The challenge of energy budget closure in Earth system models

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CDG seminar series
March, 16, 2021

Acknowledgments

Physics-Dynamics Coupling in Earth System Models (19w5153)

Organizers

Nicholas Kevlahan (McMaster University)

Peter Lauritzen (National Center for Atmospheric Research)

October 13-19, 2019

Reconciling and improving formulations for thermodynamics and conservation principles in Earth system models

P.H. Lauritzen[1], N.K.-R. Kevlahan[2], A.J. Adcroft[3], A.S. Donahue[4], T. Dubos[5], C. Eldred[6], A. Gassmann[7], B.E. Harrop[8], A.R. Herrington[1], C. Jablonowski[9], H. Johansen[15], W. Large[1], V.E. Larson[10], F. Lemarié[6], D.A. Randall[11], P.J. Rasch[8], K. Roy[12], B. Shipway[13], R. Tailleux[14], H. Wan[8]

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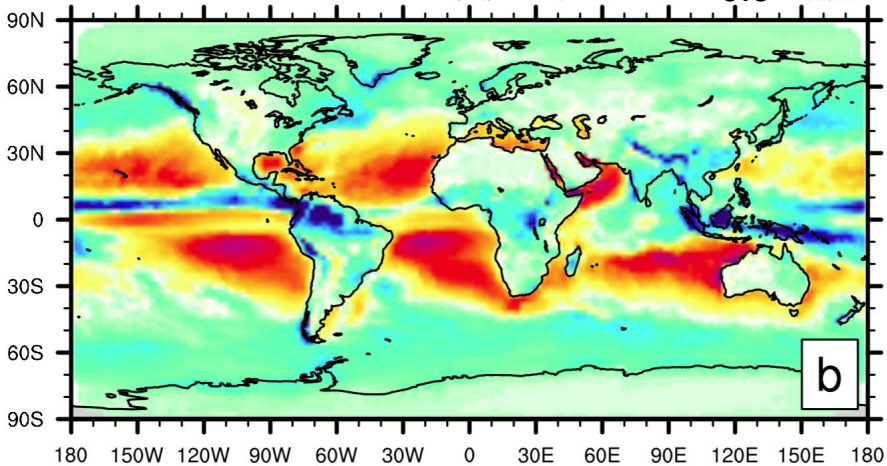# Motivation

1 degree CAM6, CESM2.2

Pressure work error TE tendency (pwork)   0.3   W/m²

global min = -76.95   global   max= 30.8
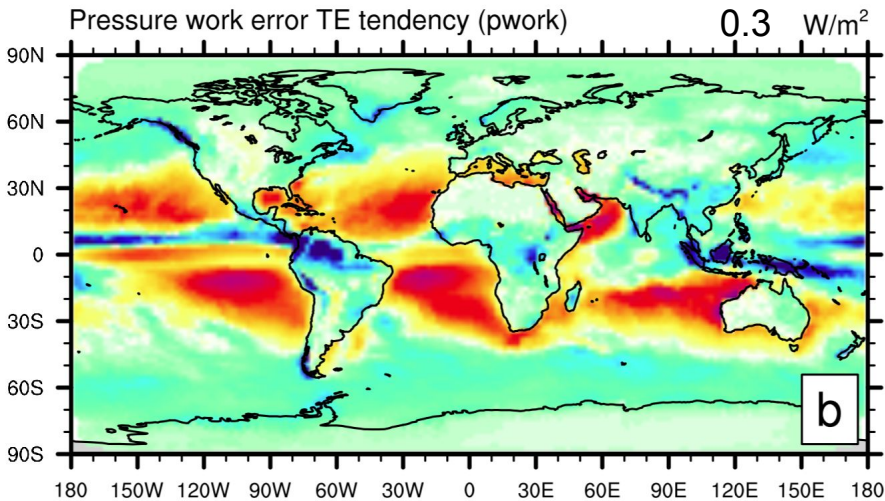
-20   -16   -12   -8   -4   0   4   8   12   16   20

Plots show one year averages using cam6_2_017 using CAM-SE-CSLAM; ¼ simulation courtesy of Adam Herrington

Lauritzen and
Williamson (2019)

Motivation
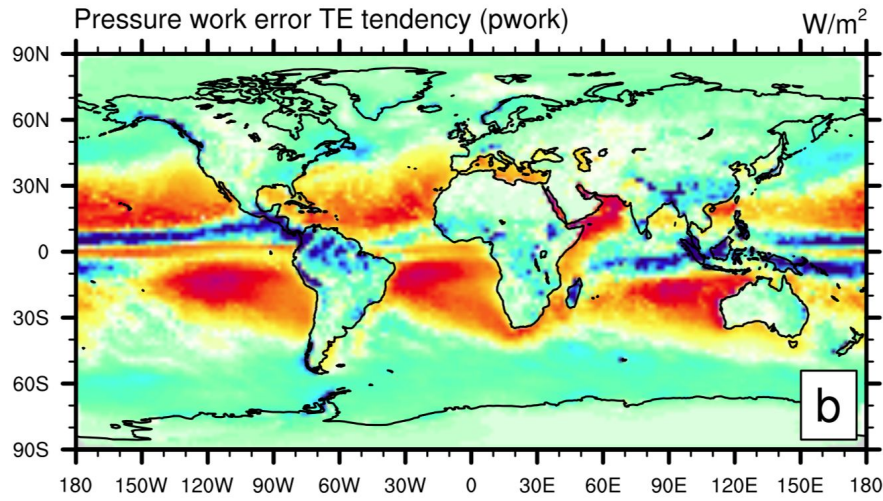
1 degree CAM6, CESM2.2

Pressure work error TE tendency (pwork)   0.3   W/m²

global min = -76.95 global max= 30.8

¼ degree CAM6, CESM2.2

Pressure work error TE tendency (pwork)   W/m²

global min = -231.4 global max=   30

Plots show one year averages using cam6_2_017 using CAM-SE-CSLAM; ¼ simulation courtesy of Adam Herrington
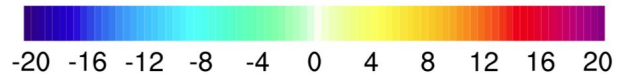
Outline

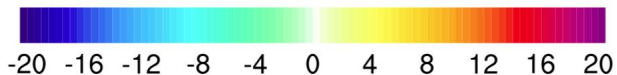$$\frac{\partial}{\partial t} \iiint \rho \left(E_{atm}\right) dV = - \oiint \mathcal{F}_{atm}^{(top)} d\sigma + \oiint \mathcal{F}_{atm}^{(bottom)} d\sigma,$$

1. What is the total energy equation for a dry atmosphere (e.g. CESM/CAM)?

   Dynamical core total energy errors and physics-dynamics coupling errors in CESM/CAM

2. What is the total energy equation for a moist atmosphere with condensates?

   What approximations do we make in CESM/CAM?

   Assess energy budget errors in CESM/CAM physics

   Path forward

$$E_{atm} = E_{feom} + E_{other}.$$

(feom=fluid equations of motion)

Un-averaged TE equations
(feom=fluid equations of motion)

The energy E_{feom} associated with the governing equations of motion is the energy of a fluid subject to gravity and rotation:

$$E_{feom} = K + I + \Phi. \tag{6}$$

The various equation sets discussed below have different expressions for kinetic energy K, internal energy I and geopotential Φ, which depend on the particular assumptions made and the choice of thermodynamics.

Common assumptions:

- Geometric   : shallow atmosphere, spherical geoid
- Dynamic   : quasi-hydrostatic, Boussinesq
- Thermodynamic : much of this talk!

Dry hydrostatic primitive equations
(feom=fluid equations of motion)

$$E_{feom} = K_h + I + \Phi$$

- K_h is horizontal kinetic energy

- In a shallow-atmosphere geometry, Φ = gz with g the constant acceleration of gravity.

- For an ideal perfect gas:

$$I = c_v^{(d)} T, \qquad (\text{dry air} = \text{ideal perfect gas})$$

TE for dry primitive equations

Assuming constant pressure at model top the hydrostatic primitive equations of motion conserve:

$$\iiint \rho^{(d)} E_{feom} dV = \iiint \rho^{(d)} \left(K_h + \underbrace{I + \frac{p^{(d)}}{\rho^{(d)}}}_{\text{specific enthalpy}} + \Phi_s \right) dA \, dz$$

(Kasahara, 1974)

$$I + p^{(d)}/\rho^{(d)} = c_p^{(d)} T, \qquad \text{(ideal perfect gas)}$$

TE for dry primitive equations

Caution: Since its mass-weighted integral coincides with total energy, it is tempting to regard

$$K_h + c_p^{(d)} T + \Phi_s$$

as total energy per unit mass. **This is incorrect!** In the derivation it has been assumed that

- Pressure is constant at model lid
- Integration by parts used

$$\iiint \rho^{(d)} E_{feom} dV = \iiint \rho^{(d)} \left(K_h + \underbrace{I + \frac{p^{(d)}}{\rho^{(d)}}}_{\text{specific enthalpy}} + \Phi_s \right) dA \, dz$$

(Kasahara, 1974)

$$I + p^{(d)}/\rho^{(d)} = c_p^{(d)} T, \qquad \text{(ideal perfect gas)}$$

Experiments with CAM: Held-Suarez (dry)

Total energy errors (CAM-SE dynamical core)

Horizontal inviscid dynamics: Energy errors resulting from solving the inviscid, adiabatic equations of motion. (zero up to time truncation errors)

Hyperviscosity: Energy errors resulting from filtering operators

Note that we use frictional heating, i.e. kinetic energy change resulting from hyperviscosity operators added locally back as heating

Vertical remapping: Mapping from Lagrangian to Eulerian reference surfaces does not conserve TE.

1 degree, 32 levels, CESM2.2 updates, FHS94
(1 month spin-up; 2 month ave)

Total adiabatic dynamics : -0.03 W/M^2

2D dynamics   : -0.015 W/M^2
Vertical remapping   : -0.015 W/M^2

Explicit diffusion total   : -0.015 W/M^2

Hypervis   : -0.16 W/M^2
Frictional heating   : 0.14 W/M^2
Hypervis + fric heat   : -0.015 W/M^2
Sponge   : -3.41e-05 W/m^2

Residual (time truncation errors)   : 4.69e-06 W/M^2

3.4. Mimetic Discretization

A numerical method is mimetic (or compatible) if key integral properties of divergence, gradient, and curl operators are mimicked in discretized space. The CAM-SE discretization satisfies the divergence/gradient adjoint relation

$$\int \phi \nabla \cdot \mathbf{v} \, dS + \int \mathbf{v} \cdot \nabla \phi \, dS = 0 \qquad (89)$$

in discretized space (Taylor & Fournier, 2010). This property can be used to show the inherent conservation properties of CAM-SE in the horizontal discretization (see Taylor, 2011, for details). Mass is conserved, and, in the absence of viscosity terms, energy is conserved with exact time stepping.

Lauritzen et al. (2018)

Lauritzen and Williamson (2019)

Experiments with CAM: Held-Suarez (dry)

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Vertical remapping: Mapping from Lagrangian to Eulerian reference surfaces does not conserve TE.

1 degree, 32 levels, CESM2.2 updates, FHS94 (1 month spin-up; 2 month ave)		1/4 degree, 32 levels, CESM2.2 updates, FHS94 (1 month spin-up; 2 month ave)	
Total adiabatic dynamics	: -0.03 W/M^2	Total adiabatic dynamics	: -0.03 W/M^2
2D dynamics	**: -0.015 W/M^2**	**2D dynamics**	**: -0.002 W/M^2**
Vertical remapping	: -0.015 W/M^2	Vertical remapping	: -0.03 W/M^2
Explicit diffusion total	**: -0.015 W/M^2**	**Explicit diffusion total**	**: -0.002 W/M^2**
Hypervis	: -0.16 W/M^2	Hypervis	: -0.0216 W/M^2
Frictional heating	: 0.14 W/M^2	Frictional heating	: 0.0198 W/M^2
Hypervis + fric heat	: -0.015 W/M^2	Hypervis + fric heat	: -0.0018 W/M^2
Sponge	: -3.41e-05 W/m^2	Sponge	: -9.94e-05 W/m^2
Residual (time truncation errors)	**: 4.69e-06 W/M^2**	**Residual (time truncation errors)**	**: 3.65e-07 W/M^2**

TE budget convergence for horizontal dynamics

Experiments with CAM: Held-Suarez (dry)

SE-CSLAM

Vertical velocity at 500 mbar pressure surface — Pa/s
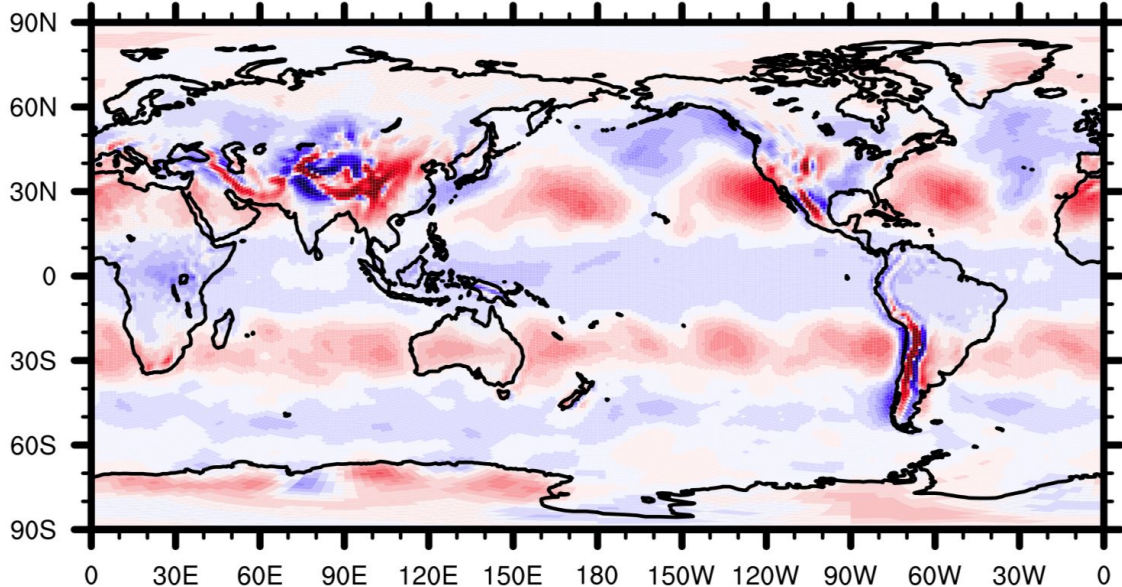
Real-world topography

⬇

Strong grid-scale forcing

⬇

Activates diffusion/viscosity operators

⬇

More total energy dissipation

Experiments with CAM: Held-Suarez with topography

Total energy errors (CAM-SE dynamical core)

Horizontal inviscid dynamics: Energy errors resulting from solving the inviscid, adiabatic equations of motion. (zero up to time truncation errors)

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Vertical remapping: Mapping from Lagrangian to Eulerian reference surfaces does not conserve TE.

	1 degree, 32 levels, CESM2.2 updates, FHS94 (1 month spin-up; 2 month ave)	1 degree, 32 levels, CESM2.2 updates, FHS94 (1 month spin-up; 2 month ave) - /w topo
Total adiabatic dynamics	: -0.03 W/M^2	: -0.14 W/M^2
2D dynamics	**: -0.015 W/M^2**	**: -0.13 W/M^2**
Vertical remapping	: -0.015 W/M^2	: -0.01W/M^2
Explicit diffusion total	**: -0.015 W/M^2**	**: -0.14 W/M^2**
Hypervis	: -0.16 W/M^2	: -0.30 W/M^2
Frictional heating	: 0.14 W/M^2	: 0.17 W/M^2
Hypervis + fric heat	: -0.015 W/M^2	: -0.14 W/M^2
Sponge	: -3.41e-05 W/m^2	: -3.1e-05 W/m^2
Residual (time truncation errors)	**: 4.69e-06 W/M^2**	**: 0.006 W/M^2**

Topography alters TE budget!

- Almost all terms increase

Experiments with CAM: Full physics

Total energy errors (CAM-SE dynamical core)

Horizontal inviscid dynamics: Energy errors resulting from solving the inviscid, adiabatic equations of motion. (zero up to time truncation errors)

Hyperviscosity: Energy errors resulting from filtering operators

Note that we use frictional heating, i.e. kinetic energy change resulting from hyperviscosity operators added locally back as heating

Vertical remapping: Mapping from Lagrangian to Eulerian reference surfaces does not conserve TE.

---

1 degree, 32 levels, CESM2.2 updates, F2000climo (1 month spin-up; 2 month ave)

Total adiabatic dynamics	: -0.24 W/M^2
2D dynamics	**: -0.12 W/M^2**
Vertical remapping	: -0.12 W/M^2
Explicit diffusion total	**: -0.13 W/M^2**
Hypervis	: -0.62 W/M^2
Frictional heating	: 0.49 W/M^2
Hypervis + fric heat	: -0.13 W/M^2
Sponge	: -0.0002 W/m^2
Residual (time truncation errors)	**: -0.005 W/M^2**

1 degree, 32 levels, CESM2.2 updates, FHS94 (1 month spin-up; 2 month ave) - /w topo

Total adiabatic dynamics	: -0.14 W/M^2
2D dynamics	**: -0.13 W/M^2**
Vertical remapping	: -0.01W/M^2
Explicit diffusion total	**: -0.14 W/M^2**
Hypervis	: -0.30 W/M^2
Frictional heating	: 0.17 W/M^2
Hypervis + fric heat	: -0.14 W/M^2
Sponge	: -3.1e-05 W/m^2
Residual (time truncation errors)	**: 0.006 W/M^2**

Full physics increases TE dissipation by ~70%:

- Vertical remap 10x larger
- Explicit diffusion approximately the same but frictional heating 2x larger

Experiments with CAM: Full physics

Total energy errors (CAM-SE dynamical core)

Horizontal inviscid dynamics: Energy errors resulting from solving the inviscid, adiabatic equations of motion. (zero up to time truncation errors)

Hyperviscosity: Energy errors resulting from filtering operators

Note that we use frictional heating, i.e. kinetic energy change resulting from hyperviscosity operators added locally back as heating

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Residual (time truncation errors) : -0.005 W/M^2

1 degree, 32 levels, CESM2.2 updates, FHS94 (1 month spin-up; 2 month ave) - /w topo

Total adiabatic dynamics : -0.14 W/M^2

2D dynamics : -0.13 W/M^2
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Hypervis + fric heat : -0.14 W/M^2
Sponge : -3.1e-05 W/m^2

Residual (time truncation errors) : 0.006 W/M^2

TE dissipation in other CESM dynamical cores:

FV and FV3: ~-1.1 W/m^2 (SE clearly an improvement)

Will have numbers for MPAS soon

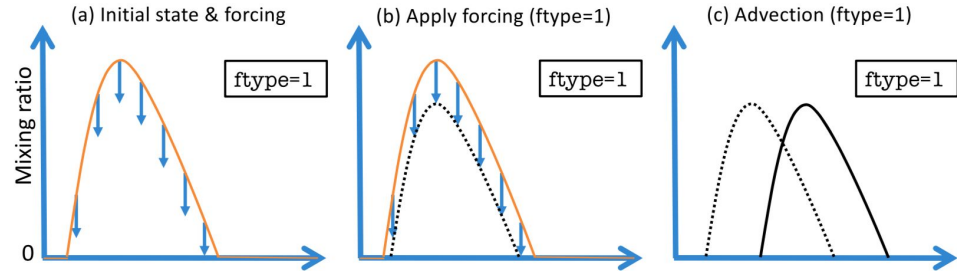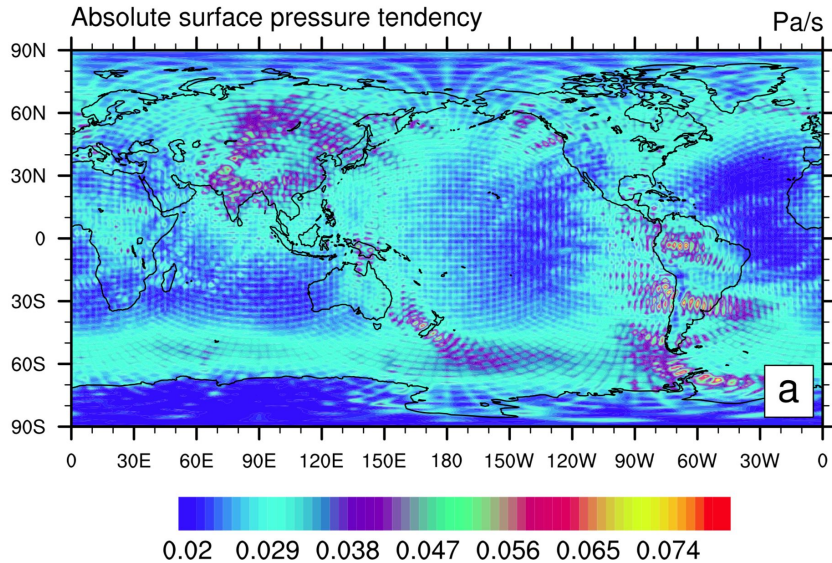Full physics increases TE dissipation by ~70%:

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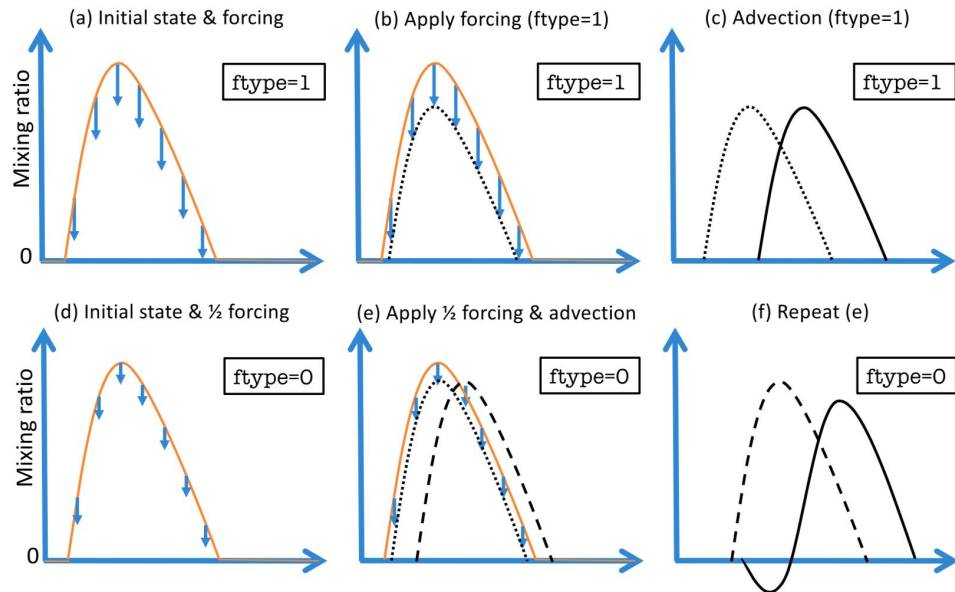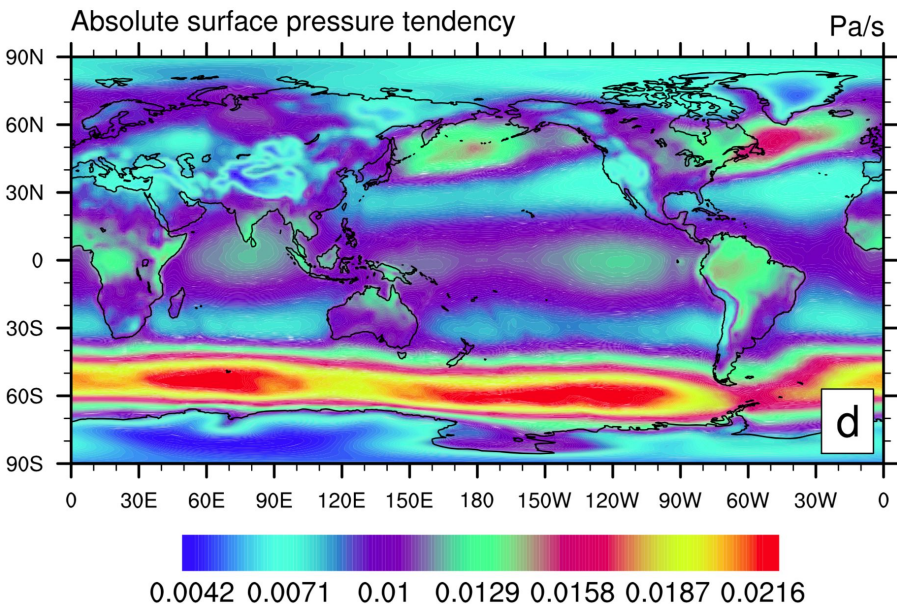# Physics-dynamics coupling

CAM-SE, cpdry, ftype=1 (state-update)

Absolute surface pressure tendency — Pa/s

a

0.02 0.029 0.038 0.047 0.056 0.065 0.074

(a) Initial state & forcing — ftype=1

(b) Apply forcing (ftype=1) — ftype=1

(c) Advection (ftype=1) — ftype=1

Mixing ratio

Lauritzen and
Williamson (2019)

Physics-dynamics coupling

CAM-SE-CSLAM, ftype=2 (combined)

Absolute surface pressure tendency
Pa/s

(a) Initial state & forcing ftype=1

(b) Apply forcing (ftype=1) ftype=1

(c) Advection (ftype=1) ftype=1

(d) Initial state & ½ forcing ftype=0

(e) Apply ½ forcing & advection ftype=0

(f) Repeat (e) ftype=0

Tracers state-updated; T and (u,v) updated with "dribbling"

Energy error is 0.05W/m^2 (CAM-SE)

Lauritzen and Williamson (2019)

Fix error with global energy fixer

$$-\partial \widehat{E}^{(\mathrm{efix})}_{\mathrm{phys}} = \partial \widehat{E}^{(\mathrm{pwork})}_{\mathrm{phys}} + \partial \widehat{E}^{(\mathrm{adiab})}_{\mathrm{dyn}} + \partial \widehat{E}^{(\mathrm{pdc})} + \partial \widehat{E}^{(\mathrm{discr})}$$

TE formula discrepancy error

- Different heat capacity for water vapor

  (SE,FV3: ~0.5 W/m^2)

- Different vertical coordinate + prognostic variables

  (e.g. CAM-MPAS)

Dynamical core:

SE   : -0.24 W/m^2
FV3 : -1.1 W/m^s
FV   : -1.1 W/m^2

Phys-dyn coupling:

SE   : -0.05 W/m^2
FV3 : 0.015 W/m^s
FV   : -   W/m^2

Lauritzen and Williamson (2019)

TE for dry primitive equations

Assuming constant pressure at model top the primitive equations of motion conserve:

$$\iiint \rho^{(d)} E_{feom} dV = \iiint \rho^{(d)} \left(K_h + \underbrace{I + \frac{p^{(d)}}{\rho^{(d)}}}_{\text{specific enthalpy}} + \Phi_s \right) dA\, dz$$

(Kasahara, 1974)

$$I + p^{(d)}/\rho^{(d)} = c_p^{(d)} T, \qquad \text{(ideal perfect gas)}$$

TE for dry primitive equations

where K_h eal perfect gas)

Assuming   rgy
(pressure-

$$\mathcal{L}_{all} \equiv \{`d, `wv`, `cl`, `ci`, `rn`, `sw`\}$$

$$\mathcal{L}_{H_2O} \equiv \{`wv`, `cl`, `ci`, `rn`, `sn`\}$$

Assumptions:

1. All constituents have the same temperature (T)
2. All constituents move with the same barycentric velocity
3. Ideal perfect gas

$\iiint$ (Kas

The perfect gas law is often rewritten in the form

$$p = \rho R^{(d)} T_v,$$

with virtual temperature

$$T_v = T\left(\frac{1 + \frac{1}{\epsilon} m^{(wv)}}{\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)}}\right), \quad m^{(\ell)} \equiv \frac{\rho^{(\ell)}}{\rho^{(d)}}$$

where $\epsilon \equiv \frac{R^{(d)}}{R^{(wv)}}$ is the ratio between the dry and water vapor gas constants

Specific Enthalpy of moist air

The specific enthalpy of an air constituent ℓ can be written on the form

$$h^{(\ell)} = h_{00}^{(\ell)} + c_p^{(\ell)}(T - T_{00}),$$

$$\rho I + p = \sum_{\ell \in \mathcal{L}_{all}} \rho^{(\ell)} \left[h_{00}^{(\ell)} + c_p^{(\ell)}(T - T_{00}) \right]$$

$$\mathcal{L}_{all} \{\text{'}d, \text{'}wv\text{'}, \text{'}cl\text{'}, \text{'}ci\text{'}, \text{'}rn\text{'}, \text{'}sw\text{'}\}$$

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right) + \rho^{(wv)} \left(h_{00}^{(wv)} + c_p^{(wv)}(T - T_{00}) \right) +$$

$$\rho^{(liq)} \left(h_{00}^{(liq)} + c_p^{(liq)}(T - T_{00}) \right) + \rho^{(ice)} \left(h_{00}^{(ice)} + c_p^{(ice)}(T - T_{00}) \right),$$

$$c_p^{(ice)} = c_p^{(sn)} = c_p^{(ci)} \text{ and } c_p^{(liq)} = c_p^{(rn)} = c_p^{(cl)}$$

Specific Enthalpy of moist air

The specific enthalpy of an air constituent ℓ can be written on the form

$$h^{(\ell)} = h_{00}^{(\ell)} + c_p^{(\ell)}(T - T_{00}),$$

Only enthalpy differences are of physical relevance

-> rewrite equations using Kirchoff's equation ...

$$\rho I + p = \sum_{\ell \in \mathcal{L}_{all}} \rho^{(\ell)} \left[h_{00}^{(\ell)} + c_p^{(\ell)}(T - T_{00}) \right]$$

$$\mathcal{L}_{all} \{\text{`}d, \text{`}wv\text{`}, \text{`}cl\text{`}, \text{`}ci\text{`}, \text{`}rn\text{`}, \text{`}sw\text{`}\}$$

$$
\begin{aligned}
\rho I + p = \ & \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right) + \rho^{(wv)} \left(h_{00}^{(wv)} + c_p^{(wv)}(T - T_{00}) \right) + \\
& \rho^{(liq)} \left(h_{00}^{(liq)} + c_p^{(liq)}(T - T_{00}) \right) + \rho^{(ice)} \left(h_{00}^{(ice)} + c_p^{(ice)}(T - T_{00}) \right),
\end{aligned}
$$

$$c_p^{(ice)} = c_p^{(sn)} = c_p^{(ci)} \text{ and } c_p^{(liq)} = c_p^{(rn)} = c_p^{(cl)}$$

Specific Enthalpy of moist air

The latent heat formulas for vaporization (liquid $\rightarrow$ water vapor):

$$L_v(T) = L_{v,00} + \left(c_p^{(wv)} - c_p^{(liq)} \right) (T - T_{00}) \, , \text{ where } L_{v,00} \equiv h_{00}^{(wv)} - h_{00}^{(liq)} \qquad (44)$$

The latent heat formulas for sublimation (solid $\rightarrow$ water vapor):

$$L_s(T) = L_{s,00} + \left(c_p^{(wv)} - c_p^{(ice)} \right) (T - T_{00}) \, , \text{ where } L_{s,00} \equiv h_{00}^{(wv)} - h_{00}^{(ice)} \, , \qquad (45)$$

The latent heat formulas for fusion (solid $\rightarrow$ liquid):

$$L_i(T) = L_{i,00} + \left(c_p^{(liq)} - c_p^{(ice)} \right) (T - T_{00}) \, , \text{ where } L_{i,00} \equiv h_{00}^{(liq)} - h_{00}^{(ice)} \, , \qquad (46)$$

(Emanuel, 1994, see, e.g., p. 114-5). Note that the latent heat of fusion, $L_i(T)$ may also be written in terms of latent heat of vaporization and sublimation

$$L_i(T) = L_s(T) - L_v(T). \qquad (47)$$

Specific enthalpy of moist air:
Reference state: 'wv', 'liq', 'ice'

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right) + \rho^{(H_2O)} \left(h_{00}^{(wv)} + c_p^{(wv)} (T - T_{00}) \right)$$

$$- \rho^{(liq)} L_v(T) - \rho^{(ice)} L_s(T).$$

(water vapor reference state)

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right) + \rho^{(H_2O)} \left(h_{00}^{(liq)} + c_p^{(liq)} (T - T_{00}) \right)$$

$$+ \rho^{(wv)} L_v(T) - \rho^{(ice)} L_i(T)$$

(liquid reference state)

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right) + \rho^{(H_2O)} \left(h_{00}^{(ice)} + c_p^{(ice)} (T - T_{00}) \right)$$

$$+ \rho^{(wv)} L_s(T) + \rho^{(liq)} L_i(T),$$

(ice reference state)

Specific enthalpy of moist air: Reference state: 'wv', 'liq', 'ice'

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \cancel{\left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right)} + \rho^{(H_2O)} \left(h_{00}^{(wv)} + c_p^{(wv)}(T - T_{00}) \right)$$
$$- \rho^{(liq)} L_v(T) - \rho^{(ice)} L_s(T).$$

(water vapor reference state)

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \cancel{\left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right)} + \rho^{(H_2O)} \left(h_{00}^{(liq)} + c_p^{(liq)}(T - T_{00}) \right)$$
$$+ \rho^{(wv)} L_v(T) - \rho^{(ice)} L_i(T)$$

(liquid reference state)

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \cancel{\left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right)} + \rho^{(H_2O)} \left(h_{00}^{(ice)} + c_p^{(ice)}(T - T_{00}) \right)$$
$$+ \rho^{(wv)} L_s(T) + \rho^{(liq)} L_i(T),$$

(ice reference state)

When taking the global integral and time-derivative of enthalpy, crossed out terms are constant if dry air mass is constant!

Specific enthalpy of moist air:
Reference state: 'wv', 'liq', 'ice'

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(H_2O)} \left(h_{00}^{(wv)} + c_p^{(wv)} (T - T_{00}) \right) - \rho^{(liq)} L_v(T) - \rho^{(ice)} L_s(T),$$
(water vapor reference state)

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(H_2O)} \left(h_{00}^{(liq)} + c_p^{(liq)} (T - T_{00}) \right) + \rho^{(wv)} L_v(T) - \rho^{(ice)} L_i(T),$$
(liquid reference state)

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(H_2O)} \left(h_{00}^{(ice)} + c_p^{(ice)} (T - T_{00}) \right) + \rho^{(wv)} L_v(T) + \rho^{(liq)} L_i(T),$$
(ice reference state)

Ice reference state is used by CAM

Water vapor reference state is a "weird" choice for coupled system

Total energy equation - no fluxes
("unapproximated")

$$\iiint \rho \left(E_{feom} \right) dA\, dz = \iiint \rho^{(d)} \Bigg[\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} \left(K + \Phi_s \right) +$$

$$c_p^{(d)} T + m^{(H_2O)} c_p^{(ice)} T$$

$$m^{(H_2O)} \left[h_{00}^{(ice)} - c_p^{(ice)} T_{00} \right]$$

$$+ m^{(wv)} L_s(T) + m^{(liq)} L_i(T) \Bigg] dA\, dz$$

(ice reference state)

If we choose zero specific enthalpy reference at 0K then **third term on r.h.s.** vanishes
(if non-zero reference state is chosen the term is cancelled by a boundary flux term)

Total energy equation - no fluxes
("unapproximated")

$$\iiint \rho \left(E_{feom} \right) dA\, dz = \iiint \rho^{(d)} \left[\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} \left(K + \Phi_s \right) + \right.$$

$$c_p^{(d)} T + m^{(H_2O)} c_p^{(ice)} T$$

$$m^{(H_2O)} \left[h_{00}^{(ice)} \xcancel{c_p^{(ice)} T_{00}} \right]$$

$$\left. + m^{(wv)} L_s(T) + m^{(liq)} L_i(T) \right] dA\, dz$$

(ice reference state)

If we choose zero specific enthalpy reference at 0K then **third term on r.h.s.** vanishes
(if non-zero reference state is chosen the term is cancelled by a boundary flux term)

Total energy equation - no fluxes
("unapproximated")

$$\iiint \rho\left(E_{feom}\right) dA\, dz = \iiint \rho^{(d)} \Bigg[\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} \left(K + \Phi_s\right) +$$

$$c_p^{(d)} T + m^{(H_2O)} c_p^{(ice)} T$$

$$m^{(H_2O)} \left[h_{00}^{(ice)} \cancel{c_p^{(ice)} T_{00}} \right]$$

$$+ m^{(wv)} L_s(T) + m^{(liq)} L_i(T) \Bigg] dA\, dz$$

If we expand **latent heat and water terms**, assume water is constant and remove constant terms in the global integral, then it can be shown that the equation can be rewritten as

$$\iiint \rho\left(E_{feom}\right) dA\, dz = \iiint \rho^{(d)} \left[\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} \left(K + \Phi_s + c_p^{(\ell)} T \right) \right] dA\, dz.$$

Used in CAM-SE in CESM2.2

Which is the same as the pseudo energy derived in Lauritzen et al. (2018)

Flux at the surface

Flux from atmosphere to surface

$$F_a = \sum_{\ell=\{'wv','liq','ice'\}} \dot{m}_a^{(\ell)} h^{(\ell)}(T_{a,surf})$$

where $\dot{m}_a^{(\ell)}$ is flux of water species ℓ from atmosphere to surface.

Processes:

'liq' : rain
'ice' : snow
'wv' : deposition to surface
       through dew and rime

$$F_a = \dot{m}_a^{(H_2O)} h_0 + \dot{m}_a^{(ice)} \left[c_p^{(ice)} T_{a,surf} \right] +$$
$$\dot{m}_a^{(wv)} \left[L_s(T_{a,surf}) + c_p^{(ice)} T_{a,surf} \right] + \dot{m}_a^{(liq)} \left[L_i(T_{a,surf}) + c_p^{(ice)} T_{a,surf} \right],$$

$$(\text{ice reference state: } h_0 \equiv h_{00}^{(ice)} - c_p^{(ice)} T_{00})$$
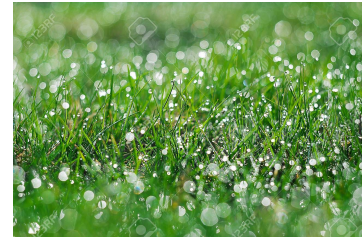
Flux at the surface

Flux from surface to atmosphere

$$F_s = \sum_{\ell = \{'wv', 'liq', 'ice'\}} \dot{m}_a^{(\ell)} h^{(\ell)}(T_{s,surf})$$

where $\dot{m}_s^{(\ell)}$ is flux of water species ℓ from surface to atmosphere.

Processes:

'liq' : spray
'ice' : snowdrift
'wv' : evaporation

$$F_s = \dot{m}_s^{(H_2O)} h_0 + \dot{m}_s^{(ice)} \left[c_p^{(ice)} T_{s,surf} \right] +$$
$$\dot{m}_s^{(wv)} \left[L_s(T_{s,surf}) + c_p^{(ice)} T_{s,surf} \right] + \dot{m}_s^{(liq)} \left[L_i(T_{s,surf}) + c_p^{(ice)} T_{s,surf} \right]$$

$$(\text{ice reference state: } h_0 \equiv h_{00}^{(ice)} - c_p^{(ice)} T_{00})$$

Net flux at the surface

Net flux into atmosphere at surface:

$$
\begin{aligned}
F_s - F_a \;=\; & h_0 \left[\dot{m}_a^{(H_2O)} - \dot{m}_s^{(H_2O)} \right] + c_p^{(ice)} \left[\dot{m}_s^{(wv)} T_{s,surf} - \dot{m}_a^{(wv)} T_{a,surf} + \right. \\
& \left. \dot{m}_s^{(liq)} T_{s,surf} - \dot{m}_a^{(liq)} T_{a,surf} + \dot{m}_s^{(ice)} T_{s,surf} - \dot{m}_a^{(ice)} T_{a,surf} \right] + \qquad (78) \\
& \dot{m}_s^{(wv)} L_s(T_{s,surf}) - \dot{m}_a^{(wv)} L_s(T_{a,surf}) + \dot{m}_s^{(ice)} L_i(T_{s,surf}) - \dot{m}_a^{(ice)} L_i(T_{a,surf})
\end{aligned}
$$

$$\text{(ice reference state: } h_0 \equiv h_{00}^{(ice)} - c_p^{(ice)} T_{00})$$

$$
\begin{aligned}
L_{s,00} &= 2834700.0 \, J \, kg^{-1} \\
L_{i,00} &= 333700.0 \, J \, kg^{-1}
\end{aligned}
$$

If we assume same temperature for atmosphere and surface as well as zero reference enthalpy then

$$F_s - F_a = c_p^{(ice)} \widetilde{T} \dot{m}^{(h_2O)} + \dot{m}^{(wv)} L_s(\widetilde{T}) + \dot{m}^{(liq)} L_i(\widetilde{T})$$

$$\text{(ice reference state: } h_0 \equiv h_{00}^{(ice)} - c_p^{(ice)} T_{00} \equiv 0 \text{ and } \widetilde{T} \equiv T_{a,surf} = T_{s,surf})$$

In the above only surface enthalpy fluxes have been discussed which ignores the turbulent (sensible) heat flux at the surface as well as the radiative flux leaving/entering the system. Hence the total net flux into the atmosphere from the surface and model top is

$$F_{net} \equiv F_s - F_a + F^{(turb)} - F^{(rad)} + F^{(\epsilon)} \tag{84}$$

where $F^{(turb)}$ which is the turbulent (sensible) heat flux from the surface to the atmosphere, $F^{(rad)}$ is radiative energy flux leaving the atmosphere and $F^{(\epsilon)}$ is a residual term that holds energy transfers that we are not accounting for.

Total energy equation

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left(m^{(d)} + m^{(H_2O)} \right) (K + \Phi_s) + c_p^{(d)} T + m^{(H_2O)} c_p^{(ice)} T + m^{(wv)} L_s(T) + m^{(liq)} L_i(T) \right\} dA\, dz$$

$$= \iint \left\{ c_p^{(ice)} \widetilde{T} \dot{m}^{(h_2O)} + \dot{m}^{(wv)} L_s(\widetilde{T}) + \dot{m}^{(liq)} L_i(\widetilde{T}) + F^{(turb)} - F^{(rad)} + F^{(\epsilon)} \right\} dA, \quad (87)$$

(assuming ice reference enthalpy: $h_0 = h_{00}^{(ice)} - c_p^{(ice)} T_{00} \equiv 0$ and $\widetilde{T} \equiv T_{a,surf} = T_{s,surf}$)

Simplified energy equation

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left(m^{(d)} + m^{(H_2O)} \right) (K + \Phi_s) + c_p^{(d)} T + m^{(H_2O)} c_p^{(ice)} T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA \, dz$$

$$= \iint \left\{ c_p^{(ice)} \widetilde{T} \dot{m}^{(h_2O)} + \dot{m}^{(wv)} L_{00,s} + \dot{m}^{(liq)} L_{00,i} + F^{(turb)} - F^{(rad)} + F^{(\epsilon)} \right\} dA, \quad (88)$$

Assume constant latent heats <=> heat capacity for water the same:

The latent heat formulas for sublimation (solid $\rightarrow$ water vapor):

$$L_s(T) = L_{s,00} + \left(c_p^{(wv)} - c_p^{(ice)} \right) (T - T_{00}), \text{ where } L_{s,00} \equiv h_{00}^{(wv)} - h_{00}^{(ice)}$$

The latent heat formulas for fusion (solid $\rightarrow$ liquid):

$$L_i(T) = L_{i,00} + \left(c_p^{(liq)} - c_p^{(ice)} \right) (T - T_{00}), \text{ where } L_{i,00} \equiv h_{00}^{(liq)} - h_{00}^{(ice)}$$

Simplified energy equation

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left(m^{(d)} + m^{(H_2O)} \right) (K + \Phi_s) + c_p^{(d)} T + m^{(H_2O)} c_p^{(ice)} T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA \, dz$$

$$= \iint \left\{ c_p^{(ice)} \widetilde{T} \dot{m}^{(h_2O)} + \dot{m}^{(wv)} L_{00,s} + \dot{m}^{(liq)} L_{00,i} + F^{(turb)} - F^{(rad)} + F^{(\epsilon)} \right\} dA, \quad (88)$$

Further simplifications in CAM:

- Assume $m^{(H_2O)} = m^{(wv)}$
  i.e. condensates do not contribute to kinetic, internal or geopotential energy (they are mass-less!)

- Assume that $m^{(H_2O)}$ during physics updates: $m^{(H_2O)} = m_{t=t^n}^{(H_2O)}$

- Discard enthalpy flux at the surface

- Use heat capacity of dry air for all forms of water

CAM energy equation

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[m^{(d)} + m_{t=t^n}^{(H_2O)} \right] (K + \Phi_s) + c_p^{(d)} T + m_{t=t^n}^{(H_2O)} c_p^{(d)} T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA\, dz$$

$$= \iint \left\{ L_{00,s} \dot{m}^{(wv)} + L_{00,i} \dot{m}^{(liq)} + F^{(turb)} - F^{(rad)} \right\} dA. \quad (90)$$

(assume $c_p^{(\ell)} = c_p^{(d)}$ for $\ell \in \mathcal{L}_{H_2O}$, $h_0 = h_{00}^{(ice)} - c_p^{(ice)} T_{00} \equiv 0$ and $\widetilde{T} \equiv T_{a,surf} = T_{s,surf}$)

Further simplifications in CAM:

- Assume $m^{(H_2O)} = m^{(wv)}$
  i.e. condensates do not contribute to kinetic, internal or geopotential energy (they are mass-less!)

- Assume that $m^{(H_2O)}$ during physics updates: $m^{(H_2O)} = m_{t=t^n}^{(H_2O)}$

- Discard enthalpy flux at the surface

- Use heat capacity of dry air for all forms of water

CAM energy equation

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[m^{(d)} + m_{t=t^n}^{(H_2O)} \right] (K + \Phi_s) + c_p^{(d)} T + m_{t=t^n}^{(H_2O)} c_p^{(d)} T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA\, dz$$

$$= \iint \left\{ L_{00,s} \dot{m}^{(wv)} + L_{00,i} \dot{m}^{(liq)} + F^{(turb)} - F^{(rad)} \right\} dA. \quad (90)$$
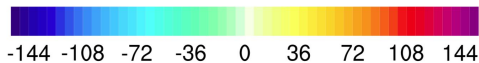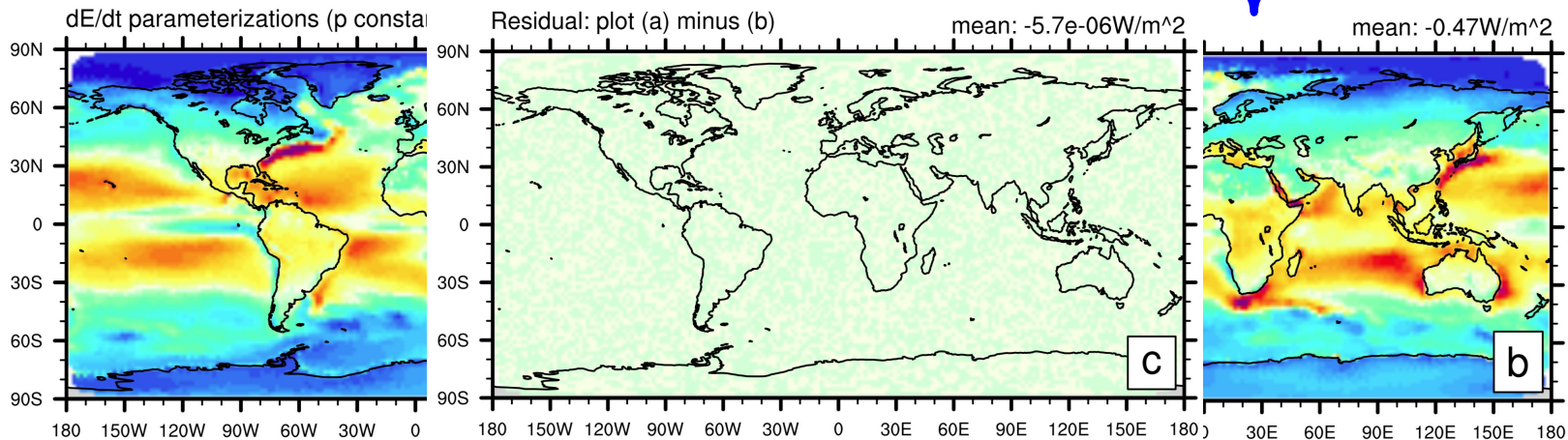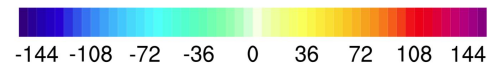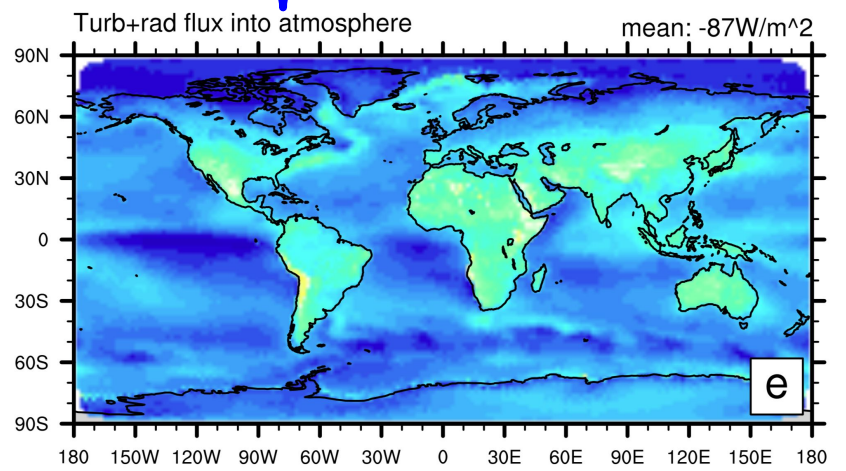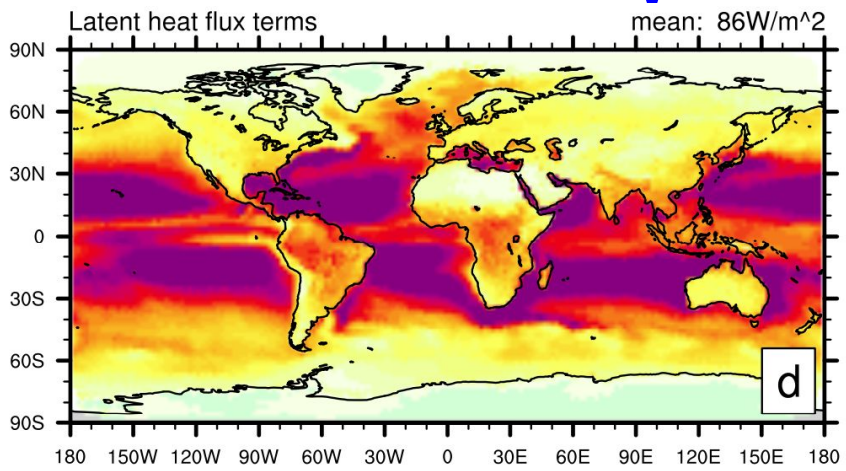
Consider a physics column (no interaction between columns so equation holds in column!):

- Phase change: E.g. water vapor to cloud liquid -> total water remains constant so K and PHIS terms remain unchanged (different story for falling precipitation)

- If there is mixing of momentum there is a corresponding T-change

- Each parameterization (in theory) satisfies this equation in CAM

- If precipitation is formed it immediately falls out of the atmosphere (but $m^{(H_2O)}$ remains constant) i.e. the precipitating water droplet or snowflake does not carry any energy in atmosphere

This system is energetically consistent - **and cleverly chosen to keep energetics simple**

CAM energy equation

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left\{\left[m^{(d)}+m_{t=t^n}^{(H_2O)}\right](K+\Phi_s)+c_p^{(d)}T+m_{t=t^n}^{(H_2O)}c_p^{(d)}T+m^{(wv)}L_{00,s}+m^{(liq)}L_{00,i}\right\}dA\,dz$$

$$=\iint\left\{\dot{m}^{(wv)}L_{00,s}+\dot{m}^{(liq)}L_{00,i}+F^{(turb)}-F^{(rad)}\right\}dA. \quad (95)$$

dE/dt parameterizations (p constant)   mean: -0.47W/m^2

Flux into atmosphere   mean: -0.47W/m^2

-144 -108 -72 -36 0 36 72 108 144

1 year ave, F2000climo, ne30pg3, CAM6
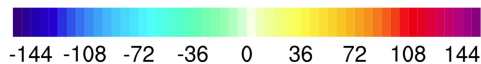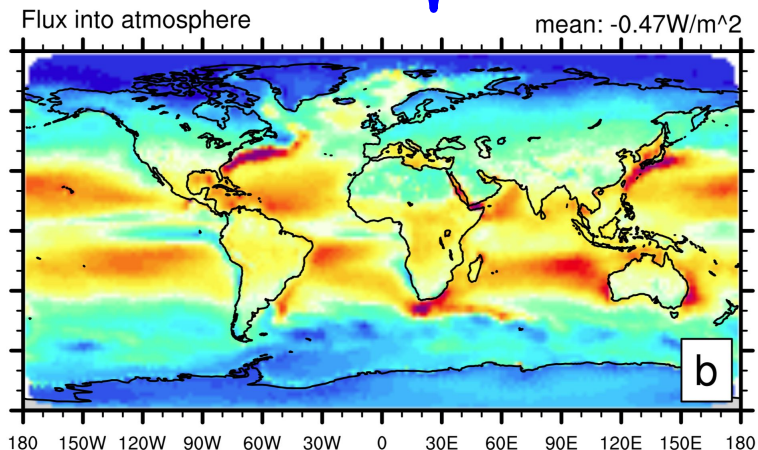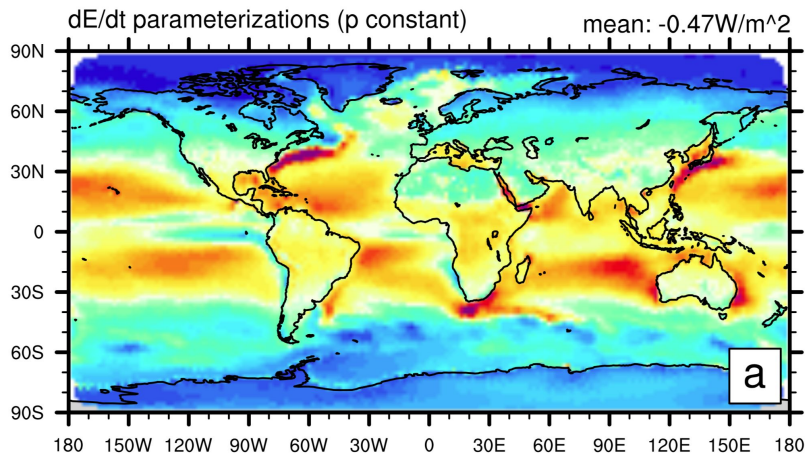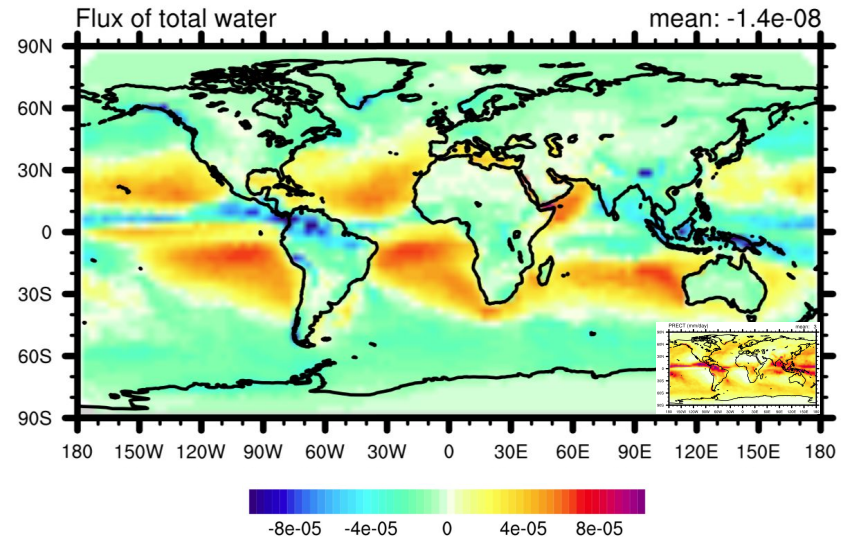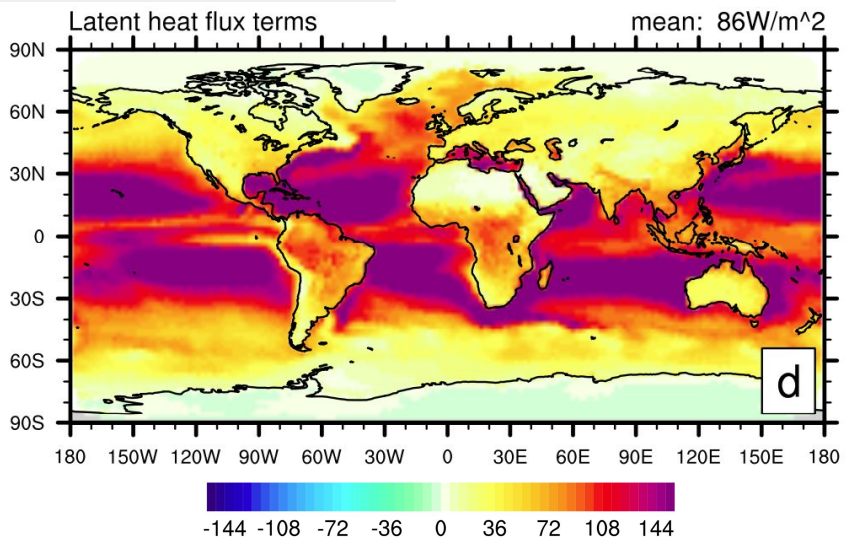
Diagnostics coded inline in CAM!

CAM energy equation

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[m^{(d)} + m_{t=t^n}^{(H_2O)} \right] (K + \Phi_s) + c_p^{(d)} T + m_{t=t^n}^{(H_2O)} c_p^{(d)} T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA \, dz$$

$$= \iint \left\{ \dot{m}^{(wv)} L_{00,s} + \dot{m}^{(liq)} L_{00,i} + F^{(turb)} - F^{(rad)} \right\} dA. \quad (95)$$

dE/dt parameterizations (p consta...   Residual: plot (a) minus (b)   mean: -5.7e-06W/m^2   mean: -0.47W/m^2

1 year ave, F2000climo, ne30pg3, CAM6

-144 -108 -72 -36 0 36 72 108 144

CAM energy equation

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[m^{(d)} + m_{t=t^n}^{(H_2O)} \right] (K + \Phi_s) + c_p^{(d)} T + m_{t=t^n}^{(H_2O)} c_p^{(d)} T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA \, dz$$
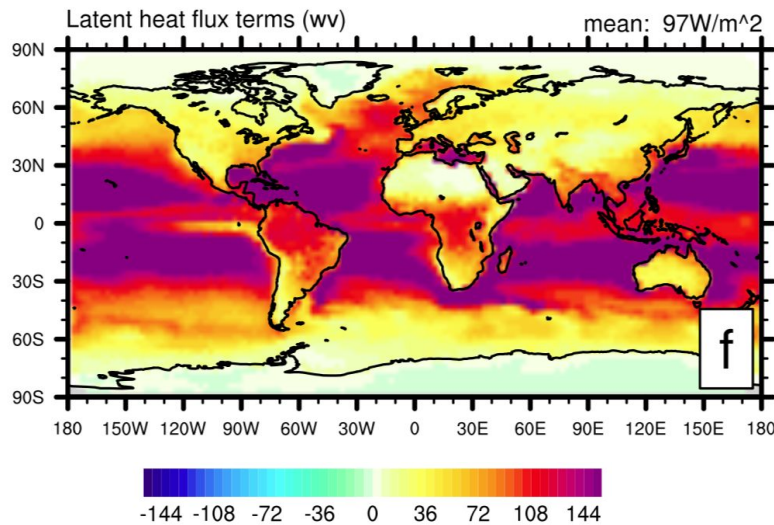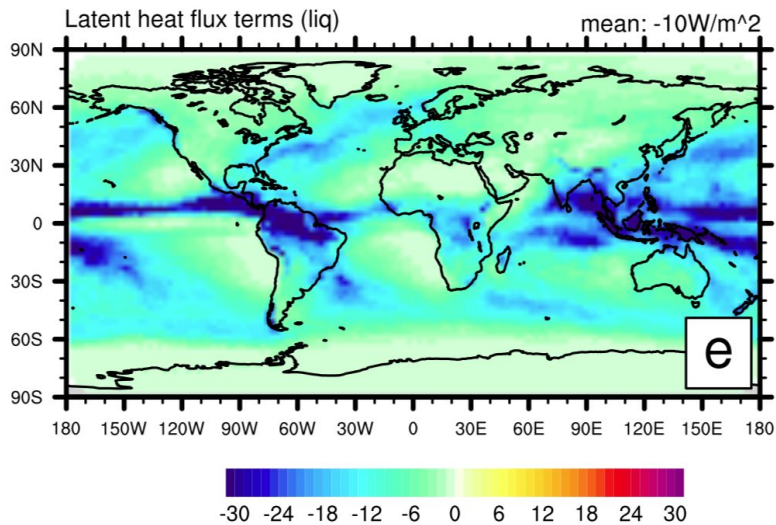
$$= \iint \left\{ \dot{m}^{(wv)} L_{00,s} + \dot{m}^{(liq)} L_{00,i} + F^{(turb)} - F^{(rad)} \right\} dA. \quad (95)$$

Latent heat flux terms — mean: 86W/m^2

Turb+rad flux into atmosphere — mean: -87W/m^2

1 year ave, F2000climo, ne30pg3, CAM6

-144 -108 -72 -36 0 36 72 108 144

CAM energy equation

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[m^{(d)} + m_{t=t^n}^{(H_2O)} \right] (K + \Phi_s) + c_p^{(d)}T + m_{t=t^n}^{(H_2O)} c_p^{(d)}T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA\, dz$$

$$= \iint \left\{ \dot{m}^{(wv)} L_{00,s} + \dot{m}^{(liq)} L_{00,i} + F^{(turb)} - F^{(rad)} \right\} dA. \quad (95)$$
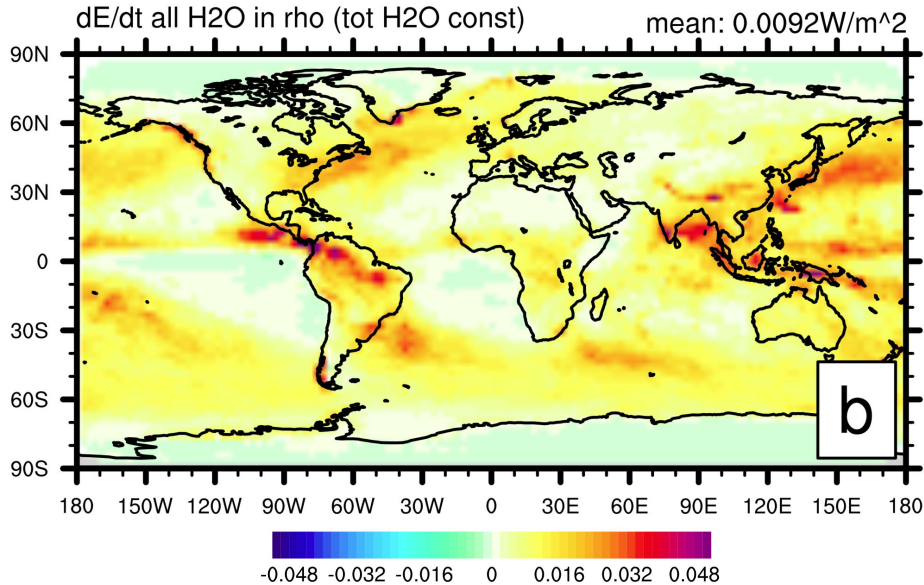
Why is latent heat flux mostly positive when water flux is both positive and negative?

Latent heat flux terms   mean: 86W/m^2

-144 -108 -72 -36 0 36 72 108 144

Flux of total water   mean: -1.4e-08

-8e-05 -4e-05 0 4e-05 8e-05

1 year ave, F2000climo, ne30pg3, CAM6

CAM energy equation

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[m^{(d)} + m_{t=t^n}^{(H_2O)} \right] (K + \Phi_s) + c_p^{(d)} T + m_{t=t^n}^{(H_2O)} c_p^{(d)} T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA\, dz$$

$$= \iint \left\{ \dot{m}^{(wv)} L_{00,s} + \dot{m}^{(liq)} L_{00,i} + F^{(turb)} - F^{(rad)} \right\} dA. \quad (95)$$

Latent heat flux terms (liq)   mean: -10W/m^2

Latent heat flux terms (wv)   mean: 97W/m^2

$$L_{s,00} = 2834700.0 \; J\,kg^{-1}$$
$$L_{i,00} = 333700.0 \; J\,kg^{-1}$$

1 year ave, F2000climo, ne30pg3, CAM6

Incl. all forms of water in $m^{(H_2O)}$

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[m^{(d)} + m_{t=t^n}^{(H_2O)} \right] (K + \Phi_s) + c_p^{(d)} T + m_{t=t^n}^{(H_2O)} c_p^{(d)} T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA \, dz$$

$$= \iint \left\{ \dot{m}^{(wv)} L_{00,s} + \dot{m}^{(liq)} L_{00,i} + F^{(turb)} - F^{(rad)} \right\} dA. \quad (95)$$

dE/dt all H2O in rho (tot H2O const)   mean: 0.0092W/m^2

b

-0.048 -0.032 -0.016   0   0.016 0.032 0.048

1 year ave, F2000climo, ne30pg3, CAM6

Instead of

$$m^{(H_2O)} = m^{(wv)}$$

lets include all forms of water:

$$m^{(H_2O)} = \sum_{\ell \in \mathcal{L}_{H_2O}} m^{(\ell)}$$

(total water still remaining constant - stays in atmosphere so energy change should stay in the atmosphere)

Using heat capacity for water vapor

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[m^{(d)} + m^{(H_2O)}_{t=t^n} \right] (K + \Phi_s) + c_p^{(d)} T + m^{(H_2O)}_{t=t^n} c_p^{(wv)} T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA \, dz$$

$$= \iint \left\{ \dot{m}^{(wv)} L_{00,s} + \dot{m}^{(liq)} L_{00,i} + F^{(turb)} - F^{(rad)} \right\} dA. \quad (93)$$

dE/dt using cpwv (const. p)   mean: 0.23W/m^2

1 year ave, F2000climo, ne30pg3, CAM6
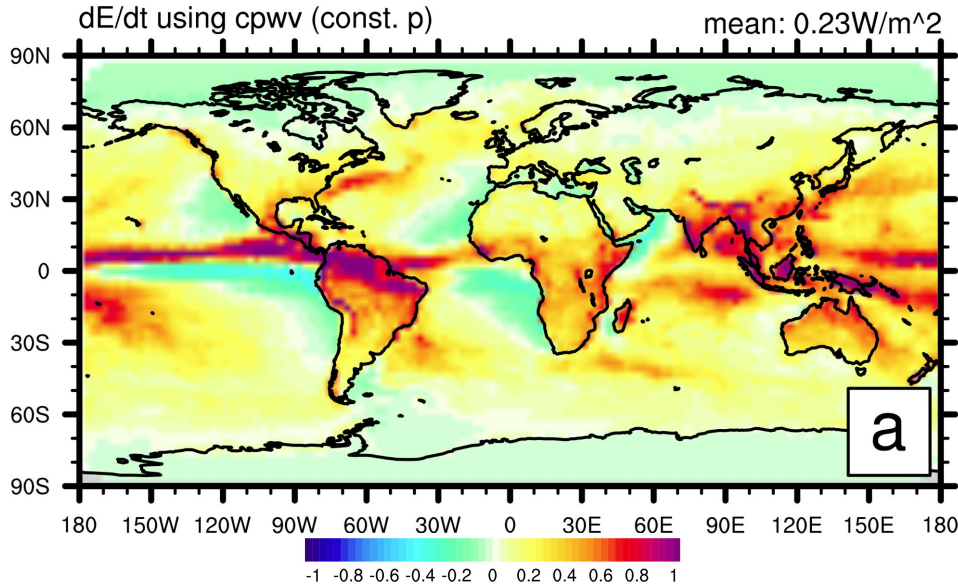
Note: this heating should remain in the atmosphere (flux terms are "balanced" in this experiment).
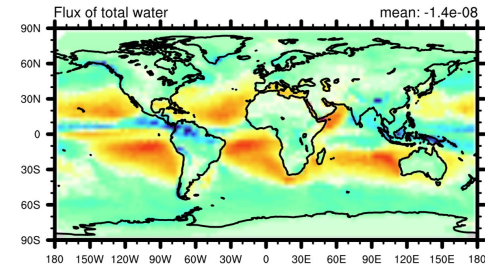
K energy associated with water change

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[m^{(d)} + m^{(H_2O)} \right] K + \left[m^{(d)} + m_{t=t^n}^{(wv)} \right] \Phi_s + c_p^{(d)} T + m_{t=t^n}^{(H_2O)} c_p^{(d)} T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA \, dz$$

$$= \iint \left\{ \dot{m}^{(wv)} L_{00,s} + \dot{m}^{(liq)} L_{00,i} + F^{(turb)} - F^{(rad)} \right\} dA. \quad (98)$$

d/dt(KE tot H2O varying)   mean: -0.0026W/m^2

Flux of total water   mean: -1.4e-08

Kinetic energy lost to surface

Very small term

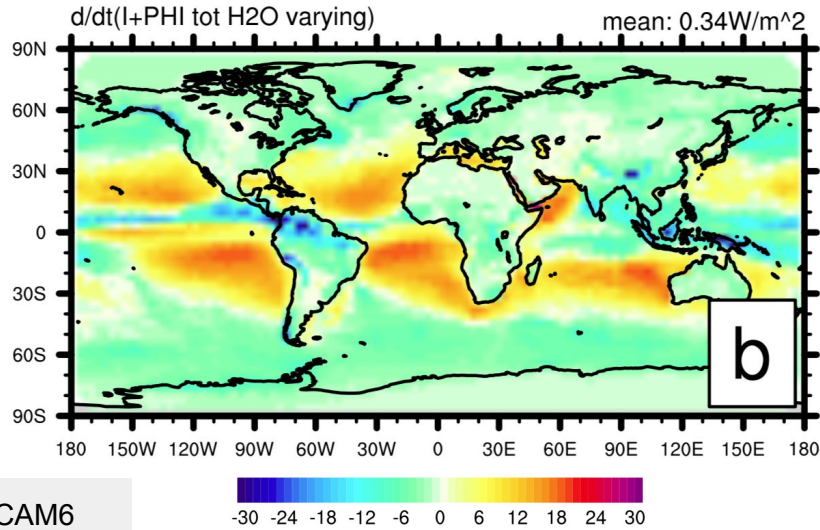Part of "dme_adjust" in CAM physics

1 year ave, F2000climo, ne30pg3, CAM6
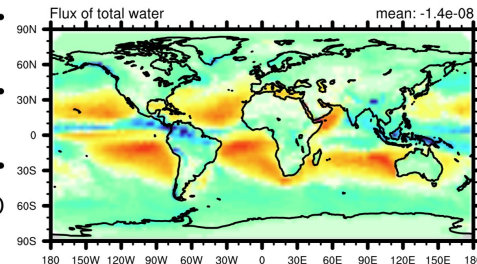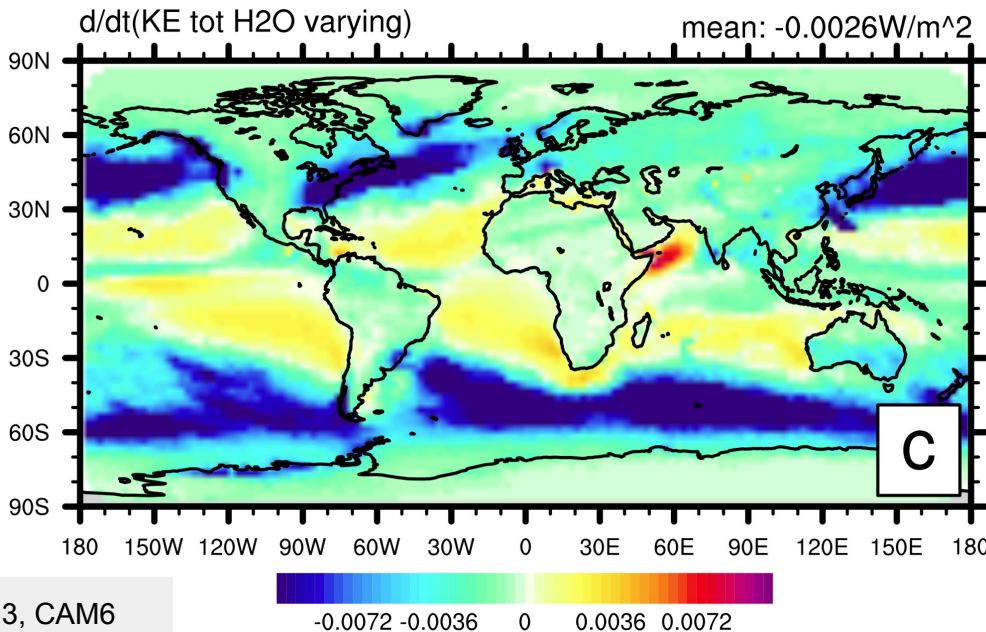
Internal and geopotential energy associated with water change

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[m^{(d)} + m^{(wv)}_{t=t^n} \right] K + \left[m^{(d)} + \underline{m^{(H_2O)}} \right] \Phi_s + c_p^{(d)} T + \underline{m^{(H_2O)}} c_p^{(d)} T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA\, dz$$

$$= \iint \left\{ \dot{m}^{(wv)} L_{00,s} + \dot{m}^{(liq)} L_{00,i} + F^{(turb)} - F^{(rad)} \right\} dA. \quad (99)$$

Much larger term and mostly due to cp*T term:

Part of "dme_adjust" in CAM physics

d/dt(I+PHI tot H2O varying)   mean: 0.34W/m^2

b

Flux of total water   mean: -1.4e-08

-30 -24 -18 -12 -6   0   6   12 18 24 30
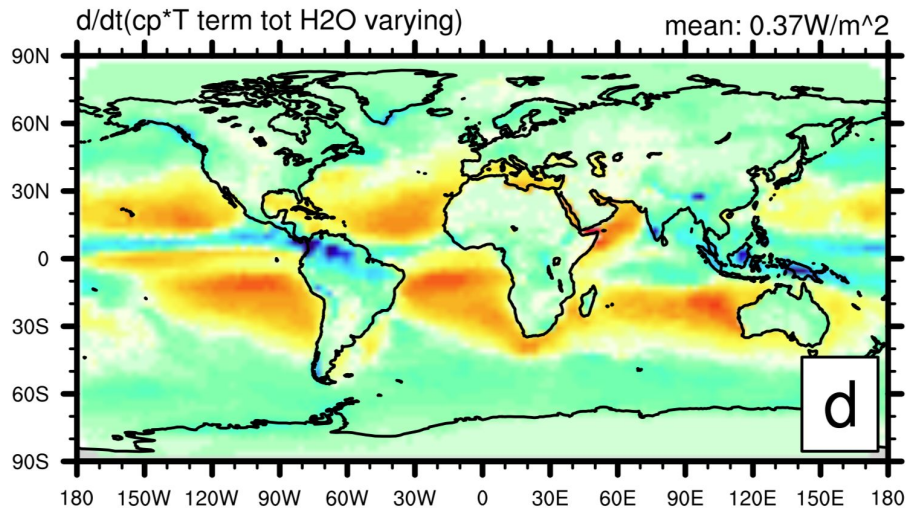
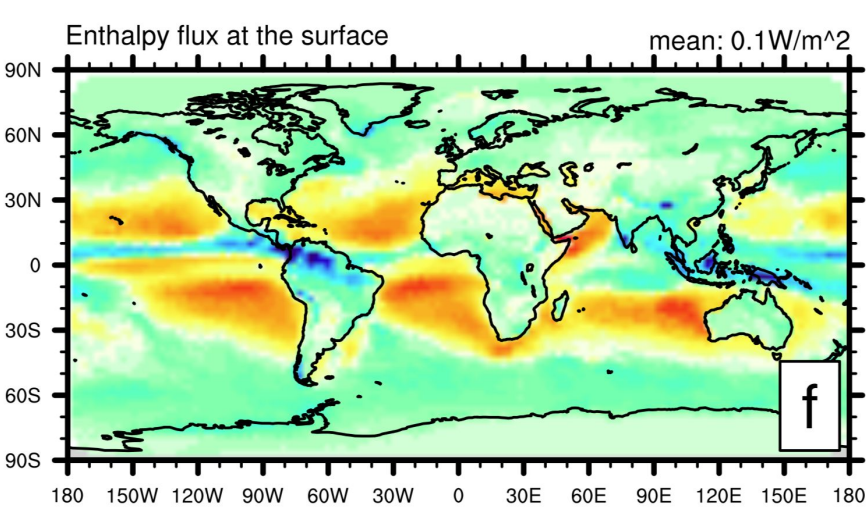1 year ave, F2000climo, ne30pg3, CAM6

Enthalpy flux at surface

using TS at the surface

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left(m^{(d)} + m^{(wv)}_{t=t^n} \right) (K + \Phi_s) + c_p^{(d)} T + m^{(wv)}_{t=t^n} c_p^{(d)} T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA \, dz$$

$$= \quad \underline{c_p^{(d)} \widetilde{T} \dot{m}^{(H_2O)}} + \dot{m}^{(wv)} L_{00,s} + \dot{m}^{(liq)} L_{00,i} + F^{(turb)} - F^{(rad)}.$$

Enthalpy flux at the surface   mean: 0.1W/m^2

f

1 year ave, F2000climo, ne30pg3, CAM6

Enthalpy flux at surface

using TS at the surface

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left(m^{(d)} + m^{(wv)}_{t=t^n} \right) (K + \Phi_s) + c_p^{(d)} T + m^{(wv)}_{t=t^n} c_p^{(d)} T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA\, dz$$

$$= \quad \underline{c_p^{(d)} \widetilde{T} \dot{m}^{(H_2O)}} + \dot{m}^{(wv)} L_{00,s} + \dot{m}^{(liq)} L_{00,i} + F^{(turb)} - F^{(rad)}.$$

Enthalpy flux at the surface   mean: 0.1W/m^2
d/dt(cp*T term tot H2O varying)   mean: 0.37W/m^2

-30 -24 -18 -12 -6 0 6 12 18 24 30

1 year ave, F2000climo, ne30pg3, CAM6

Enthalpy flux at surface

using TS at the surface

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left(m^{(d)} + m^{(wv)}_{t=t^n} \right) (K + \Phi_s) + c_p^{(d)} T + m^{(wv)}_{t=t^n} \cdots \right\} dA\, dz$$
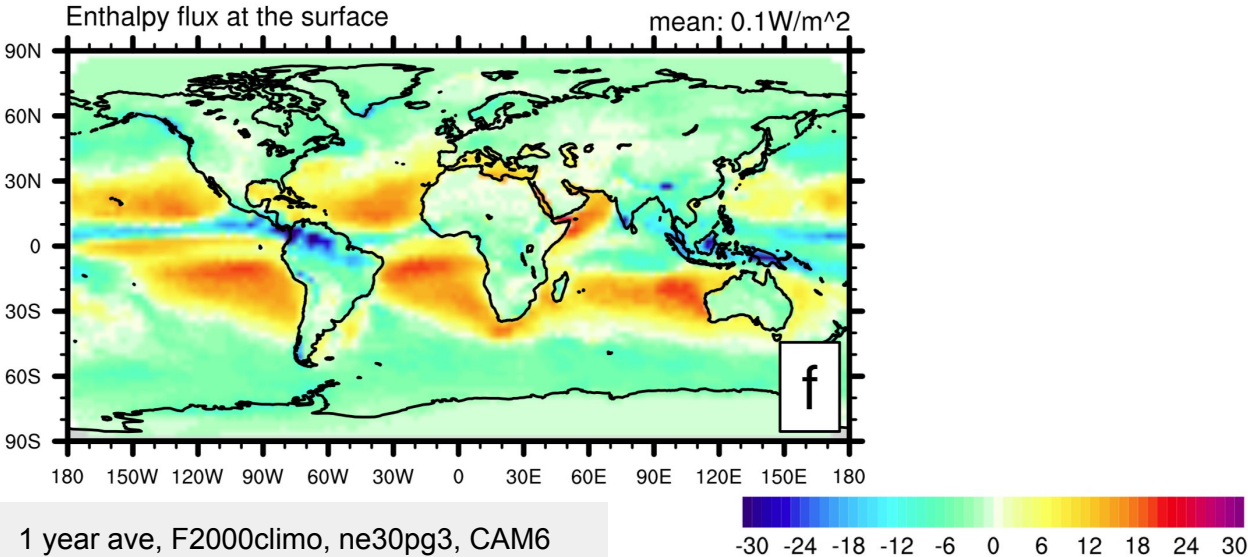
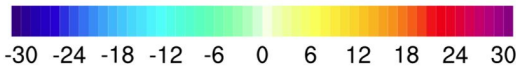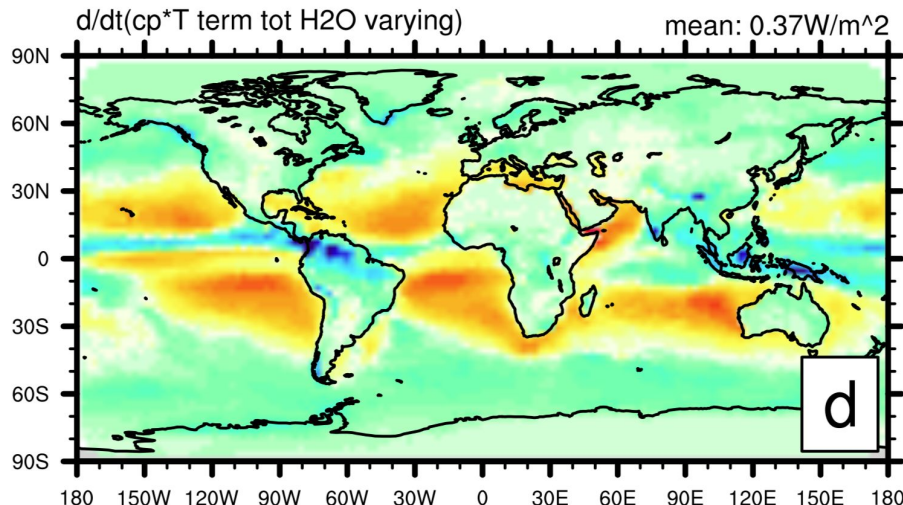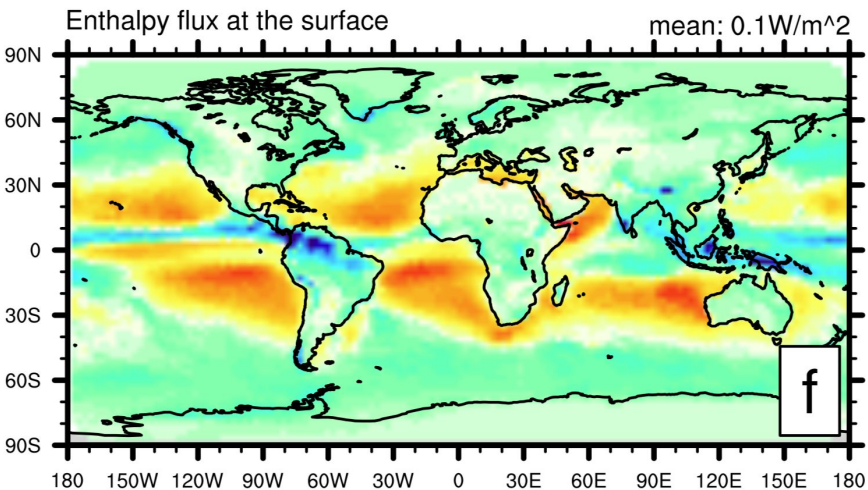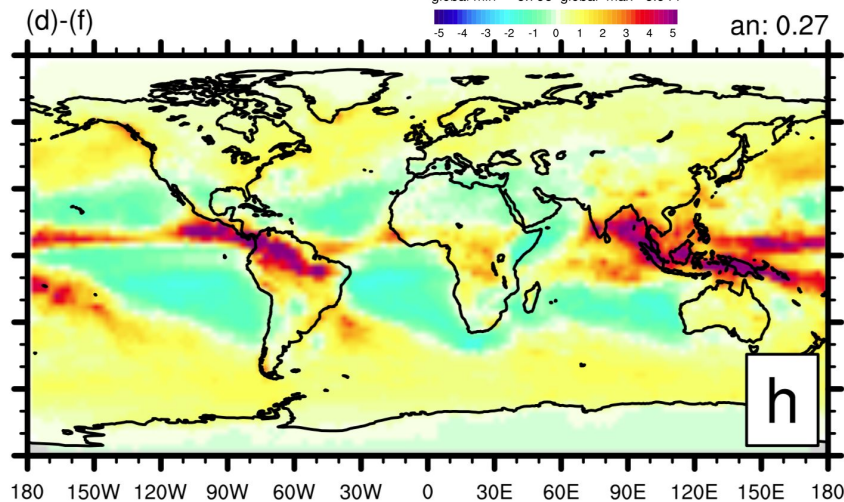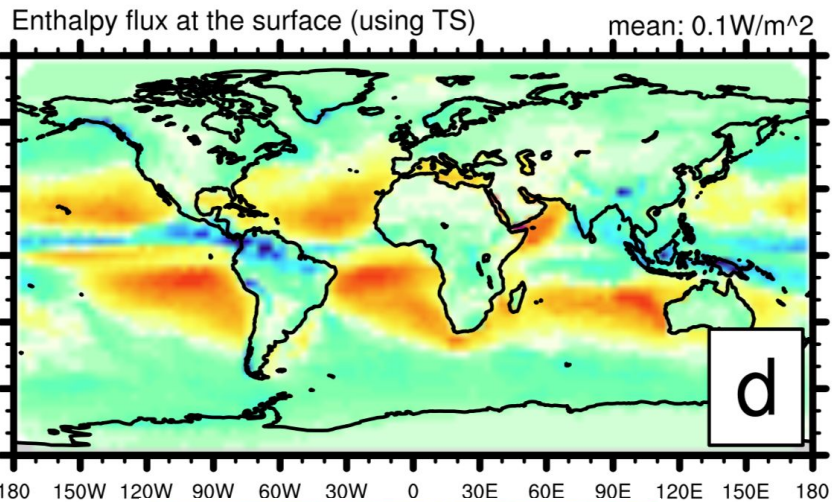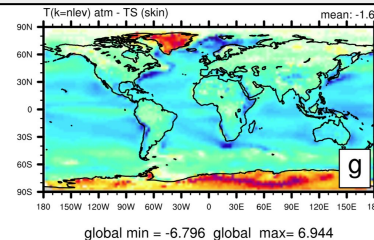$$= \quad c_p^{(d)} \widetilde{T} \dot{m}^{(H_2O)} + \dot{m}^{(wv)} L_{00,s} \cdots).$$

Local error is reduced significantly:

Pacific evaporation zones 20W/m^2 imbalance reduced to <1W/m^2

Enthalpy flux at the surface — mean: 0.1W/m^2 — f

d/dt(cp*T term tot H2O varying) — mean: 0.37W/m^2 — d

1 year ave, F2000climo, ne30pg3, CAM6

-30 -24 -18 -12 -6 0 6 12 18 24 30

They match up pretty well in terms of pattern but not entirely in magnitude! Why?

- Some of the energy probably stays in the atmosphere: frictional heating of falling rain and rain slowing down winds!

- We use same temperature in surface enthalpy (wv enters atmosphere with same T as precipitation leaves atmosphere)

$$
\begin{aligned}
F_s - F_a =\ & h_0 \left[\dot{m}_a^{(H_2O)} - \dot{m}_s^{(H_2O)} \right] + c_p^{(ice)} \left[\dot{m}_s^{(wv)} T_{s,surf} - \dot{m}_a^{(wv)} T_{a,surf} + \right. \\
& \dot{m}_s^{(liq)} T_{s,surf} - \dot{m}_a^{(liq)} T_{a,surf} + \dot{m}_s^{(ice)} T_{s,surf} - \dot{m}_a^{(ice)} T_{a,surf} \bigg] \qquad (79) \\
& \dot{m}_s^{(wv)} L_s(T_{s,surf}) - \dot{m}_a^{(wv)} L_s(T_{a,surf}) + \dot{m}_s^{(ice)} L_i(T_{s,surf}) - \dot{m}_a^{(ice)} L_i(T_{a,surf})
\end{aligned}
$$

T(k=nlev) atm - TS (skin)   mean: -1.6

global min = -6.796 global max= 6.944

-5 -4 -3 -2 -1 0 1 2 3 4 5

Enthalpy flux at the surface (using TS)   mean: 0.1W/m^2

18 24 30

(d)-(f)   an: 0.27

-2 -1.6 -1.2 -0.8 -0.4 0 0.4 0.8 1.2 1.6 2

1 year ave, F2000climo, ne30pg3, CAM6

Enthalpy flux at surface

using TS at the surface
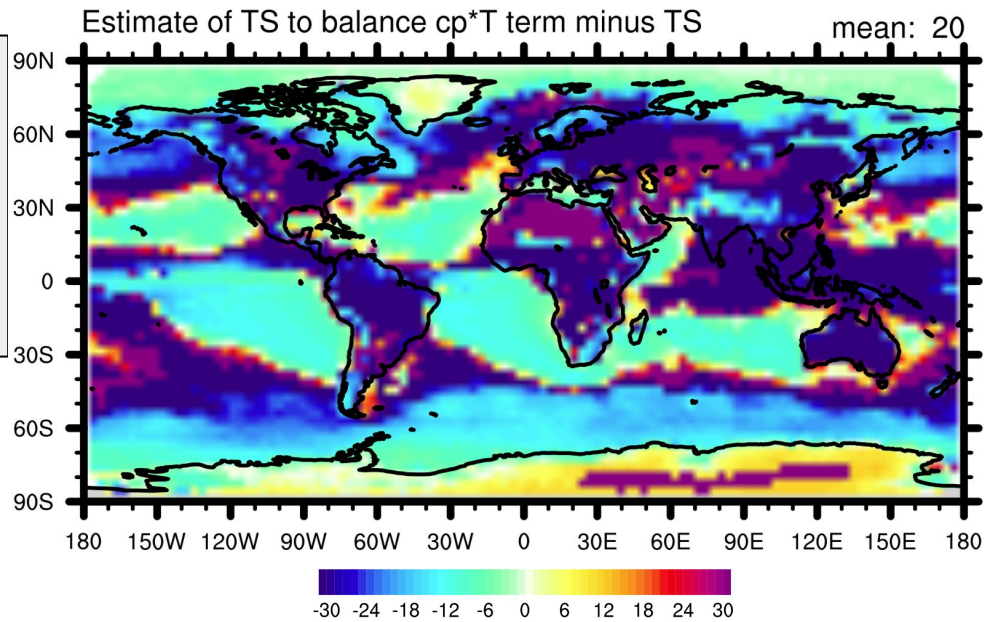
$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left(m^{(d)} + m^{(wv)}_{t=t^n} \right) (K + \Phi_s) + c_p^{(d)} T + m^{(H_2O)} c_p^{(d)} T + m^{(wv)} L_{00,s} + m^{(liq)} L_{00,i} \right\} dA\, dz$$

$$= c_p^{(d)} \widetilde{T} \dot{m}^{(H_2O)} + \dot{m}^{(wv)} L_{00,s} + \dot{m}^{(liq)} L_{00,i} + F^{(turb)} - F^{(rad)}.$$

How much should TS be increased/decreased by for surface enthalpy and "cp*T" term to balance?

(purple area is where water flux is below threshold value so discard those areas)

Over ocean $\boxed{\widetilde{T}}$ should be ~10+K less than TS in model!

Estimate of TS to balance cp*T term minus TS   mean: 20

Enthalpy flux

using TS at th...

TS (skin)-T(k=nlev) atm   mean: 1.6

global min = -6.944   global max= 6.796

-4 -3.2 -2.4 -1.6 -0.8 0 0.8 1.6 2.4 3.2 4

$$c_p \cdots ,s + m^{(liq)} L_{00,i} \Big\} dA\, dz$$

$$\cdots (turb) - F^{(rad)}.$$

Estimate of TS to balance cp*T term minus TS   mean: 20

-30 -24 -18 -12 -6 0 6 12 18 24 30

Pacific evaporation zones:

Atmosphere is colder than ocean

-> consistent with $\boxed{\widetilde{T}}$ <TS for enthalpy term.

Large precip zones:

Measurements and theory for the surface rainfall temperature show that it is approximately equal to the surface wet-bulb temperature (Byers et al., 1949; Kinzer & Gunn, 1951; Gosnell et al., 1995), which will be warmer than the temperature at which condensation occurs, on average.

E.g. pacific warm pool: droplets 5K < SST=TS (Anderson, 1998)

-> consistent with $\boxed{\widetilde{T}}$ < TS for enthalpy term
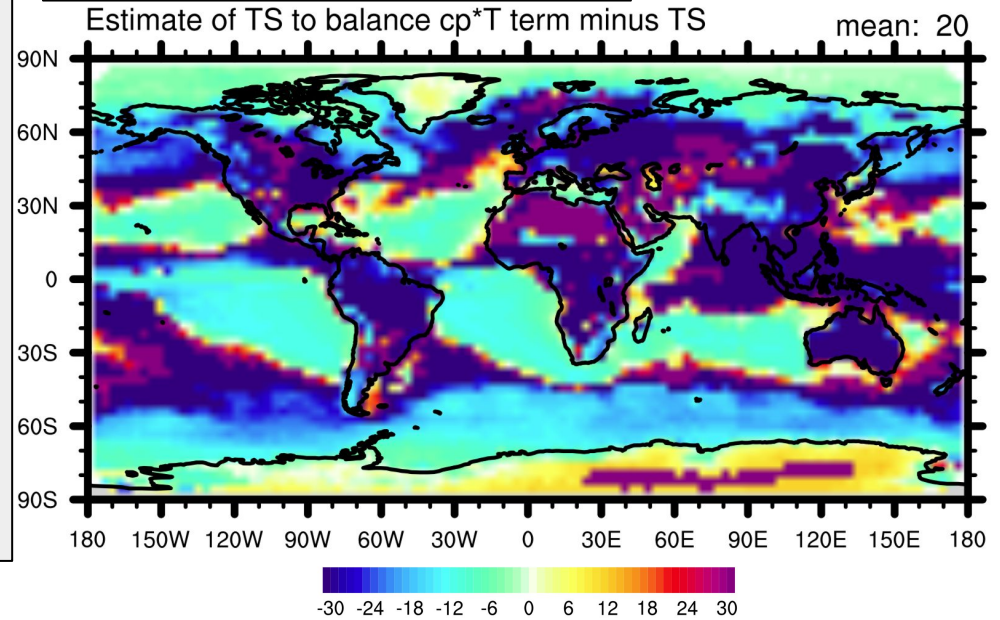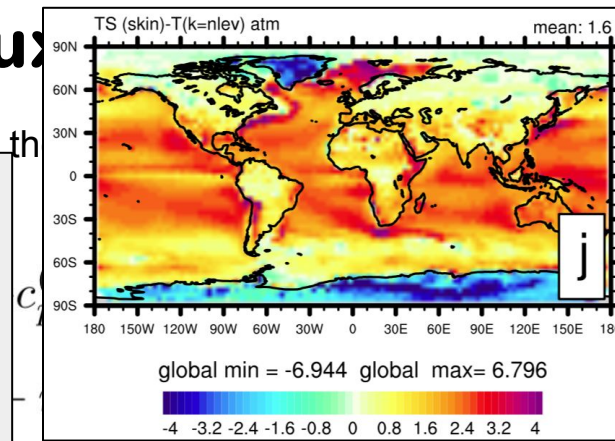
Summary

- Importance of doing total energy budgets - there can be compensating errors as in CAM-SE:

      dE/dt energy fixer (efix)   : -0.02 W/m^2
      dE/dt dry mass adjustment   : 0.31 W/m^2
      dE/dt total adiabatic dynamics   : -0.24 W/m^2
      dE/dt phys-dyn coupling error   : -0.05 W/m^2

- Dynamical core errors are large

- Physics: Surface enthalpy flux is a large term and should not be neglected
  (and, if included, will reduce dry-mass adjustment error); but not straightforward how to formulate term!

Possible path forward

(with many "details" to work out)

Need continued research on energy conserving discretizations for dynamical cores

Move towards more consistent/accurate thermodynamics in physics:

- Do not ignore surface enthalpy flux term (implementation challenge?)

- Changing to more comprehensive energy formula requires changing the static energy formula used in physics (latent heats should be functions of T, density should incl. all water, move to dry mixing ratios, ...)

- Changes to individual parameterizations: all should use same/consistent thermodynamics (extent of problem unknown, e.g. CLUBB uses different thermodynamic variables than CAM)

- Ideally parameterizations should call libraries or be passed functions to compute thermodynamic variables (can we move to a coding paradigm supporting this?)

- Consider specifying thermodynamic quantities in terms of thermodynamic potentials from which they can be derived