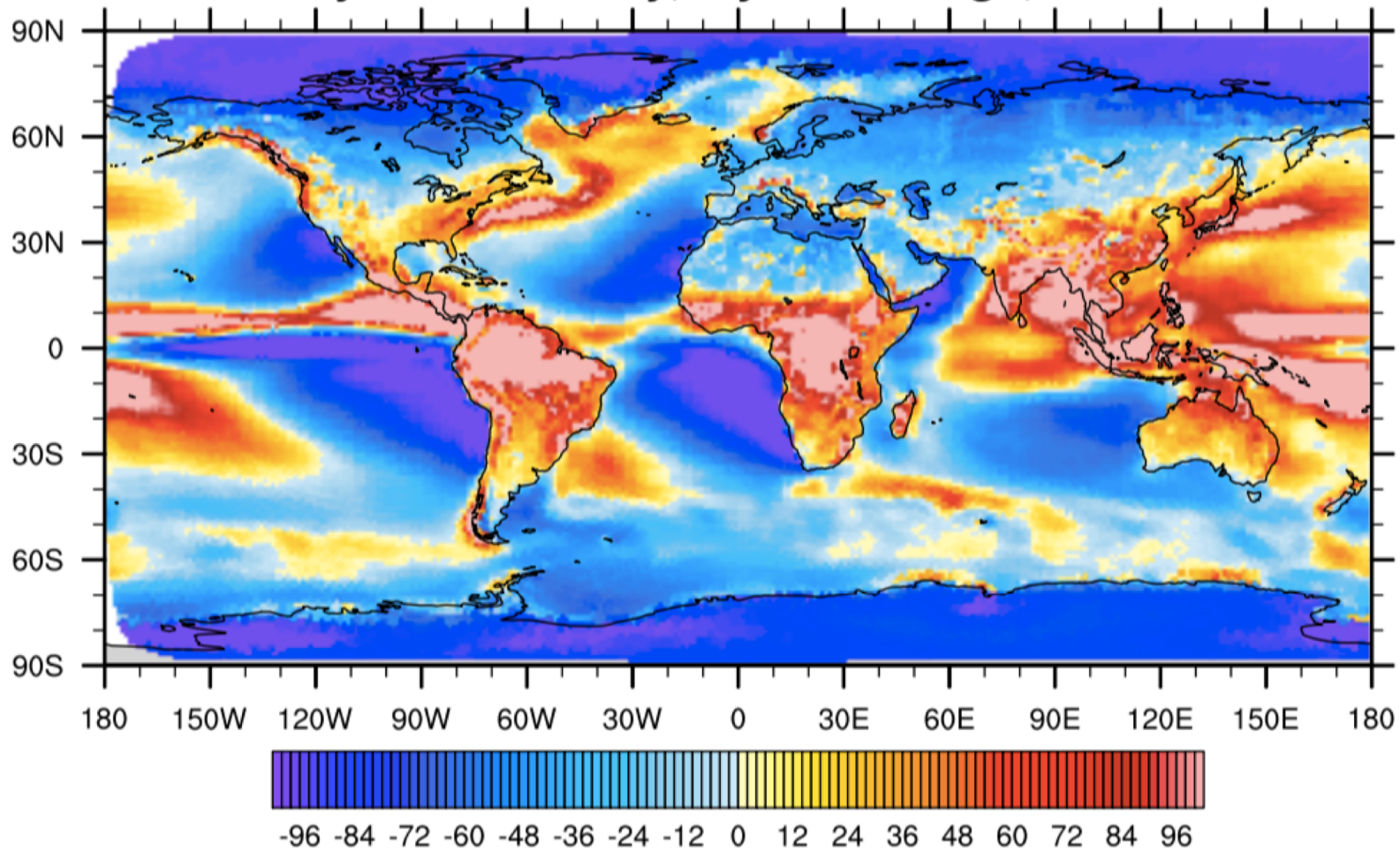
Pressure work: $\sim 0.3 \text{ W/m}^2$

TE formula discr.: $\sim 0.6 \text{ W/m}^2$

**Dycore: $\sim -0.6 \text{ W/m}^2$ (decreases to -0.3 W/m^2 with smoother topography)
(frictional heating is $\sim -0.6 \text{ W/m}^2$)**

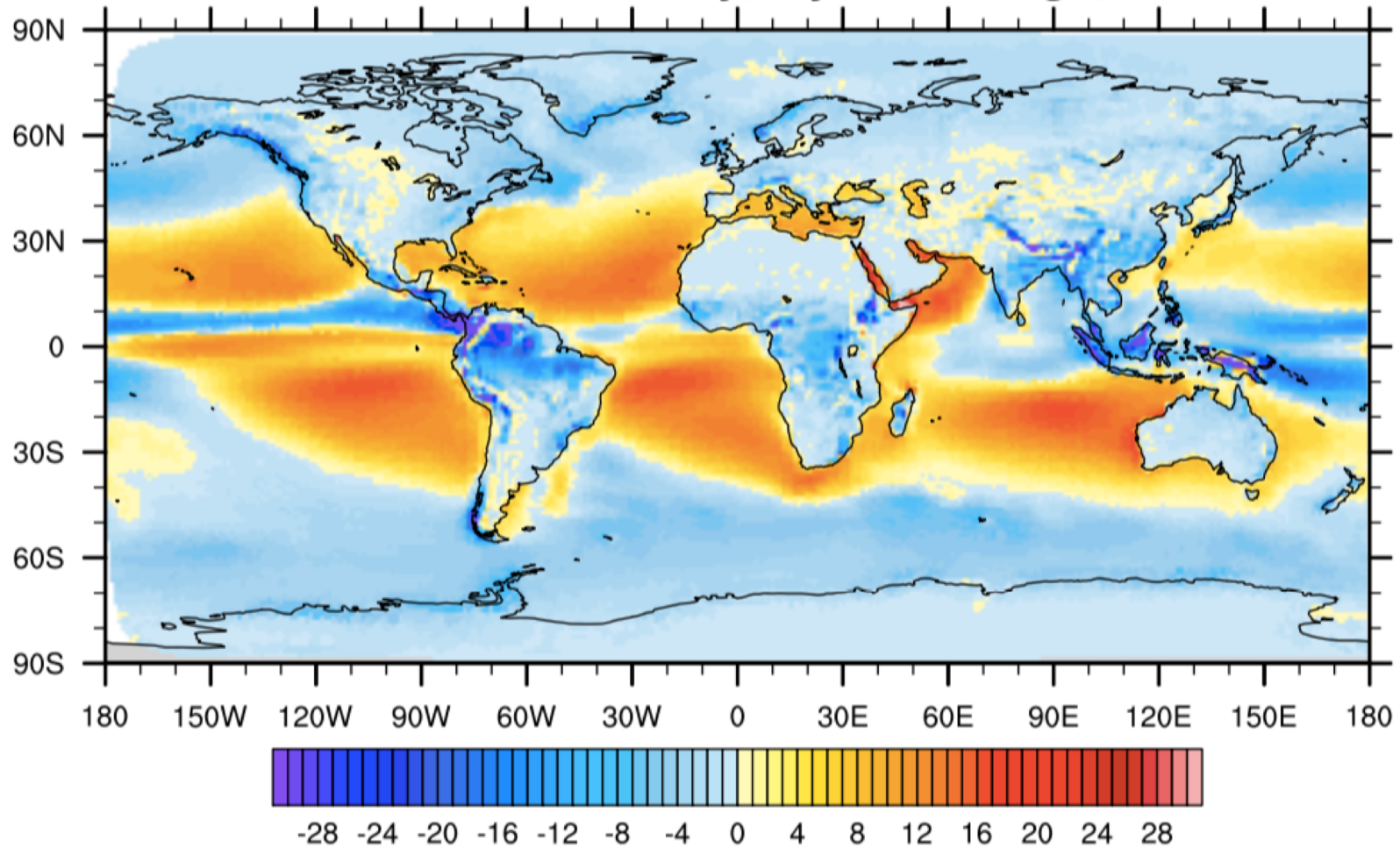
PDC errors (“dribbling”): $\sim 0.5 \text{ W/m}^2$
(Lauritzen and Williamson., 2019)

Physics tendency, 1 year average, W/m^2



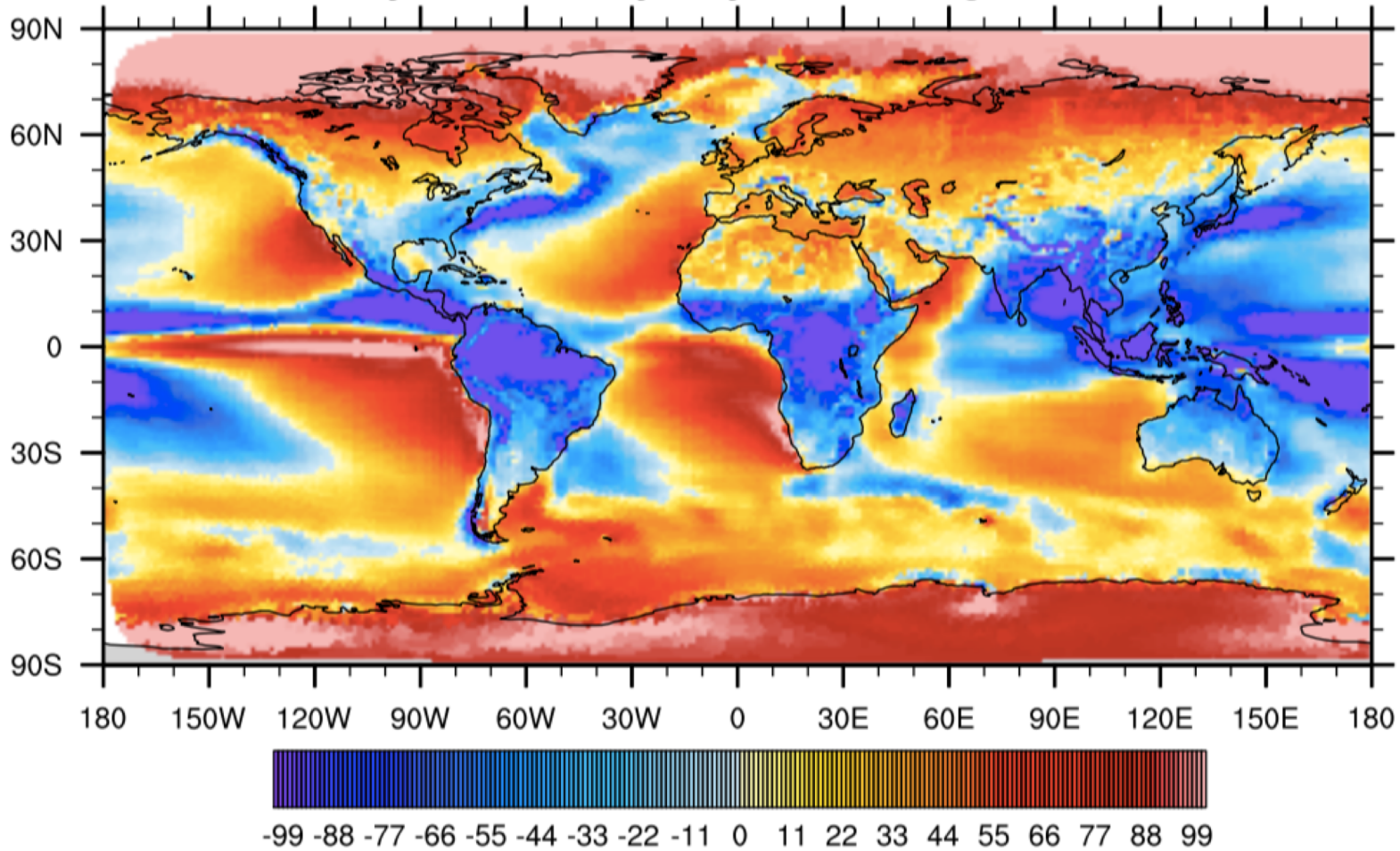
NOT SPURIOUS

Pressure work tendency, 1 year average, W/m^2



SPURIOUS in each column

2D dyn tendency, 1 year average, W/m²



Not SPURIOUS locally
- should integrate to 0

Summary

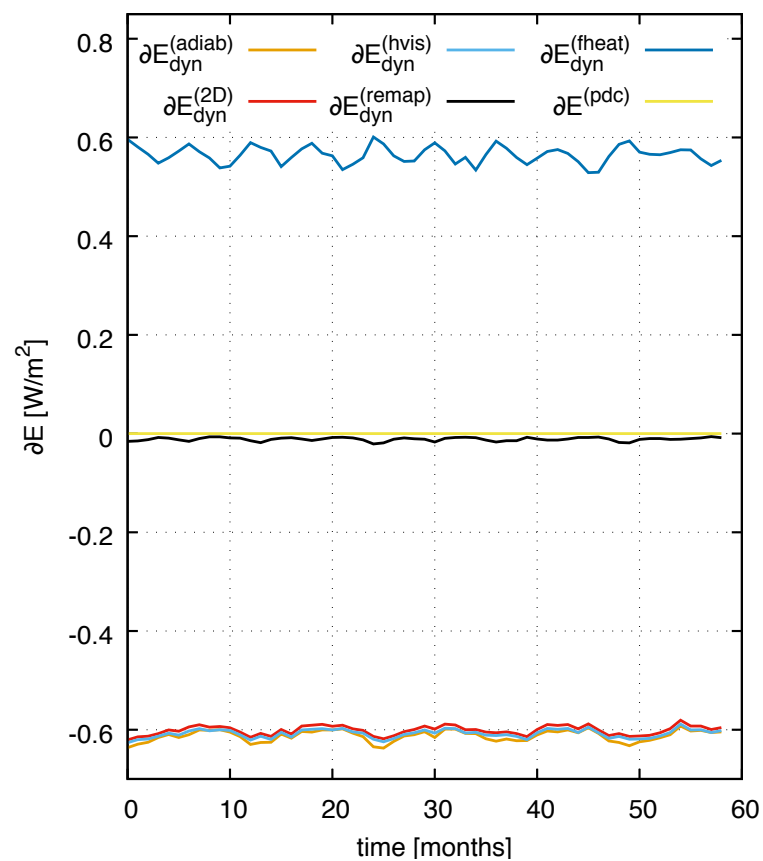
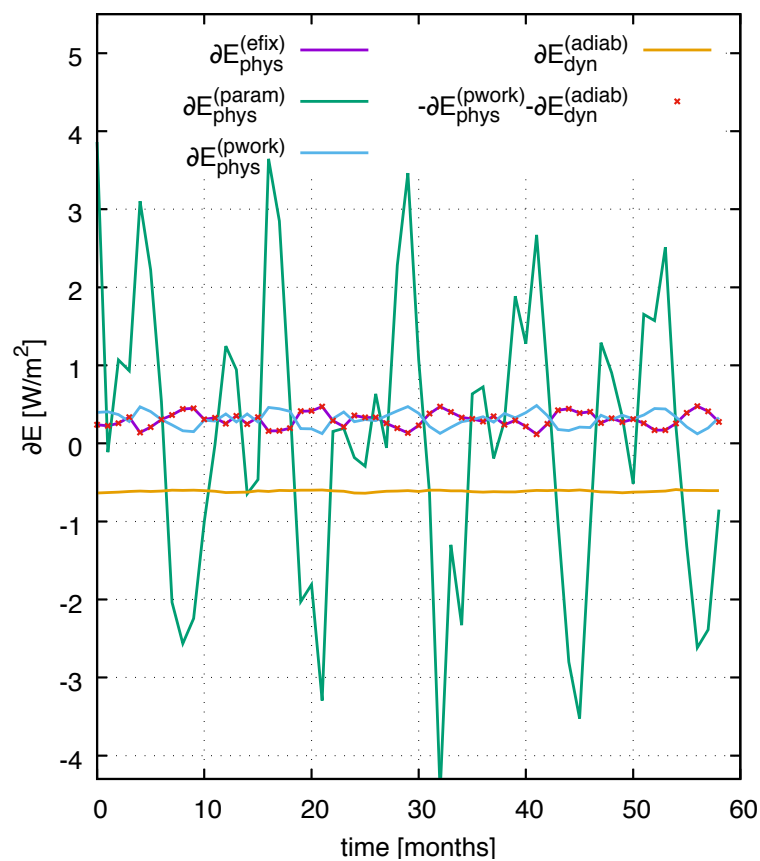
- Total energy errors in numerical discretizations (dynamical core), physics-dynamics coupling and pressure work are $\sim -0.6 - 0.3 \text{ W/m}^2$
- Local errors can be an order of magnitude larger (at least)!

Outlook

- In next-generation models we should consider formulating physics in dry pressure coordinates (so that coordinate surfaces stay fixed during physics updates)
- Can we close the total energy budget locally in models?
- Integrating weather-climate models: parameterizations for weather models are, in general, not formulated to have a closed TE budget. Major challenge?



F2000climo, CAM-SE, lcpdry, ftype=1



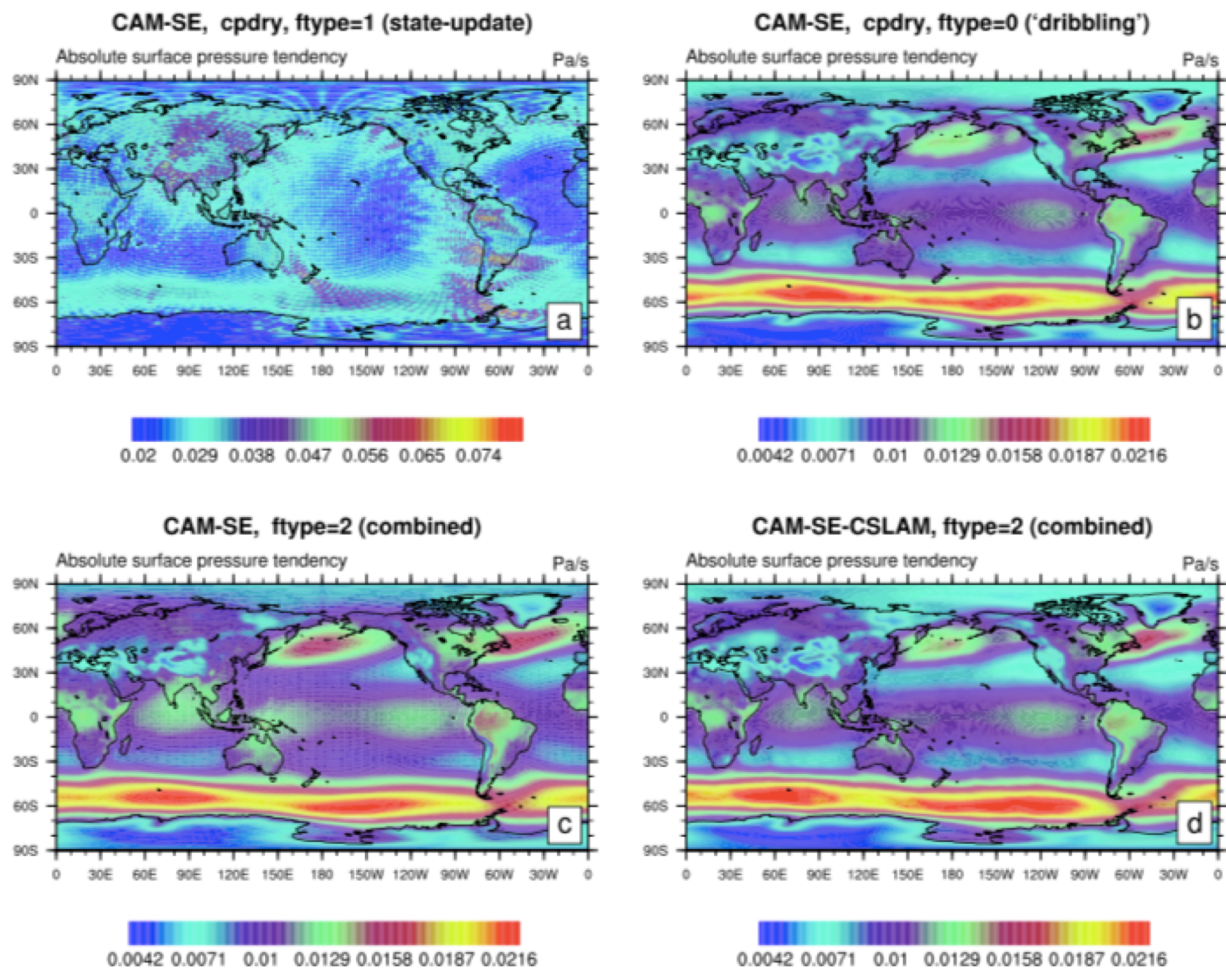


Figure 3. One year average of the absolute surface pressure tendency for (a) the TE consistent configuration, (b) 'dribbling' physics-dynamics coupling, (c) ftype=2 physics-dynamics coupling and (d) CSLAM version of CAM-SE, respectively. (a) has a closed physics-dynamics coupling budget but spurious noise, (b) has no spurious noise but the mass-budget in physics-dynamics coupling is not closed (see Figure 6), (c) has a closed mass budget in physics-dynamics coupling but some spurious noise at element boundaries which is eliminated when using CAM-SE-CSLAM (d). Note, the smallest value in panel (a) is the largest in panels (b), (c) and (d).

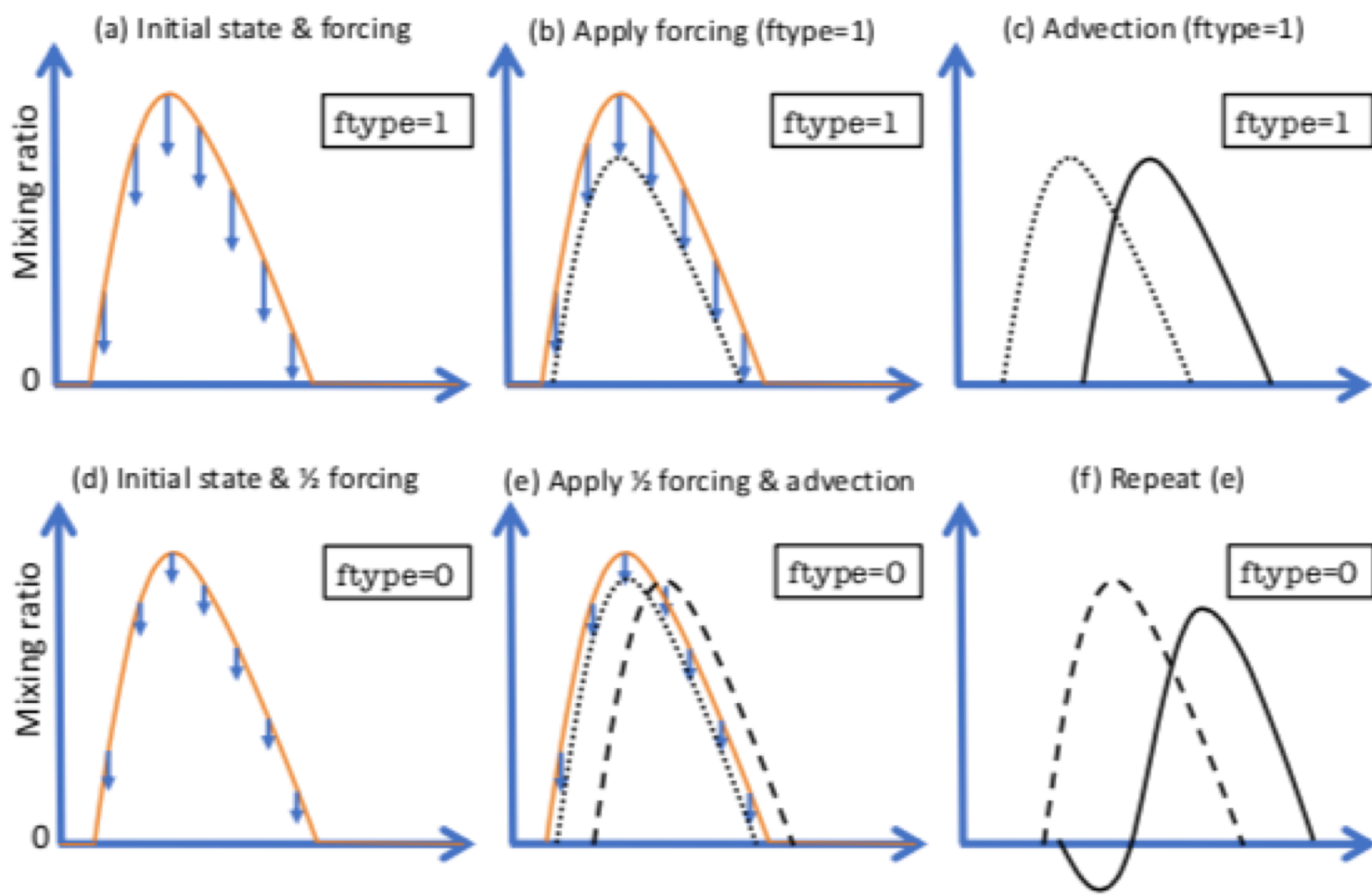


Figure 5. A schematic of state-update ($f_{type}=1$; row 1) and 'dribbling' ($f_{type}=0$; row 2) physics-dynamics coupling algorithms. See Section 3.2 for details.

F2000climo, CAM-SE, cpdry, ftype=0 ('dribbling')

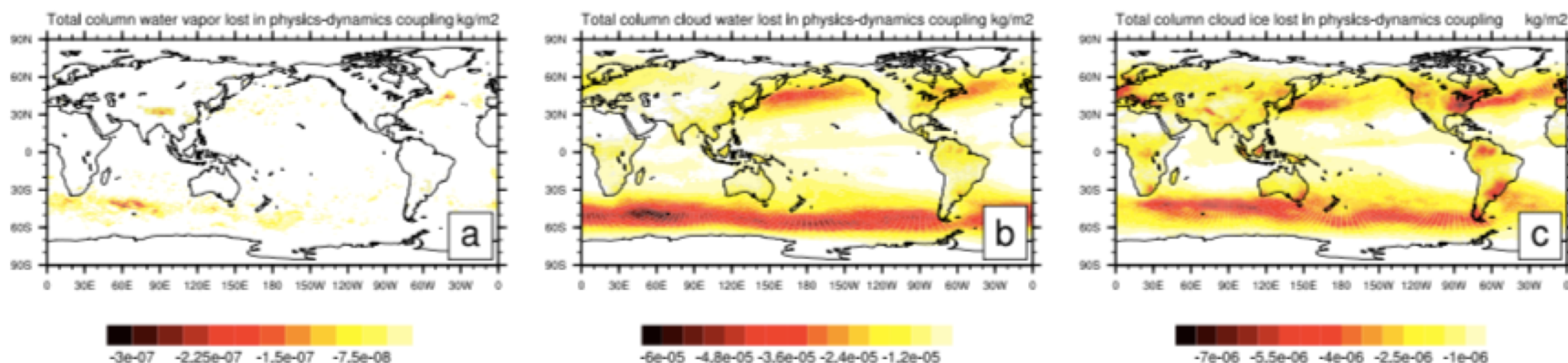


Figure 6. One year average of mass [kg/m^2] 'clipped' in physics-dynamics coupling (so that state is not driven negative) when using $f_{type}=0$ ('dribbling') physics-dynamics coupling for (a) water vapor, (b) cloud liquid and (c) cloud ice, respectively. Interestingly the element boundaries systematically show in the plots which is likely related to the anisotropy of the quadrature grid [Herrington *et al.*, 2018].