CAM-SE: Lecture I

Peter Hjort Lauritzen

Atmospheric Modeling and Predictability Section
Climate and Global Dynamics Laboratory
National Center for Atmospheric Research

2nd WCRP Summer School on Climate Model Development: Scale aware parameterization for representing sub-grid scale processes
January 23, 2018
National Institute for Space Research, Center for Weather Forecasting and Climate Studies, Cachoeira Paulista,
São Paulo, Brazil.

JAMES
JOURNAL OF ADVANCES IN MODELING EARTH SYSTEMS

NCAR CESM2.0 release of CAM-SE: A reformulation of the spectral-element dynamical core in dry-mass vertical coordinates with comprehensive treatment of condensates and energy

P.H. Lauritzen[1*], R.D. Nair[1], A.R. Herrington[2], P. Callaghan[1], S. Goldhaber[1], J.M. Dennis[1], J.T. Bacmeister[1], B.E. Eaton[1], C.M. Zarzycki[1], Mark A. Taylor[3], P.A. Ullrich[4], T. Dubos[5], A. Gettelman[1], R.B. Neale[1], B. Dobbins[1], K.A. Reed[2], C. Hannay[1], B. Medeiros[1], J.J. Benedict and J.J. Tribbia[1]

[1]National Center for Atmospheric Research, Boulder, Colorado, USA
[2]School of Marine and Atmospheric Sciences, Stony Brook University, State University of New York, Stony Brook, New York
[3]Sandia National Laboratories, Albuquerque, New Mexico, USA
[4]Department of Land, Air and Water Resources, University of California, Davis, California, USA
[5]Ecole Polytechnique, UMR 8539, Laboratoire de Météorologie Dynamique/IPSL, Palaiseau, France
[6]Rosenstiel School of Marine and Atmospheric Science, University of Miami, Miami, Florida, USA

CAM-HOMME and CAM-SE

- HOMME = HOMME (High-Order Method Modeling Environment)

  CAM used to pull the spectral-element (SE) dynamical core from a separate repository called HOMME

- Now the SE dycore is residing in the CAM repository and we have made numerous modifications (refer to this model version as CAM-SE):

  - science changes: rigorous treatment of condensates and associated energies, capability to separate physics and dynamics grid, finite-volume advection
  - code optimization
  - massive code clean-up
  - code will be released with CESM2.0 and scientifically supported with CESM2.1 (will be used for high-res CMIP6 simulations)

C… -SE

In this talk I will focus mainly on the continuous equations

- **HOMME … ironment)**

  CAM use… re from a separate repositor…

- **Now the SE dycore is re… in the CAM repository and we have made numerous modifications (re… to this model version as CAM-SE):**

  - science changes: rigorous treatment of condensates and associated energies, capability to separate physics and dynamics grid, finite-volume advection
  - code optimization
  - massive code clean-up
  - code will be released with CESM2.0 and scientifically supported with CESM2.1 (will be used for high-res CMIP6 simulations)

C… -SE

Next talk is specifically about the spectral-element discretization

- HOMME … ironment)

  CAM use… re from a separate repositor…

- Now the SE dycore is re… in the CAM repository and we have made numerous modifications (re… to this model version as CAM-SE):

  - science changes: rigorous treatment of condensates and associated energies, capability to separate physics and dynamics grid, finite-volume advection
  - code optimization
  - massive code clean-up
  - code will be released with CESM2.0 and scientifically supported with CESM2.1 (will be used for high-res CMIP6 simulations)

CAM-HO

- HOMME = HOMME (High

  CAM used to pull the spect e
  repository called HOMME

- Now the SE dycore is residing in the CA sitory and we have made numerous modifications (refer to this mod rsion as CAM-SE):

  - science changes: rigorous treatment of condensates and associated energies, capability to separate physics and dynamics grid, finite-volume advection
  - code optimization
  - massive code clean-up
  - code will be released with CESM2.0 and scientifically supported with CESM2.1 (will be used for high-res CMIP6 simulations)

Separate talk (Thursday)

Outline

- **Representation of water phases**
- Ideal gas law and virtual temperature for moist air containing condensates
- Dry-mass floating Lagrangian vertical coordinate
- Adiabatic frictionless equations of motion
- Viscosity
- Conservation properties: Axial angular momentum and total energy

A1. Representation of water phases in terms of dry and wet (specific) mixing ratios

Define the dry mixing ratios for the water variables (vapor 'wv', cloud liquid 'cl', cloud ice 'ci', rain 'rn' and snow 'sw')

$$m^{(\ell)} \equiv \frac{\rho^{(\ell)}}{\rho^{(d)}}, \text{ where } \ell = \text{`}wv\text{`}, \text{`}cl\text{`}, \text{`}ci\text{`}, \text{`}rn\text{`}, \text{`}sw\text{`},$$

where $\rho^{(d)}$ is the mass of dry air per unit volume of moist air and $\rho^{(\ell)}$ is the mass of the water substance of type ℓ per unit volume of moist air.

SI unit for density: kg/m^3

$$\mathcal{L}_{all} = \{\text{`}d\text{`}, \text{`}wv\text{`}, \text{`}cl\text{`}, \text{`}ci\text{`}, \text{`}rn\text{`}, \text{`}sw\text{`}\}$$

$$\mathcal{L}_{water} = \{\text{`}wv\text{`}, \text{`}cl\text{`}, \text{`}ci\text{`}, \text{`}rn\text{`}, \text{`}sw\text{`}\}$$

$$\mathcal{L}_{cond} = \{\text{`}cl\text{`}, \text{`}ci\text{`}, \text{`}rn\text{`}, \text{`}sw\text{`}\}$$

A1. Representation of water phases in terms of dry and wet (specific) mixing ratios

Define the dry mixing ratios for the water variables (vapor 'wv', cloud liquid 'cl', cloud ice 'ci', rain 'rn' and snow 'sw')

$$m^{(\ell)} \equiv \frac{\rho^{(\ell)}}{\rho^{(d)}}, \text{ where } \ell = \text{`}wv\text{`}, \text{`}cl\text{`}, \text{`}ci\text{`}, \text{`}rn\text{`}, \text{`}sw\text{`},$$

where $\rho^{(d)}$ is the mass of dry air per unit volume of moist air and $\rho^{(\ell)}$ is the mass of the water substance of type ℓ per unit volume of moist air.

The density of a unit volume of moist air is related to the dry air density through

$$\rho = \rho^{(d)} \left(\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} \right).$$

SI unit for density: kg/m^3

A1. Representation of water phases in terms of dry and wet (specific) mixing ratios

Mixing ratios can also be specified in terms of density per density of moist air, in other words, specific/moist mixing ratios

$$q^{(\ell)} \equiv \frac{\rho^{(\ell)}}{\rho},$$

It is straight forward to convert between moist and dry mixing ratios

$$m^{(\ell)} = \frac{q^{(\ell)}}{1 - \sum_{\ell \in \mathcal{L}_{water}} q^{(\ell)}},$$

$$q^{(\ell)} = \frac{m^{(\ell)}}{\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)}}.$$

A1. Representation of water phases in terms of dry and wet (specific) mixing ratios

$$m^{(\ell)} \equiv \frac{\rho^{(\ell)}}{\rho^{(d)}} \qquad q^{(\ell)} \equiv \frac{\rho^{(\ell)}}{\rho}$$

What is an important difference between specific mixing ratio and dry mixing ratio?

A1. Representation of water phases in terms of dry and wet (specific) mixing ratios

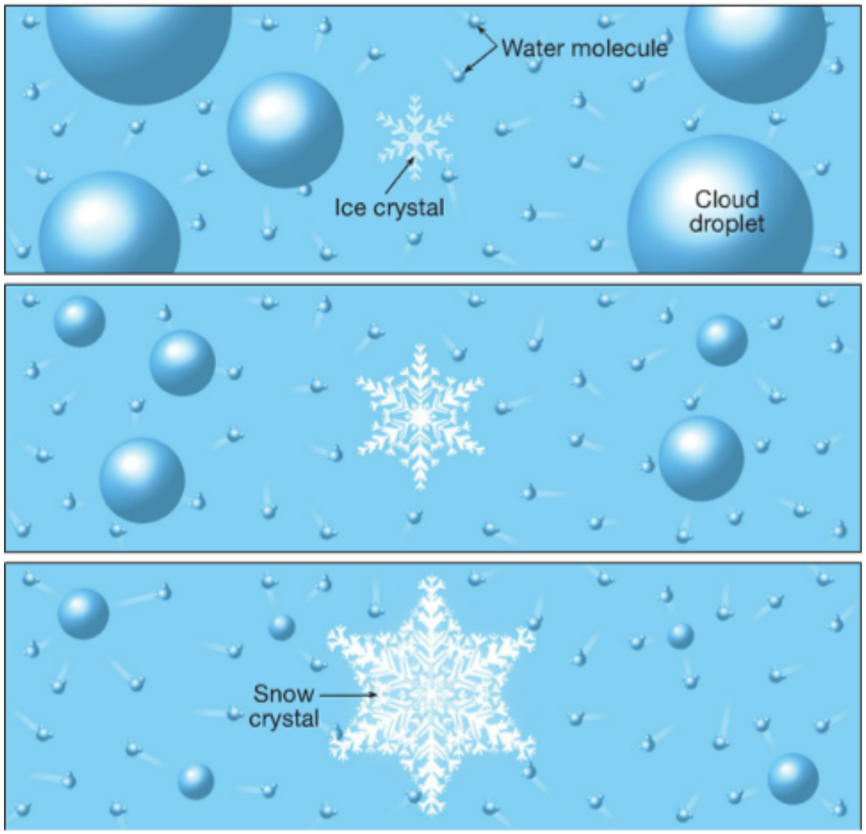
What is an important difference between dry air+water vapor and cloud ice, liquid, snow, rain, …?

$$\mathcal{L}_{all} = \{\text{`}d\text{`}, \text{`}wv\text{`}, \text{`}cl\text{`}, \text{`}ci\text{`}, \text{`}rn\text{`}, \text{`}sw\text{`}\}$$

$$\mathcal{L}_{water} = \{\text{`}wv\text{`}, \text{`}cl\text{`}, \text{`}ci\text{`}, \text{`}rn\text{`}, \text{`}sw\text{`}\}$$

$$\mathcal{L}_{cond} = \{\text{`}cl\text{`}, \text{`}ci\text{`}, \text{`}rn\text{`}, \text{`}sw\text{`}\}$$

A1. Representation of water phases in terms of dry and wet (specific) mixing ratios



What is an important difference between dry air+water vapor and cloud ice, liquid, snow, rain, ...?

Dry air+water vapor are gases!

$$\mathcal{L}_{all} = \{'d', 'wv', 'cl', 'ci', 'rn', 'sw'\}$$

$$\mathcal{L}_{water} = \{'wv', 'cl', 'ci', 'rn', 'sw'\}$$

$$\mathcal{L}_{cond} = \{'cl', 'ci', 'rn', 'sw'\}$$

Outline

- Representation of water phases
- **Ideal gas law and virtual temperature for moist air containing condensates**
- Dry-mass floating Lagrangian vertical coordinate
- Adiabatic frictionless equations of motion
- Viscosity
- Conservation properties: Axial angular momentum and total energy

A2. Ideal gas law and virtual temperature

Derivation of the ideal gas law for a volume V of air containing condensates:

- **Gaseous components of air occupy $V^{(gas)}$ and condensates $V^{(cond)}$: $V=V^{(gas)}+V^{(cond)}$**

- **The ideal gas law applies to the gaseous component of air only, i.e. the partial pressure of dry air $p^{(d)}$ (the pressure dry air would exert if it alone would occupy V)**

$$p^{(d)}V^{(gas)} = N^{(d)}k_B T$$

where k_B is the Boltzmann constant, T temperature and $N^{(d)}$ is # of molecules of dry air

$$N^{(d)} = \frac{V\rho^{(d)}}{\mathcal{M}^{(d)}}$$

where $M^{(d)}$ is molar mass of dry air.

A2. Ideal gas law and virtual temperature

Derivation of the ideal gas law for a volume V of air containing condensates:

- **Gaseous components of air occupy $V^{(gas)}$ and condensates $V^{(cond)}$: $V=V^{(gas)}+V^{(cond)}$**

- **The ideal gas law applies to the gaseous component of air only, i.e. the partial pressure of dry air $p^{(d)}$ (the pressure dry air would exert if it alone would occupy V)**

$$p^{(d)}V^{(gas)} = V\rho^{(d)}R^{(d)}T,$$

where $R^{(d)}$ is dry air gas constant

$$R^{(d)} \equiv \frac{k_B}{\mathcal{M}^{(d)}}$$

- **Can do same derivation for water vapor …**

A2. Ideal gas law and virtual temperature

Using Dalton's law of partial pressures we get

$$p = \frac{V}{V^{(gas)}} \left(\rho^{(d)} R^{(d)} T + \rho^{(wv)} R^{(wv)} T \right)$$

Move $R^{(d)}$ **outside parenthesis, define** $\epsilon \equiv \frac{R^{(d)}}{R^{(wv)}}$, **substitute** $m^{(\ell)} \equiv \frac{\rho^{(\ell)}}{\rho^{(d)}}$ **for 'wv',**

multiply by ρ / ρ **and use** $\rho = \rho^{(d)} \left(\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} \right)$ **and simplify**

$$p = \frac{V}{V^{(gas)}} \rho R^{(d)} \left(\frac{1 + \frac{1}{\epsilon} m^{(wv)}}{\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)}} \right) T.$$

A2. Ideal gas law and virtual temperature

Using Dalton's law of partial pressures we get

$$p = \frac{V}{V^{(gas)}} \left(\rho^{(d)} R^{(d)} T + \rho^{(wv)} R^{(wv)} T \right)$$

Move $R^{(d)}$ outside parenthesis, define $\epsilon \equiv \frac{R^{(d)}}{R^{(wv)}}$, substitute $m^{(\ell)} \equiv \frac{\rho^{(\ell)}}{\rho^{(d)}}$ for 'wv',

mulitply by ρ / ρ and use $\rho = \rho^{(d)} \left(\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} \right)$ and simplify

$$p = \frac{V}{V^{(gas)}} \rho R^{(d)} \left(\frac{1 + \frac{1}{\epsilon} m^{(wv)}}{\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)}} \right) T.$$

Virtual temperature

Temperature dry air would have, at a given density in order to exert the same pressure as moist air at temperature T

A2. Ideal gas law and virtual temperature

Using Dalton's law of partial pressures we get

$$p = \frac{V}{V^{(gas)}}\left(\rho^{(d)}R^{(d)}T + \rho^{(wv)}R^{(wv)}T\right)$$

Move $R^{(d)}$ **outside parenthesis, define** $\epsilon \equiv \frac{R^{(d)}}{R^{(wv)}}$, **substitute** $m^{(\ell)} \equiv \frac{\rho^{(\ell)}}{\rho^{(d)}}$ **for 'wv',**

mulitply by ρ/ρ **and use** $\rho = \rho^{(d)}\left(\sum_{\ell\in\mathcal{L}_{all}} m^{(\ell)}\right)$ **and simplify**

Henceforth we will assume that $V \cong V^{(gas)}$

$$p = \frac{V}{V^{(gas)}}\rho R^{(d)}\left(\frac{1+\frac{1}{\epsilon}m^{(wv)}}{\sum_{\ell\in\mathcal{L}_{all}} m^{(\ell)}}\right)T$$

Virtual temperature

Temperature dry air would have, at a given density in order to exert the same pressure as moist air at temperature T

A2. Ideal gas law and virtual temperature

In all, the ideal gas law for a volume V of air containing condensates can be written as

$$p = \rho R^{(d)} T_v$$

where virtual temperature is given by

$$T_v = T \left(\frac{1 + \frac{1}{\epsilon} m^{(wv)}}{\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)}} \right)$$

WE HAVE ASSUMED THAT THE VOLUME OF CONDENSATES IS NEGLITIBLE!

A3. Thermodynamic equation for moist air

Let S **denote the entropy of moist** $(S^{(l)}$ **entropy of component** $l)$

$$S = \sum_{\ell \in \mathcal{L}_{all}} S^{(\ell)}$$

From chain rule applied to each component of moist air separately

$$\begin{aligned} dS &= \sum_{\ell \in \mathcal{L}_{all}} dS^{(\ell)}, \\ &= \sum_{\ell \in \mathcal{L}_{all}} \left[\frac{\partial S^{(\ell)}}{\partial E^{(\ell)}} dE^{(\ell)} + \frac{\partial S^{(\ell)}}{\partial V^{(\ell)}} dV^{(\ell)} \right], \end{aligned}$$

where $E^{(l)}$ **is internal energy of** l**. Now assume** $dV^{(\ell)} = 0$ **for condensates ...**

A3. Thermodynamic equation for moist air

By manipulating this equation, assuming no phase changes, assuming that condensates are incompressible, and that the volume of condensates is zero then the thermodynamic equation can be written as

$$\delta T - \frac{RT}{c_p\, p}\delta p = \frac{dQ}{c_p}$$

where

$$R = \frac{\sum_{\ell \in \mathcal{L}_{all}} R^{(\ell)} m^{(\ell)}}{\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)}} \qquad c_p = \frac{\sum_{\ell \in \mathcal{L}_{all}} c_p^{(\ell)} m^{(\ell)}}{\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)}}$$

and $dQ = \frac{T \sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} ds^{(\ell)}}{\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)}}$ is the amount of heat per unit mass supplied reversibly to moist air.

A1-A3: The story so far

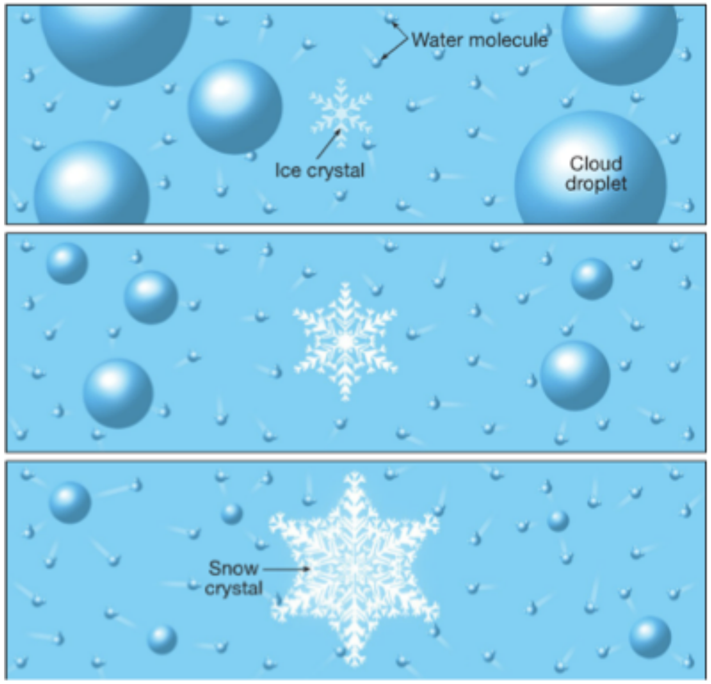
We have discussed the

- representation of the components of moist air
- ideal gas law for moist air
- thermodynamic equation for moist air

where moist air = dry air, water vapor, cloud liquid, cloud ice, rain and snow

We have assumed that condensates are incompressible and occupy zero volume

Now I'll introduce the vertical coordinate used in CAM-SE …



Outline

- Representation of water phases
- Ideal gas law and virtual temperature for moist air containing condensates
- **Dry-mass floating Lagrangian vertical coordinate**
- Adiabatic frictionless equations of motion
- Viscosity
- Conservation properties: Axial angular momentum and total energy

B1. Vertical coordinate: Dry mass

Let $M_s^{(d)}$ be the mass of a column of dry air per unit area [*kg/m*[2]*],* **i.e.**

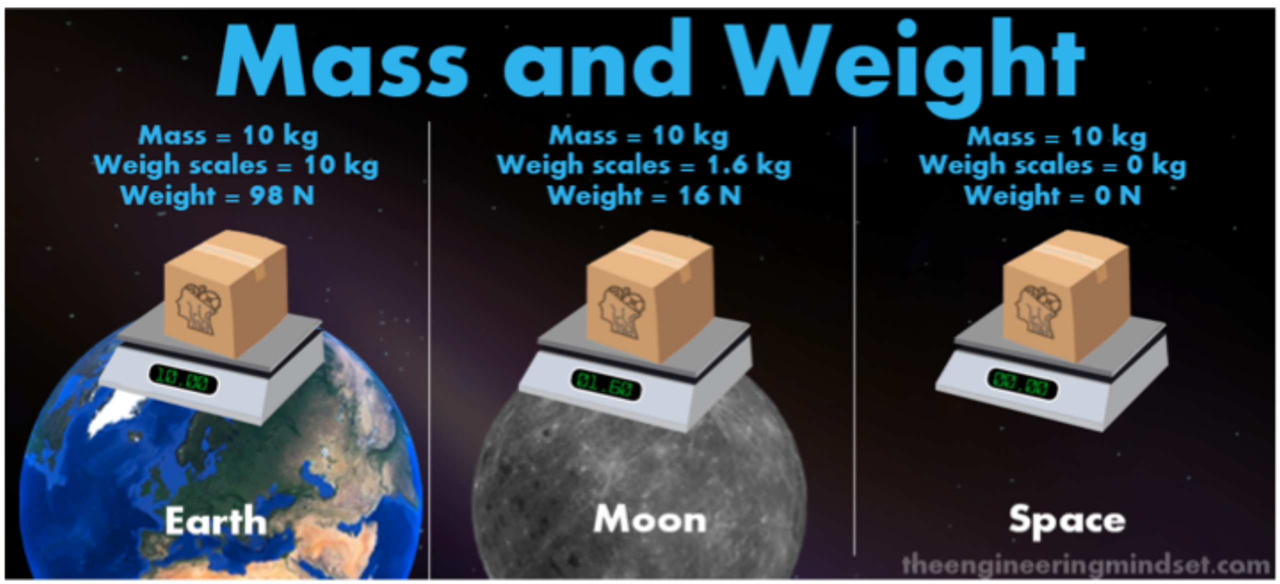
the weight of dry air at the surface is $g\,M_s^{(d)}$ [*kg/(m s*[2]*)≡ Pa ≡ N*]**.**

B1. Vertical coordinate

Let $M_s^{(d)}$ be the mass of a column of dry air per unit area [*kg/m*^{2}], i.e.

the weight of dry air at the surface is $g\, M_s^{(d)}$ [$kg/(m\ s^2) \equiv Pa \equiv N$].

Note: mass is invariant whereas weight (force exerted) depends on gravitational field.



B1. Vertical coordinate

Let $M_s^{(d)}$ be the mass of a column of dry air per unit area [*kg/m*^{2}], i.e.

the weight of dry air at the surface is $g\,M_s^{(d)}$ [$kg/(m\,s^2) \equiv Pa \equiv N$].

Note: mass is invariant whereas weight (for ... d.

Why aren't we calling this partial dry pressure $p^{(d)}$ at the surface

B1. Partial pressure of dry air and mass of dry air

The hydrostatic (moist) pressure at a given height (per unit area)

$$\begin{aligned} p(z) &= -g \int_{z'=z}^{z'=\infty} \rho \, dz', \\ &= -g \int_{z'=z}^{z'=\infty} \rho^{(d)} \left(\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} \right) dz', \\ &= g \sum_{\ell \in \mathcal{L}_{all}} M^{(\ell)}(z), \end{aligned}$$

where we have defined the mass of each component of moist air

$$M^{(\ell)}(z) = - \int_{z'=z}^{z'=\infty} \rho^{(d)} \, m^{(\ell)} \, dz'.$$

B1. Partial pressure of dry air and mass of dry air

The hydrostatic (moist) pressure at a given height (per unit area)

$$p^{(d)}(z) + p^{(wv)}(z) = -g \int_{z'=z}^{z'=\infty} \rho \, dz',$$

$$= -g \int_{z'=z}^{z'=\infty} \rho^{(d)} \Big(\sum_{\ell \in \mathcal{L}_{a}} \cdots$$

$$= g \sum_{\ell \in \mathcal{L}_{all}} M^{(\ell)}(z),$$

Using Dalton's law of partial pressures

$$p^{(d)}(z) \neq g M^{(d)}(z)$$

Weight of dry air is not necessarily equal to the partial pressure of dry air

When is it equal?

where we have defined the mass of each component of moist air

$$M^{(\ell)}(z) = -\int_{z'=z}^{z'=\infty} \rho^{(d)} \, m^{(\ell)} \, dz'.$$

B1. Vertical coordinate

Let $M_s^{(d)}$ be the mass of a column of dry air per unit area [*kg/m²*], i.e.

the weight of dry air at the surface is $g\,M_s^{(d)}$ [*kg/(m s²)≡ Pa ≡ N*].

Note: mass is invariant whereas weight (force exerted) depends on gravitational field.

Consider a general terrain following vertical coordinate that is a function of $M_s^{(d)}$

$$\eta^{(d)} = h(M^{(d)}, M_s^{(d)})$$

where

$$h(M_s^{(d)}, M_s^{(d)}) = 1 \qquad h(M_t^{(d)}, M_s^{(d)}) = 0$$

surface top

B1. Vertical coordinate

Let $M_s^{(d)}$ be the mass of a column of dry air per unit area [*kg/m²*], i.e.

the weight of dry air at the surface is $g\,M_s^{(d)}$ [*kg/(m s²)* ≡ *Pa* ≡ *N*].

Note: mass is invariant whereas weight (force exerted) depends on gravitational field.

Consider a general terrain following vertical coordinate that is a function of $M_s^{(d)}$

$$M_{k+1/2}^{(d)} = A_{k+1/2} M_t^{(d)} + B_{k+1/2} M_s^{(d)}$$

where the A's and B's are the "usual" hybrid coefficients (terrain-following at surface and transitioning to constant dry mass levels aloft).

Vertical staggering: u,v,T defined at full levels and mass defined at interfaces

B1. Vertical coordinate

Let $M_s^{(d)}$ be the mass of a column of dry air per unit area [kg/m^2], i.e.

the weight of dry air at the surface is $g\,M_s^{(d)}$ [$kg/(m\ s^2) \equiv Pa \equiv N$].

Note: mass is invariant whereas weight (force exerted) depends on gravitational field.

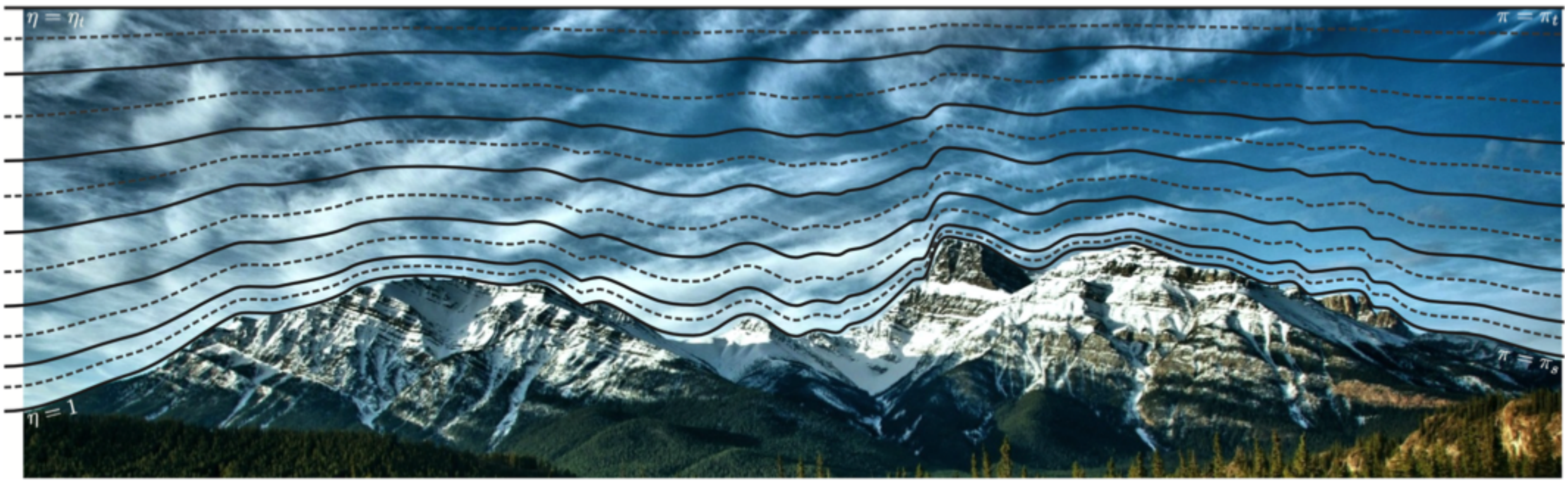
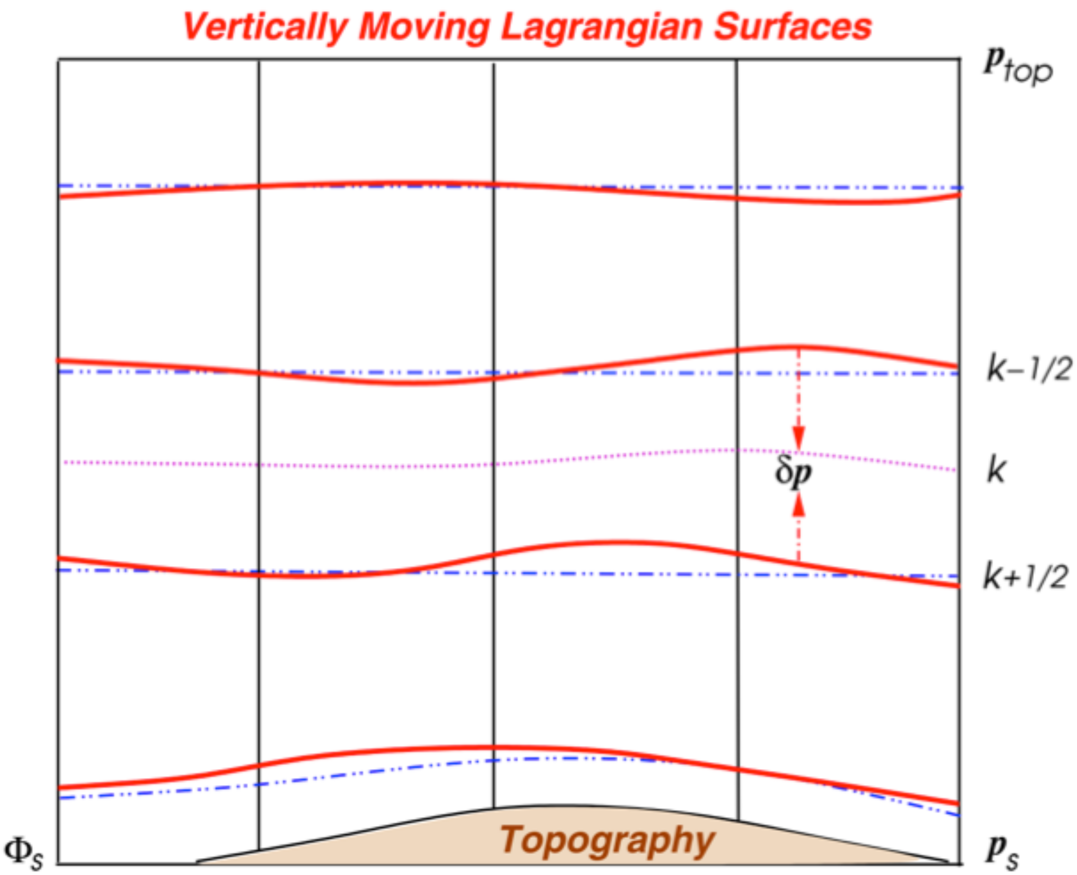


Figure courtesy of David Hall (CU Boulder).

B1. Floating Lagrangian vertical coordinate



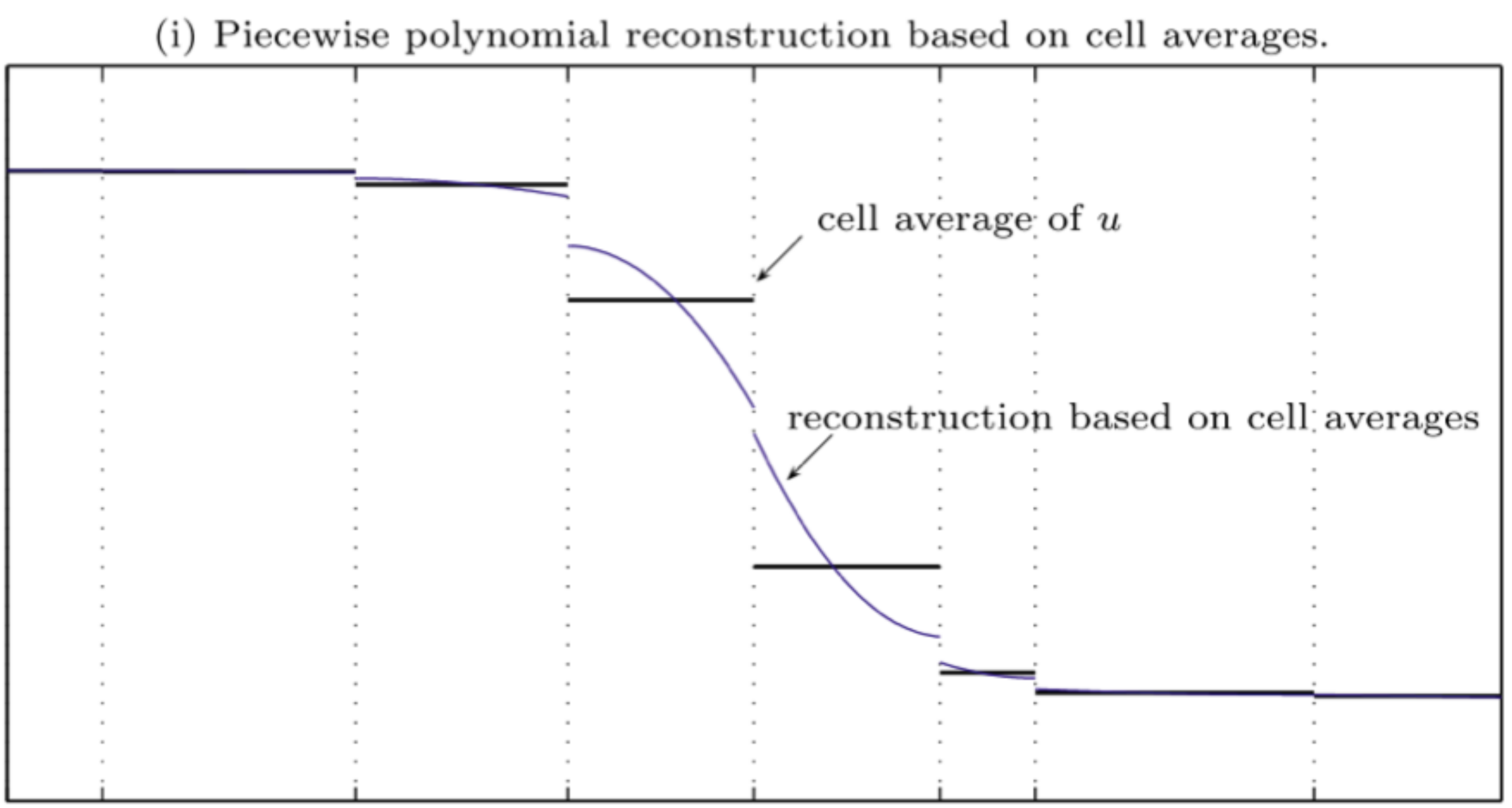
R.D. Nair et al. / Computers & Fluids 38 (2009) 309–319

Lagrangian surfaces are material surfaces with no flux of mass across them => they move up and down with flow convergence and divergence

Advantage: 2D operators in solver

To avoid excessive deformation of the vertical levels the prognostic variables are periodically mapped back to the Eulerian reference vertical coordinate

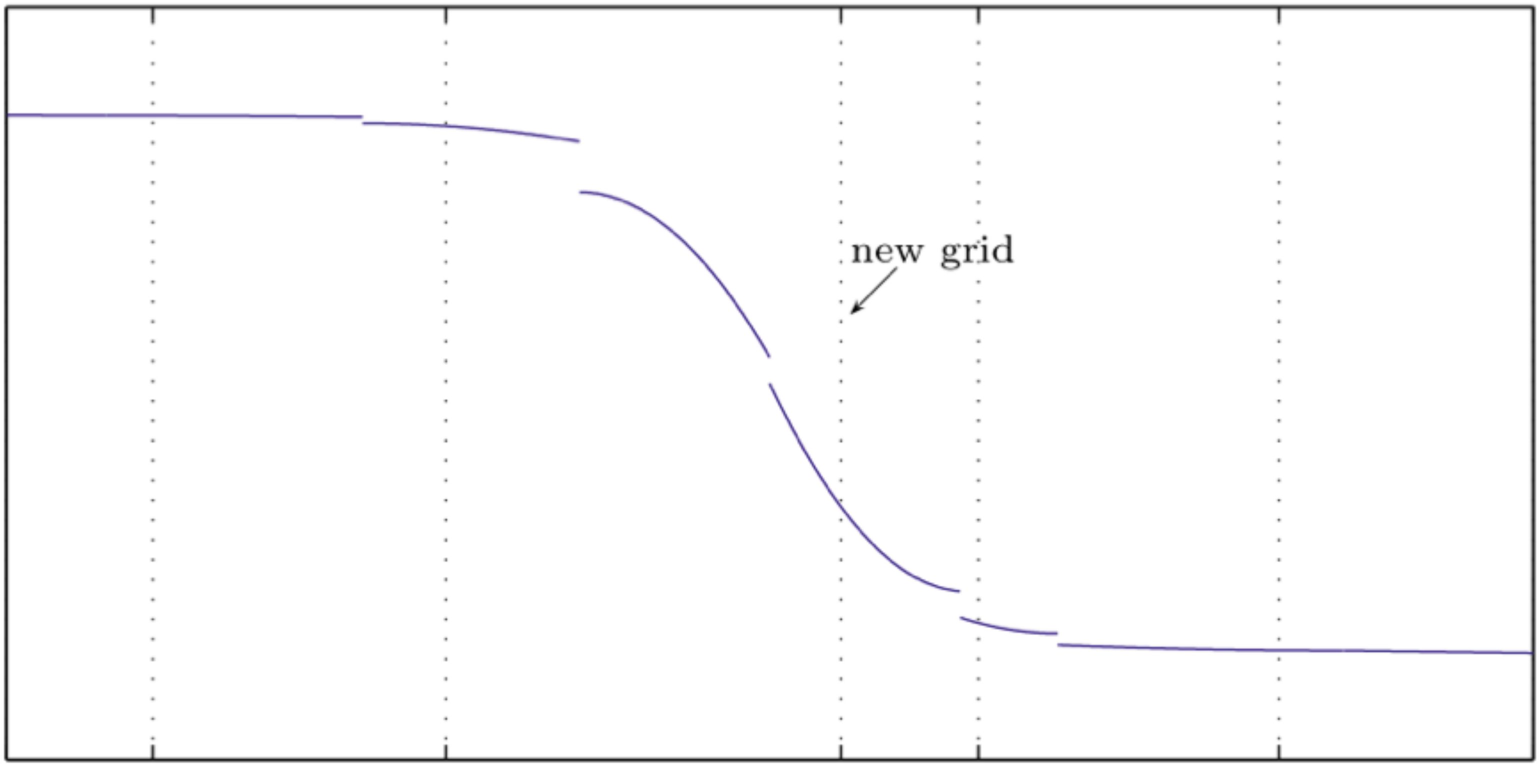
B1. Floating Lagrangian vertical coordinate



L. White et al. / Journal of Computational Physics 228 (2009) 8665–8692

B1. Floating Lagrangian vertical coordinate

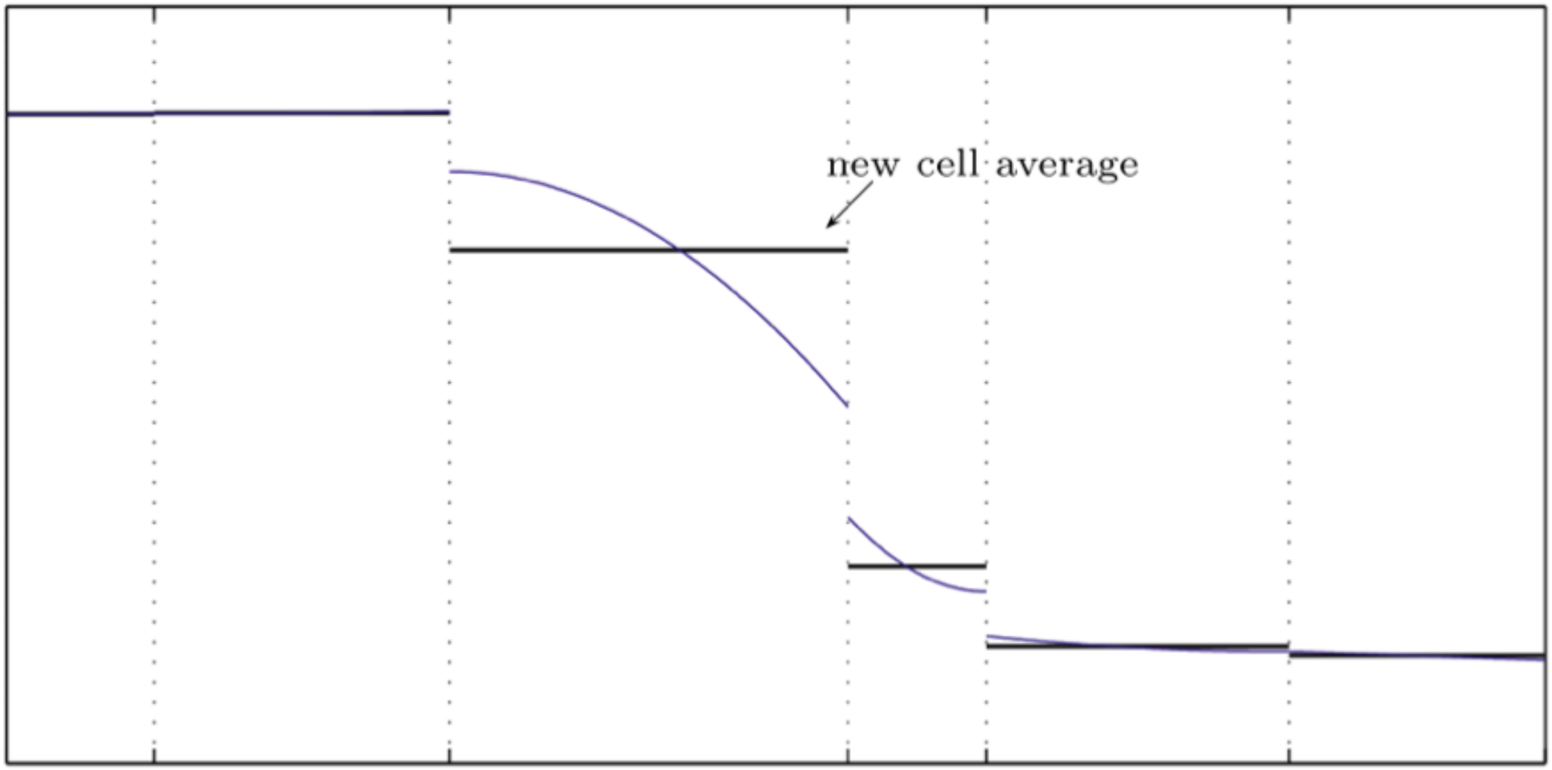
(ii) A new grid is considered and superimposed on the reconstructed profile.



L. White et al. / Journal of Computational Physics 228 (2009) 8665–8692

B1. Floating Lagrangian vertical coordinate

(iii) Cell averages are computed by integration. Reconstruction is repeated.



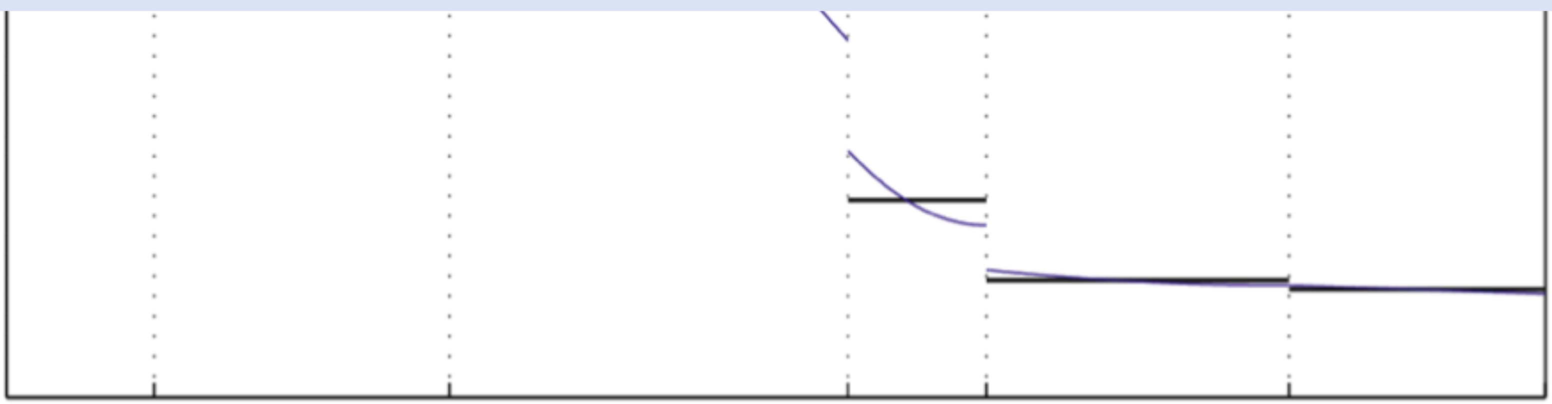
L. White et al. / Journal of Computational Physics 228 (2009) 8665–8692

B1. Floating Lagrangian vertical coordinate

We map tracer mass, mass-weighted (u,v) and internal energy

=> The vertical remapping process conserves mass, angular momentum and internal energy.

Aside: It is possible to conserve total energy instead of internal energy in the remapping process but that approach is "ill-conditioned"



L. White et al. / Journal of Computational Physics 228 (2009) 8665–8692

Outline

- Representation of water phases
- Ideal gas law and virtual temperature for moist air containing condensates
- Dry-mass floating Lagrangian vertical coordinate
- **Adiabatic frictionless equations of motion**
- Viscosity
- Conservation properties: Axial angular momentum and total energy

C1. Adiabatic & frictionless equations of motion

The $\eta^{(d)}$-coordinate adiabatic and frictionless atmospheric primitive equations assuming floating Lagrangian vertical coordinates [*Starr*, 1945; *Lin*, 2004] can be written in vector invariant form as

$$\frac{\partial \mathbf{v}}{\partial t} + (\zeta + f)\,\hat{\vec{k}} \times \mathbf{v} + \nabla_{\eta^{(d)}}\left(\frac{1}{2}\mathbf{v}^2 + \Phi\right) + \frac{1}{\rho}\nabla_{\eta^{(d)}} p = 0,$$

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla_{\eta^{(d)}} T - \frac{1}{c_p \rho}\omega = 0,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}} m^{(\ell)}\right) + \nabla_{\eta^{(d)}}\cdot\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}} m^{(\ell)}\mathbf{v}\right) = 0, \quad \ell \in \mathcal{L}_{all},$$

where Φ is the geopotential height ($\Phi = g\,z$), $\hat{\vec{k}}$ is the unit vector normal to the surface of the sphere, $\mathbf{v} = (u, v)$ is the velocity vector with u being the zonal velocity component and v the meridional velocity component, $\zeta = \hat{\vec{k}} \cdot \nabla\times\mathbf{v}$ is vorticity, f Coriolis parameter, and $\omega = dp/dt$ is the (moist) pressure vertical velocity with $d/dt = \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\eta_d}$ being the material/total derivative along $\eta^{(d)}$.

C1. Adiabatic & frictionless equations of motion

The $\eta^{(d)}$-coordinate adiabatic and frictionless atmospheric primitive equations assuming floating Lagrangian vertical coordinates [*Starr*, 1945; *Lin*, 2004] can be written in vector invariant form as

$$\frac{\partial \mathbf{v}}{\partial t} + (\zeta + f)\,\hat{\vec{k}} \times \mathbf{v} + \nabla_{\eta^{(d)}}\left(\frac{1}{2}\mathbf{v}^2 + \Phi\right) + \frac{1}{\rho}\nabla_{\eta^{(d)}} p = 0,$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla_{\eta^{(d)}} T - \frac{1}{c_p \rho}\omega = 0,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}} m^{(\ell)}\right) + \nabla_{\eta^{(d)}} \cdot \left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}} m^{(\ell)} \mathbf{v}\right) = 0, \quad \ell \in \mathcal{L}_{all},$$

The prognostics variables are momentum (zonal and meridional components), temperature (T), dry-mass layer thickness, and tracer mixing ratios.

How do we compute pressure, density, geopotential, omega?

C1. Adiabatic & frictionless equati…

Th… onless atmosph… suming f… es [*Starr*, 1945… vector in…

Hydrostatic balance:

$$\frac{\partial \Phi}{\partial \eta^{(d)}} = -\frac{R^{(d)} T_v}{p} \frac{\partial p}{\partial \eta^{(d)}}$$

Hydrostatic balance:

$$\frac{\partial p}{\partial \eta^{(d)}} = g \frac{\partial M^{(d)}}{\partial \eta^{(d)}} \left(\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} \right)$$

Ideal gas law:

$$p = \rho R^{(d)} T_v$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\zeta + f)\,\hat{k} \times \mathbf{v} + \nabla_{\eta^{(d)}} \left(\frac{1}{2}\mathbf{v}^2 + \Phi \right) + \frac{1}{\rho} \nabla_{\eta^{(d)}} p = 0,$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla_{\eta^{(d)}} T - \frac{1}{c_p \rho} \omega = 0,$$

$$\frac{\partial}{\partial t} \left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}} m^{(\ell)} \right) + \nabla_{\eta^{(d)}} \cdot \left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}} m^{(\ell)} \mathbf{v} \right) = 0, \quad \ell \in \mathcal{L}_{all},$$

$$\omega(\eta^{(d)}) = \frac{dp}{dt}(\eta^{(d)}),$$

$$= \int_{\eta^{(d)}}^{\eta^{(d)}=0} \frac{d}{dt} \left(\frac{\partial p}{\partial \eta^{(d)}} \right) d\eta^{(d)},$$

$$= \int_{\eta^{(d)}}^{\eta^{(d)}=0} \frac{\partial}{\partial t} \left(\frac{\partial p}{\partial \eta^{(d)}} \right) d\eta^{(d)} + \int_{\eta^{(d)}}^{\eta^{(d)}=0} \mathbf{v} \cdot \nabla_{\eta^{(d)}} \left(\frac{\partial p}{\partial \eta^{(d)}} \right) d\eta^{(d)}$$

The … meridional components), tem… mixing ratios.

How … omega?

Outline

- Representation of water phases
- Ideal gas law and virtual temperature for moist air containing condensates
- Dry-mass floating Lagrangian vertical coordinate
- Adiabatic frictionless equations of motion
- **Viscosity**
- Conservation properties: Axial angular momentum and total energy

C2. Hyperviscosity ∇^4

The spectral-element method does not have implicit diffusion. Hyperviscosity operators are applied to the prognostic variables to

- **dissipate energy near the grid scale**
- **damps the propagation of spurious grid-scale modes [Ainsworth and Wajid, 2009]**
- **smoothes the solution at element boundaries where the basis-functions are least smooth (C^0-continuous) – I'll come back to that ...**

$$\frac{\partial \mathbf{v}}{\partial t} + (\zeta + f)\,\hat{\vec{k}} \times \mathbf{v} + \nabla_{\eta^{(d)}}\left(\frac{1}{2}\mathbf{v}^2 + \Phi\right) + \frac{1}{\rho}\nabla_{\eta^{(d)}} p =$$ Vector viscosity

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla_{\eta^{(d)}} T - \frac{1}{c_p \rho}\omega = \nu_T \nabla^4 T$$

$$\frac{\partial}{\partial t}\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}}\right) + \nabla_{\eta^{(d)}}\cdot\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}}\mathbf{v}\right) = \nu_p \nabla^4 \left\{\frac{\partial M^{(d)}}{\partial \eta^{(d)}}\right\}$$

C2. Hyperviscosity ∇^4

The spectral-element method does not have implicit diffusion.
Hyperviscosity operators are applied to the prognostic variables to

- **dissipate energy near the grid scale**
- **damps the propagation of spurious grid-scale modes [Ainsworth and Wajid, 2009]**
- **smoothes the solution at element boundaries where the basis-functions are least smooth (C^0-continuous).**

$$\frac{\partial \mathbf{v}}{\partial t} + (\zeta + f)\,\hat{\vec{k}} \times \mathbf{v} + \nabla_{\eta^{(d)}}\left(\frac{1}{2}\mathbf{v}^2 + \Phi\right) + \frac{1}{\rho}\nabla_{\eta^{(d)}} p =$$ Vector viscosity

Use vector identity: $\nabla^2 \mathbf{v} = \nabla(\nabla \cdot \mathbf{v}) - \nabla \times (\nabla \times \mathbf{v})$ -> split into divergent and rotational term

We can damp divergence and vorticity with different damping coefficients

$$\nu_{div}\,\nabla(\nabla \cdot \mathbf{v}) - \nu_{vor}\,\nabla \times (\nabla \times \mathbf{v})$$

Operators are applied iteratively for 4th-order damping

2.6.2 *Frictional heating*

Let $\delta\mathbf{v}$ be the change in the velocity vector due to diffusion of momentum. Then the change in kinetic energy due to hyperviscosity applied to $\mathbf{v}$ is $\frac{1}{2}\rho\mathbf{v}\cdot\delta\mathbf{v}$. This kinetic energy is converted to a heating rate by adding a heating term $\delta\mathcal{T}$ in the thermodynamic equation corresponding to the kinetic energy change

$$\rho c_p \delta\mathcal{T} = -\frac{1}{2}\rho\mathbf{v}\cdot\delta\mathbf{v} \Rightarrow \delta\mathcal{T} = -\frac{1}{c_p}(\mathbf{v}\cdot\delta\mathbf{v}),$$

[p.71 in *Neale et al.*, 2012]. As shown in the results section 4.2 this term is rather large and therefore important for good energy conservation characteristics of the dynamical core.

to

th and Wajid, 2009]

unctions are least

$$\frac{\partial\mathbf{v}}{\partial t} + (\zeta + f)\,\hat{\vec{k}}\times\mathbf{v} + \nabla_{\eta^{(d)}}\left(\frac{1}{2}\mathbf{v}^2 + \Phi\right) + \frac{1}{\rho}\nabla_{\eta^{(d)}}p =$$

Vector viscosity

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla_{\eta^{(d)}}T - \frac{1}{c_p\rho}\omega = \nu_T\nabla^4 T$$

$$\frac{\partial}{\partial t}\left(\frac{\partial M^{(d)}}{\partial\eta^{(d)}}\right) + \nabla_{\eta^{(d)}}\cdot\left(\frac{\partial M^{(d)}}{\partial\eta^{(d)}}\mathbf{v}\right) = \nu_p\nabla^4\left\{\frac{\partial M^{(d)}}{\partial\eta^{(d)}}\right\}$$

The following hyperviscosity coefficients are used in CAM-SE:

$$\nu_T = \nu_{vor} = 0.150 \times \left(\frac{30}{N_e} 1.1 \times 10^5\right)^3 \frac{m^4}{s},$$

$$\nu_p = \nu_{div} = 0.751 \times \left(\frac{30}{N_e} 1.1 \times 10^5\right)^3 \frac{m^4}{s},$$

where $N_e = 30$ and $N_e = 120$ for the 1° and 1/4° horizontal resolution configurations. Note that mass-wind consistency may be violated if $\nu_p \neq \nu_{div}$. The term inside the parenthesis is the average grid spacing in kilometers and the scaling with resolution is what is used with MPAS [Model for Prediction Across Scales; *Skamarock et al.*, 2014]. The damping of temperature and vorticity in CAM-SE is similar to the damping in MPAS, i.e. MPAS uses a coefficient of 0.05 in front of the $(\cdot)^3$ term and CAM-SE uses 0.751.

to

th and Wajid, 2009]

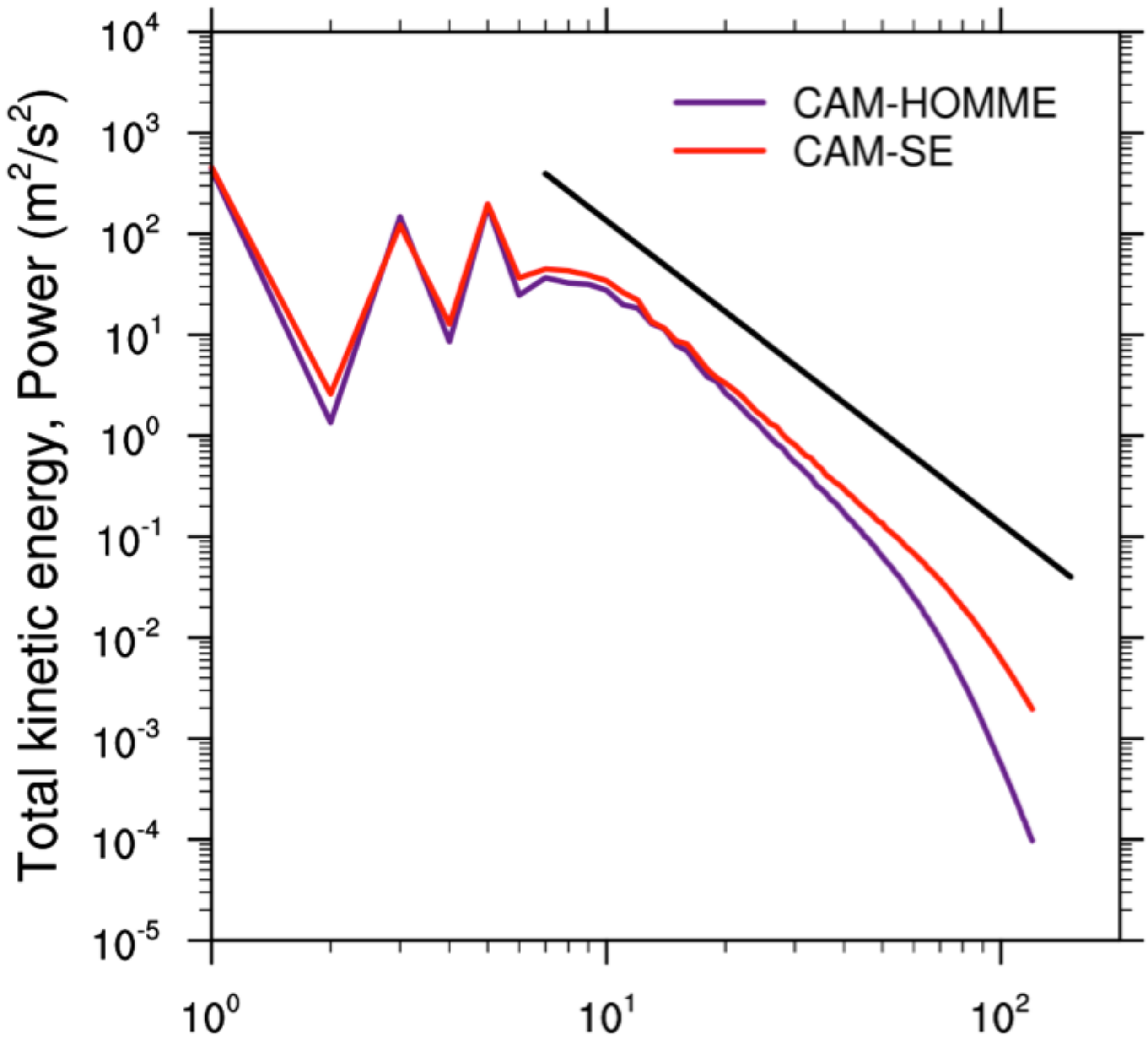
unctions are least

$$\frac{\partial \mathbf{v}}{\partial t} + (\zeta + f)\,\hat{\vec{k}} \times \mathbf{v} + \nabla_{\eta^{(d)}}\left(\frac{1}{2}\mathbf{v}^2 + \Phi\right) + \frac{1}{\rho}\nabla_{\eta^{(d)}} p =$$

Vector viscosity

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla_{\eta^{(d)}} T - \frac{1}{c_p \rho}\omega = \nu_T \nabla^4 T$$

$$\frac{\partial}{\partial t}\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}}\right) + \nabla_{\eta^{(d)}} \cdot \left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}}\mathbf{v}\right) = \nu_p \nabla^4 \left\{\frac{\partial M^{(d)}}{\partial \eta^{(d)}}\right\}$$



Total kinetic energy, Power (m²/s²)
CAM-HOMME
CAM-SE
10⁴
10³
10²
10¹
10⁰
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁰
10¹
10²

C3. On what iso-surfaces to apply hyperviscosity?

The horizontal hyperviscosity operator can be applied on η_d-surfaces, $\nabla^4 = \nabla^4_{\eta_d}$, but it may be advantageous to apply the hyperviscosity operator on approximate dry-mass surfaces

$$\nu\nabla^4\Xi = \nu\nabla^4_{\eta_d} - \nu\frac{\partial\Xi}{\partial M^{(d)}}\nabla^4_{\eta_d}M^{(d)}, \qquad \Xi = \mathbf{v}, T,$$

[p.58 in *Neale et al.*, 2012] to reduce spurious diffusion over steep topography. In theory the damping of dry-mass layer thickness should be zero if hyperviscosity is applied on dry-mass surfaces. However, for stability it is necessary to damp dry-mass layer thickness, but instead of applying ∇^4 to $\frac{\partial M^{(d)}}{\partial\eta^{(d)}}$ it is applied to the difference between $\frac{\partial M^{(d)}}{\partial\eta^{(d)}}$ and a smoothed version of $\frac{\partial M^{(d)}}{\partial\eta^{(d)}}$ referred to as $\left(\frac{\partial M^{(d)}}{\partial\eta^{(d)}}\right)^{(ref)}$. The reference/smoothed dry-mass layer thickness is defined in Appendix A.2.

Outline

- Representation of water phases
- Ideal gas law and virtual temperature for moist air containing condensates
- Dry-mass floating Lagrangian vertical coordinate
- Adiabatic frictionless equations of motion
- Viscosity
- **Conservation properties: Axial angular momentum and total energy**

C4. Conservation properties of eqn's of motion: AAM

Definition of axial angular momentum (AAM):

$$\mathcal{M} = (u + \Omega r \cos\varphi)\, r \cos\varphi$$

where Ω angular velocity, Φ is latitude. It can be shown that
(see Appendix in Lauritzen et al., 2018)

$$\frac{\partial}{\partial t}\int_{\eta=0}^{\eta=1}\iint_S \left[g\left(\sum_{\ell\in\mathcal{L}_{all}} m^{(\ell)}\right)\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}}\right)\mathcal{M}\right] dA\, d\eta^{(d)} = -\iint_S \left[p_s \frac{\partial z_s}{\partial \lambda}\right] dA,$$

where $dA = r^2 \cos\varphi d\lambda d\varphi$.

-> In the absence of mountains (z_s=0) AAM is conserved for the continuous equations of motion. Spurious sources/sinks of AAM in dynamical core should be << "physical" source/sinks from the parameterizations (e.g. drag parameterizations).

C4. A simple test case to assess global AAM conservation

Held-Suarez forcing: flat-Earth (no mountain torque), physics replaced by simple boundary layer friction and relaxation of temperature towards zonally symmetric reference profile

$$\frac{\partial v}{\partial t} = \cdots - k_v(\sigma) v$$

$$\frac{\partial T}{\partial t} = \cdots - k_T(\phi,\sigma)\left[T - T_{eq}(\phi,p)\right]$$

$$T_{eq} = \max\left\{200\mathrm{K}, \left[315\mathrm{K} - (\Delta T)_y \sin^2\phi - (\Delta\theta)_z \log\left(\frac{p}{p_0}\right)\cos^2\phi\right]\left(\frac{p}{p_0}\right)^{\kappa}\right\}$$

$$k_T = k_a + (k_s - k_a)\max\left(0, \frac{\sigma - \sigma_b}{1 - \sigma_b}\right)\cos^4\phi$$

$$k_v = k_f \max\left(0, \frac{\sigma - \sigma_b}{1 - \sigma_b}\right)$$

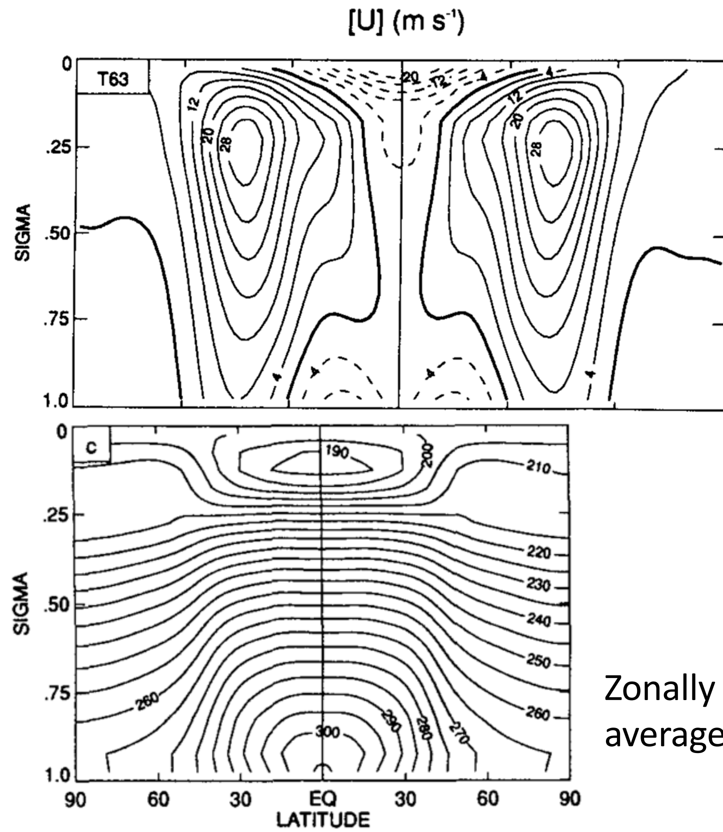
$\sigma_b = 0.7$ $\qquad$ $k_f = 1\ \mathrm{day}^{-1}$,

$k_a = 1/40\ \mathrm{day}^{-1}$ $\qquad$ $k_s = 1/4\ \mathrm{day}^{-1}$

$(\Delta T)_y = 60\mathrm{K}$ $\qquad$ $(\Delta\theta)_z = 10\mathrm{K}$

$p_0 = 1000\ \mathrm{mb}$ $\qquad$ $\kappa = \frac{R}{c_p} = \frac{2}{7}$ $\qquad$ $c_p = 1004\ \mathrm{J\ kg^{-1}\ K^{-1}}$

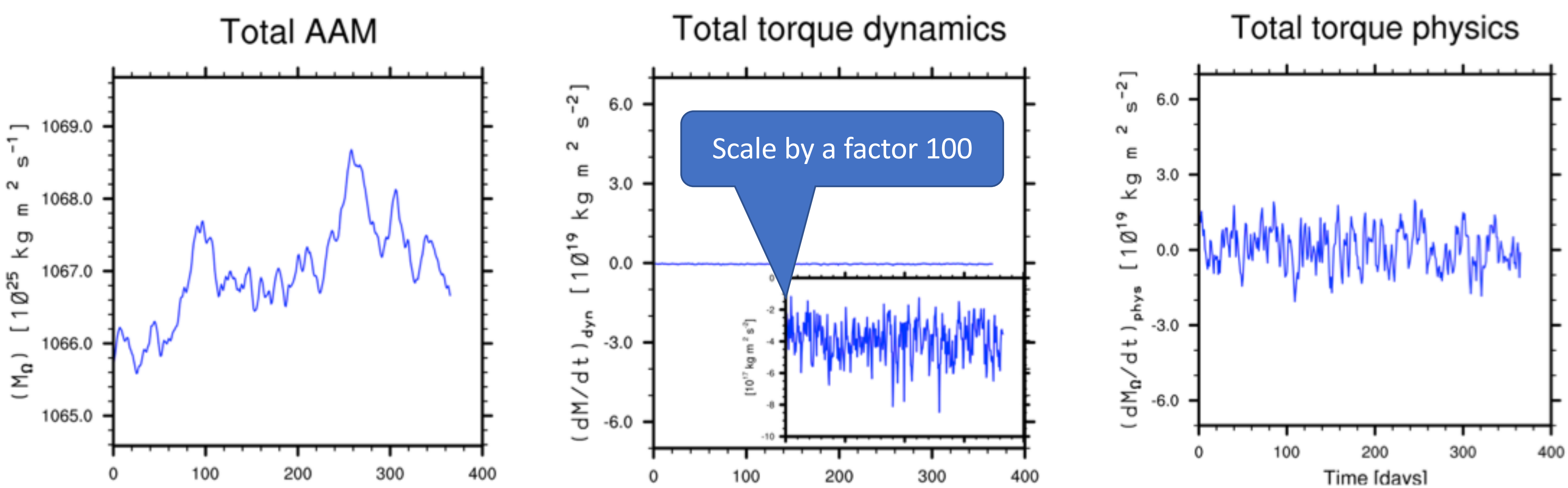
$\Omega = 7.292 \times 10^{-5}\ \mathrm{s}^{-1}$ $\qquad$ $g = 9.8\ \mathrm{m\ s^{-2}}$ $\qquad$ $a_e = 6.371 \times 10^6\ \mathrm{m}$.



Zonally and time averaged zonal velocity component u

Zonally and time averaged T

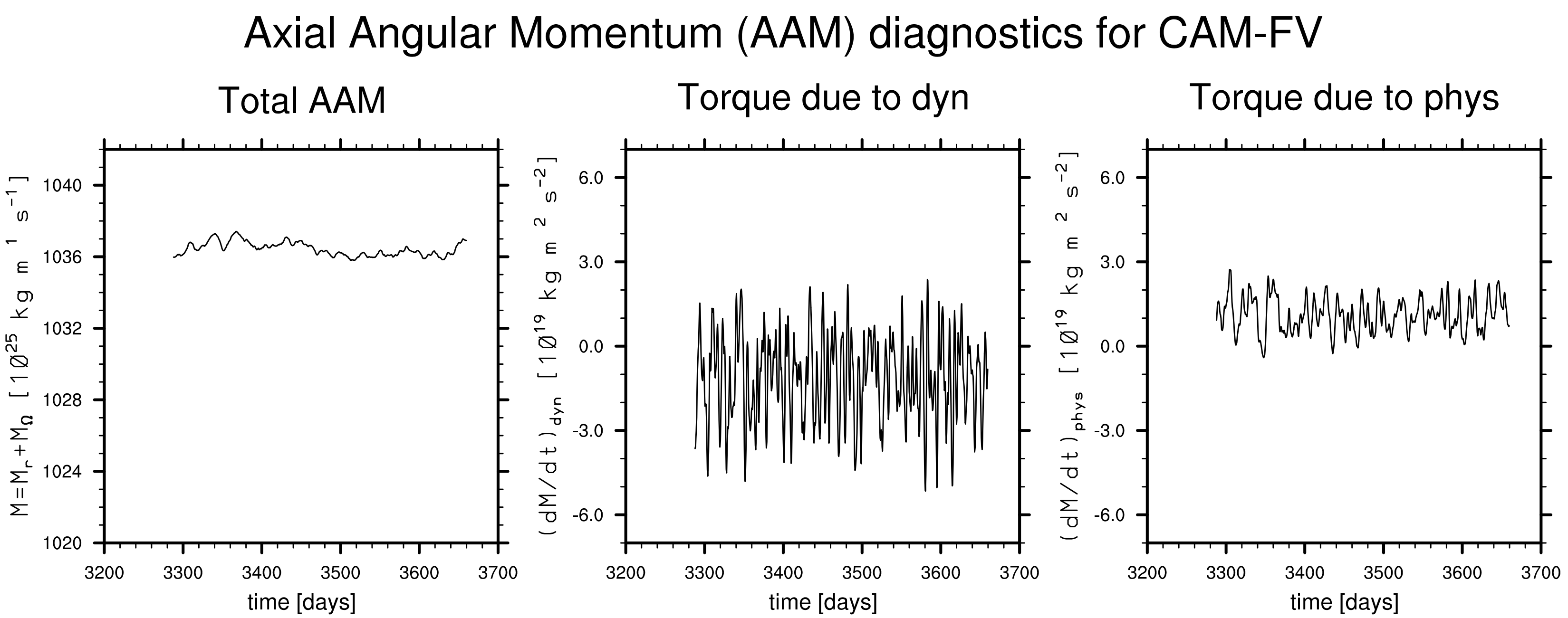
C4. Axial angular momentum conservation with CAM-SE



Angular momentum diagnostics for CAM6 Aqua-planet simulations using CAM-SE

For a detailed analysis of AAM conservation with CAM-HOMME see Lauritzen et al. (2014; doi:10.1002/2013MS000268)

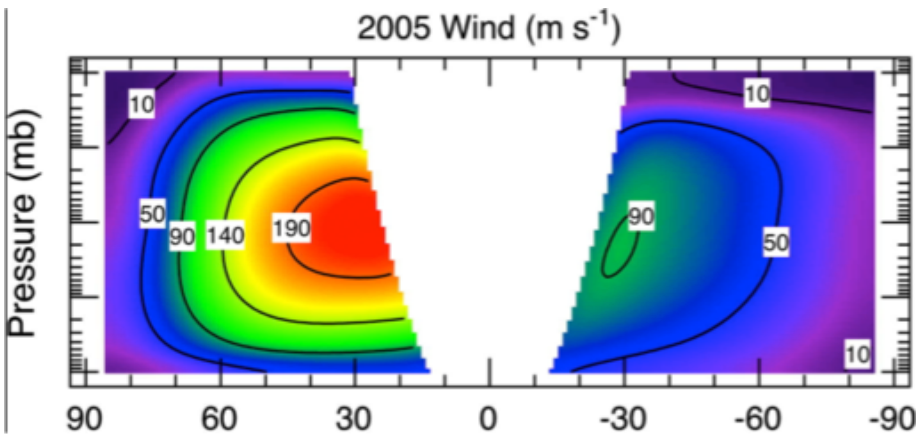
C4. A simple test case to assess global AAM conservation



Source-sink of axial angular momentum from the dynamical core are the same order of magnitude of "physical" sources-sinks from parameterizations!

C4. Example of superrotating atmospheres

Observations - Titan



Achterberg et al. (2008) using observations averaged from July 2004 through March 2006

CAM-FV Titan (Finite-Volume)

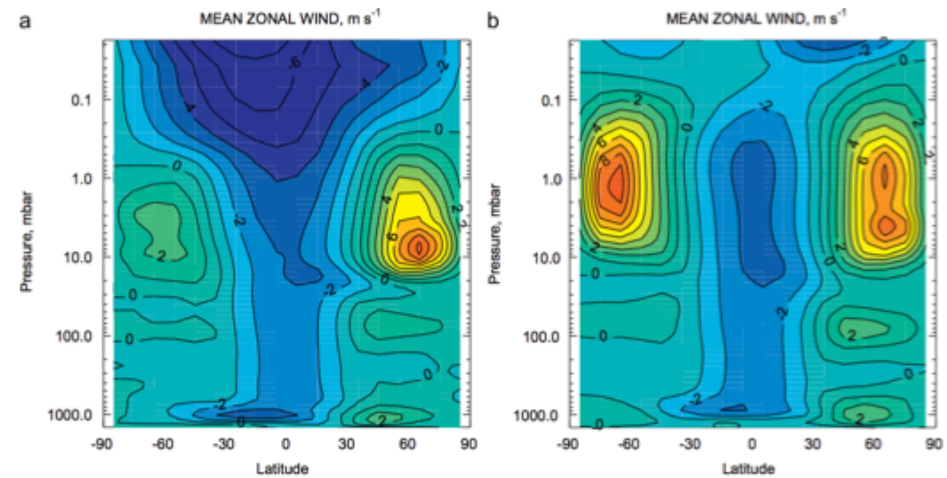


Fig. 12. Mean zonal wind field during (a) northern winter ($L_s = 293°$), and (b) at $L_s = 16°$, i.e., just after the northern vernal equinox. The contour interval is 1 m s^{-1}.

Friedson et al. (2009); https://doi.org/10.1016/j.pss.2009.05.006

Lebonnois et al. (2012) showed that different dynamical cores under same simple thermal forcing (Venus-like) performed very differently in terms of simulating super rotation which correlated exactly with lack of axial angular momentum in the dycore!

C5. Total energy conservation (continuous eqn's of motion)

Total energy equation integrated over the entire atmosphere can be written as

$$\frac{\partial}{\partial t}\int_{\eta=0}^{\eta=1}\iint_{S}\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}}\right)\sum_{\ell\in\mathcal{L}_{all}}\left[m^{(\ell)}\left(K+c_p^{(\ell)}T+\Phi_s\right)\right]dA\,d\eta^{(d)}=0$$

Derivation based on Kasahara (1974) but for moist air including condensates (for a detailed derivation see Lauritzen et al. (2018)).

Note that total energy splits into contribution for dry air, water vapor, and condensates.

Aside: CAM physics energy fixer uses

$$\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}}\right)\left(1+m^{(wv)}\right)\left[\left(K+c_p^{(d)}T+\Phi_s\right)\right]$$

which does not incl. condensates and uses same c_p for dry air and water vapor.

C5. Total energy conservation (continuous eqn's of motion)

Total energy equation integrated over the entire atmosphere can be written as

$$\frac{\partial}{\partial t}\int_{\eta=0}^{\eta=1}\iint_S \left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}}\right)\sum_{\ell\in\mathcal{L}_{all}}\left[m^{(\ell)}\left(K+c_p^{(\ell)}T+\Phi_s\right)\right]dA\,d\eta^{(d)}=0$$

Derivation based on Kasahara (1974) but for moist air including c... for a detailed derivation see Lauritzen et al. (2018)).

Note that total energy splits into contribution for dry air, wa... tes.

Difference amounts to 0.5 W/M²

Aside: CAM physics energy fixer uses

$$\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}}\right)\left(1+m^{(wv)}\right)\left[\left(K+c_p^{(d)}T+\Phi_s\right)\right]$$

which does not incl. condensates and uses same c_p for dry air and water vapor.

Total energy conservation in the atmosphere as a whole

For a coupled climate model total energy conservation is important (otherwise climate will drift)

=> Need to satisfy

$$\frac{d}{dt}\left(K+c_pT+\Phi\right)=\frac{1}{\rho}\frac{\partial p}{\partial t}+F_{net}$$

where K kinetic energy, \rho is density, p pressure, T temperature, \Phi geopotential height and F_{net} are net fluxes computed by parameterization (e.g., heating and momentum forcing).

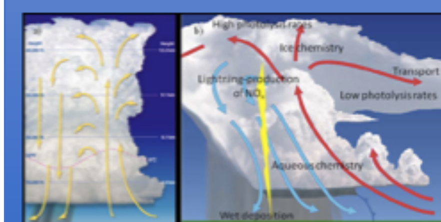
Dynamical core module

The $\eta^{(d)}$-coordinate adiabatic and frictionless atmospheric primitive equations assuming floating Lagrangian vertical coordinates [*Starr*, 1945; *Lin*, 2004] can be written in vector invariant form as

$$\frac{\partial \mathbf{v}}{\partial t}+(\zeta+f)\,\hat{k}\times\mathbf{v}+\nabla_{\eta^{(d)}}\left(\frac{1}{2}\mathbf{v}^2+\Phi\right)+\frac{1}{\rho}\nabla_{\eta^{(d)}}p=0,$$
$$\frac{\partial T}{\partial t}+\mathbf{v}\cdot\nabla_{\eta^{(d)}}T-\frac{1}{c_p\rho}\omega=0,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}}m^{(\ell)}\right)+\nabla_{\eta^{(d)}}\cdot\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}}m^{(\ell)}\mathbf{v}\right)=0,\quad \ell\in\mathcal{L}_{all},$$

where Φ is the geopotential height ($\Phi = g\,z$), $\hat{k}$ is the unit vector normal to the surface of the sphere, $\mathbf{v}=(u,v)$ is the velocity vector with u being the zonal velocity component and v the meridional velocity component, $\zeta=\hat{k}\cdot\nabla\times\mathbf{v}$ is vorticity, f Coriolis parameter, and $\omega = dp/dt$ is the (moist) pressure vertical velocity with $d/dt=\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla_{\eta_d}$ being the material/total derivative along $\eta^{(d)}$.

Physics module



Physics-dynamics coupling layer

Total e… …servation in the atmosphere as a whole

Frictional heating rate is calculated from K energy tendency produced from momentum diffusion and added to T

For a coupled climate model total energy conservation is important (otherwise climate will drift)

=> Need to satisfy

$$\frac{d}{dt}\left(K + c_p T + \Phi\right) = \frac{1}{\rho}\frac{\partial p}{\partial t} + F_{net}$$

where K kinetic energy, \rho is density, p pressure, T temperature, \Phi geopotential height and F_{net} are net fluxes computed by parameterization (e.g., heating and momentum forcing).

Dynamical core module

The $\eta^{(d)}$-coordinate adiabatic and frictionless atmospheric primitive equations assuming floating Lagrangian vertical coordinates [*Starr*, 1945; *Lin*, 2004] can be written in vector invariant form as

$$\frac{\partial \mathbf{v}}{\partial t} + (\zeta + f)\hat{k} \times \mathbf{v} + \nabla_{\eta^{(d)}}\left(\frac{1}{2}\mathbf{v}^2 + \Phi\right) + \frac{1}{\rho}\nabla_{\eta^{(d)}} p = 0,$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla_{\eta^{(d)}} T - \frac{1}{c_p \rho}\omega = 0,$$

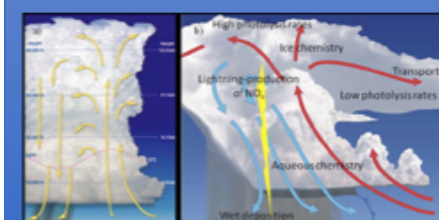
$$\frac{\partial}{\partial t}\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}} m^{(\ell)}\right) + \nabla_{\eta^{(d)}} \cdot \left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}} m^{(\ell)} \mathbf{v}\right) = 0, \quad \ell \in \mathcal{L}_{all},$$

where Φ is the geopotential height ($\Phi = g z$), $\hat{k}$ is the unit vector normal to the surface of the sphere, $\mathbf{v} = (u, v)$ is the velocity vector with u being the zonal velocity component and v the meridional velocity component, $\zeta = \hat{k} \cdot \nabla \times \mathbf{v}$ is vorticity, f Coriolis parameter, and $\omega = dp/dt$ is the (moist) pressure vertical velocity with $d/dt = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\eta_d}$ being the material/total derivative along $\eta^{(d)}$.

The dynamical core may not conserve energy due to inherent numerical dissipation, non-conservation due to time truncation errors, etc.

Physics-dynamics coupling layer

Physics module



Total energy conservation in the atmosphere as a whole

Dynamical core module

The $\eta^{(d)}$-coordinate adiabatic and frictionless atmospheric primitive equations assuming floating Lagrangian vertical coordinates [*Starr*, 1945; *Lin*, 2004] can be written in vector invariant form as

$$\frac{\partial \mathbf{v}}{\partial t} + (\zeta + f)\,\hat{k} \times \mathbf{v} + \nabla_{\eta^{(d)}}\left(\frac{1}{2}\mathbf{v}^2 + \Phi\right) + \frac{1}{\rho}\nabla_{\eta^{(d)}} p = 0,$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla_{\eta^{(d)}} T - \frac{1}{c_p \rho}\omega = 0,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}} m^{(\ell)}\right) + \nabla_{\eta^{(d)}} \cdot \left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}} m^{(\ell)} \mathbf{v}\right) = 0, \quad \ell \in \mathcal{L}_{all},$$

where Φ is the geopotential height ($\Phi = g\,z$), $\hat{k}$ is the unit vector normal to the surface of the sphere, $\mathbf{v} = (u, v)$ is the velocity vector with u being the zonal velocity component and v the meridional velocity component, $\zeta = \hat{k} \cdot \nabla \times \mathbf{v}$ is vorticity, f Coriolis parameter, and $\omega = dp/dt$ is the (moist) pressure vertical velocity with $d/dt = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\eta_d}$ being the material/total derivative along $\eta^{(d)}$.

The dynamical core may not conserve energy due to inherent numerical dissipation, non-conservation due to time truncation errors, etc.

For a coupled climate model total energy conservation is important (otherwise climate will drift)

=> Need to satisfy

$$\frac{d}{dt}\left(K + c_p T + \Phi\right) = \frac{1}{\rho}\frac{\partial p}{\partial t} + F_{net}$$

where K kinetic energy, \rho is density, p pressure, T temperature, \Phi geopotential height and F_{net} are net fluxes computed by parameterization (e.g., heating and momentum forcing).

Energy conservation can be violated in physics-dynamics coupling if the physics tendencies are added during the time-stepping (underlying pressure changes!)

Physics-dynamics coupling layer

Physics module

CAM physics does not change surface pressure – under that assumption each paramerization conserves energy (i.e. energy change due to state variables changing is exactly balanced by net fluxes).

However, changes in water variables does change pressure => When pressure is updated energy conservation is violated

Energy budgets in CAM-SE

Dynamical core module

- **Rate of energy change due to explicit dissipation (hyperviscosity)**

  dE/dt = 0.0729 W/m^2

- **Frictional heating rate is calculated from K tendency produced from momentum diffusion and added to T:**

  dE/dt = 0.6997 W/m^2

- **Vertical remapping**

  dE/dt = -0.1547 W/m^2

Total loss of energy in dynamics

dE/dt = -0.0723 W/m^2

Rate of energy change due to “dribbling” physics tendencies in the dynamics

dE/dt = 0.056 W/m^2

Physics-dynamics coupling layer

Physics module

- **“physical” changes in energy due to water change**

  dE/dt = -0.0016 W/m^2

- **Change in energy due to change in pressure due to water vapor change (“dme_adjust”)**

  dE/dt = 0.2667 W/m^2

- **Energy fixer**

  dE/dt = -0.1843

  (= loss in dynamics + dme_adjust)

Energy budgets in CAM-SE

Our current "workhorse" model CAM-FV looses 1.07 W/m²

- **Frict… calculated from K tende… luced from momentum d… and added to T:**

  dE/d… 997 W/m²

- **Vertical remapp…**

  dE/dt = … 1547 W/m²

Total loss of energ… in dynamics

dE/dt = -0.0723 W/m²

Rate of energy change due to "dribbling" physics tendencies in the dynamics

dE/dt = 0.056 W/m²

Physics-dynamics coupling layer

Physics module

- **"physical" changes in energy due to water change**

  dE/dt = -0.0016 W/m²

- **Change in energy due to change in pressure due to water vapor change ("dme_adjust")**

  dE/dt = 0.2667 W/m²

- **Energy fixer**

  dE/dt = -0.1843

  (= loss in dynamics + dme_adjust)

Summary

- Derived equations of motion where condensates are thermodynamically active
- Introduced floating Lagrangian dry-mass vertical coordinate
- Discussed conservation properties

More information: **http://www.cgd.ucar.edu/cms/pel**
Email: **pel@ucar.edu**