



# *The Challenge of Energy Budget Closure in Earth System Models*

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# When Kevin called me to his office ...

## Applications of an Updated Atmospheric Energetics Formulation

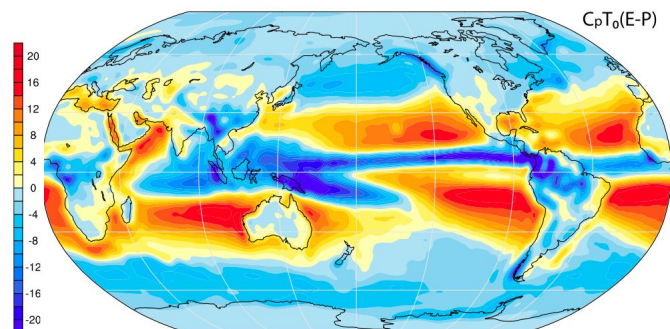
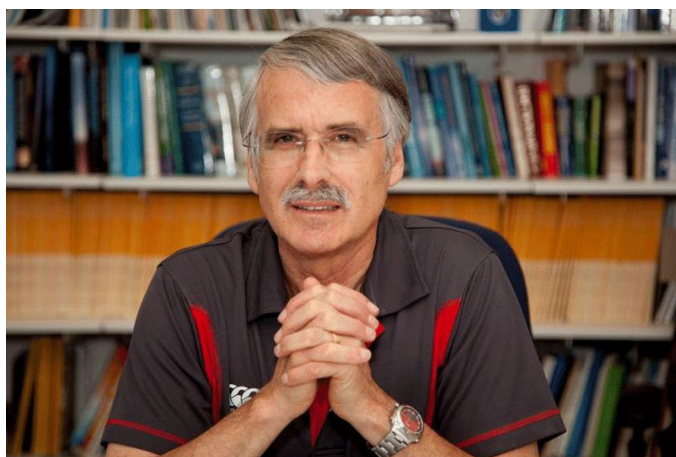
KEVIN E. TRENBERTH<sup>a</sup> AND JOHN T. FASULLO

*National Center for Atmospheric Research, Boulder, Colorado*

(Manuscript received 8 December 2017, in final form 24 April 2018)

### ABSTRACT

As observations and atmospheric reanalyses have improved, the diagnostics that can be computed with confidence also increase. Accordingly, a new formulation of the energetics of the atmosphere is laid out, with a view to advancing diagnostic studies of Earth's energy budget and flows. It is utilized to produce assessments of the vertically integrated divergences in both the atmosphere and ocean. Careful conservation of mass is required, with special attention given to the hydrological cycle and redistribution of mass associated with precipitation and evaporation, and a new method for ensuring this is developed. It guarantees that the atmospheric divergence is associated with moisture and precipitation, unlike previous methods. A new term, identified as associated with the enthalpy of precipitation, is included in a preliminary way. It is sensitive to the formulation, and the use of temperature in degrees Celsius instead of Kelvin greatly reduces errors and produces the extra term with values up to about  $\pm 5 \text{ W m}^{-2}$ . New results for 2000 to 2016 are presented for the vertical-mean and annual-mean diabatic atmospheric heating, atmospheric moistening, and total atmospheric energy divergence. Results for the atmospheric divergence are combined with top-of-atmosphere radiation observations to deduce total surface energy fluxes. Along with estimates of changes in ocean heat content, the Atlantic Ocean meridional heat transports are recomputed for March 2000 through 2013. The new results are compared with previous estimates and an assessment is made of the effects of the new mass balance, change in temperature scale, and the extra precipitation enthalpy term.





# For all “details” see:

Reconciling and improving formulations for thermodynamics and conservation principles in Earth System Models (ESMs)

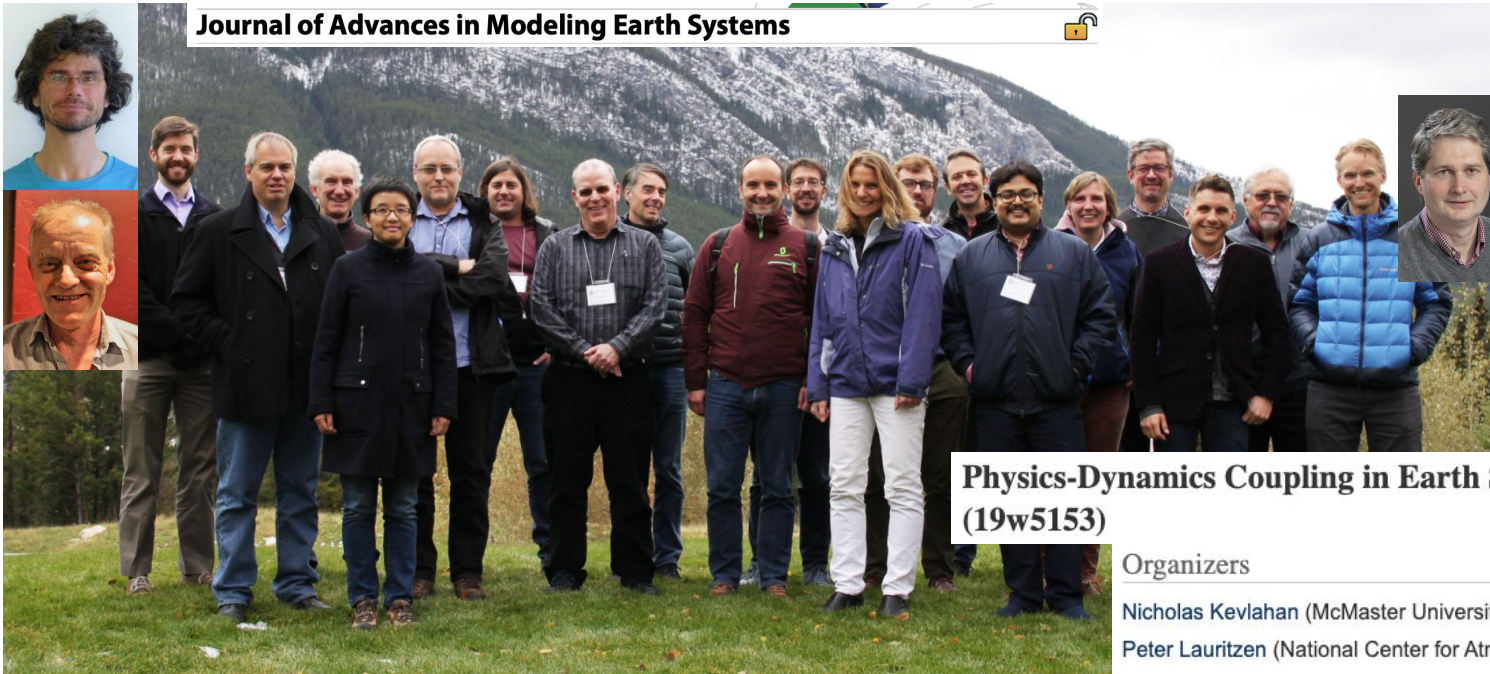


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Physics-Dynamics Coupling in Earth System Models  
(19w5153)

Organizers

Nicholas Kevlahan (McMaster University)

Peter Lauritzen (National Center for Atmospheric Research)



Assume:

- Primitive equations (hydrostatic, shallow atmosphere, ideal gas)
- Assume model top pressure is constant
- All components of moist air have the **same temperature** and move with the **same horizontal velocity**
- Assume that water entering the atmosphere (evaporation, snow drift, sea spray) has **same temperature** as water leaving the atmosphere (dew, liquid and frozen precipitation) **DEFINITELY NOT ALWAYS ACCURATE!**

Then it can be shown that the following globally integrated total energy equation holds:

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ K + \Phi_s + c_p^{(d)} T + \sum_{\ell \in \mathcal{L}_{H_2O}} m^{(\ell)} \left[ K + \Phi_s + c_p^{(\ell)} (T - T_{00}) + h_{00}^{(ice)} \right] + m^{(wv)} L_{s,00} + m^{(liq)} L_{f,00} \right\} dA dz$$

$$= \iint \left\{ \sum_{\ell \in \mathcal{L}_{H_2O}} F_{net}^{(\ell)} \left[ \tilde{K}_s + \Phi_s + c_p^{(\ell)} (\tilde{T}_s - T_{00}) + h_{00}^{(ice)} \right] + F_{net}^{(wv)} L_{s,00} + F_{net}^{(liq)} L_{f,00} + F_{net}^{(turb,rad)} \right\} dA. \quad (94)$$

(ice reference enthalpy,  $\tilde{T}_s \equiv T_{atm,s} = T_{surf,s}$ )

symbol	description	unit
$c_p^{(\ell)}$	heat capacity at constant pressure of species $\ell$	J/K/kg
$F_{net}^{(\ell)}$	net flux of water species $\ell$ into the atmosphere	kg/m <sup>2</sup> /s
$F_{net}^{(turb,rad)}$	Radiative and sensible/turbulent fluxes into atmosphere (90)	J/m <sup>2</sup> /s
$h_{00}^{(ice)}$	reference enthalpy for water form $\ell$	J/kg
$m^{(\ell)}$	dry mixing ratio ( $\equiv \rho^{(\ell)}/\rho^{(d)}$ )	kg/kg
$K$	specific horizontal kinetic energy ( $\equiv \frac{1}{2} \tilde{v}^2$ )	m <sup>2</sup> /s <sup>2</sup>
$L_{f,00}$	latent heat of fusion	J/K
$L_{s,00}$	latent heat of sublimation	J/K
$L_{w,00}$	latent heat of vaporization	J/K
$\Phi_s$	surface geopotential	m <sup>2</sup> /s <sup>2</sup>
$\rho_d$	dry air density	kg/m <sup>3</sup>
$T$	temperature	K
$\tilde{T}_s$	common temperature at surface	K
$\tilde{v}$	horizontal velocity vector	m/s

Now also assume that the energy equation is valid for grid mean values in the model (**QUESTIONABLE ASSUMPTION!**)



# Total energy equation



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Many models make these assumptions:

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$$= \iint \left\{ \sum_{\ell \in \mathcal{L}_{H_2O}} \bar{F}_{net}^{(\ell)} \left[ \bar{\tilde{K}}_s + \bar{\Phi}_s + c_p^{(\ell)} (\bar{\tilde{T}}_s - T_{00}) + h_{00}^{(ice)} \right] + \bar{F}_{net}^{(wv)} L_{s,00} + \bar{F}_{net}^{(liq)} L_{f,00} + \bar{F}_{net}^{(turb,rad)} \right\} dA.$$

(94) ←

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Now also assume that the energy equation is valid for grid mean values in the model (**QUESTIONABLE ASSUMPTION!**)



# Total energy

Imbalance of incl. all forms of water in CAM's parameterization total energy equation:



Assume:

- Primitive equations (hydrostatic, shallow)
- Assume model top pressure is constant
- All components of moist air have the same
- Assume that water entering the atmosphere is leaving the atmosphere (dew, liquid and

Then it can be shown that the following global

Many models make these assumptions:

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \bar{K} + \bar{\Phi}_s + c_p^{(d)} \bar{T} + \mathcal{L}_{H_2O} = 'wv' + \bar{m} \right.$$

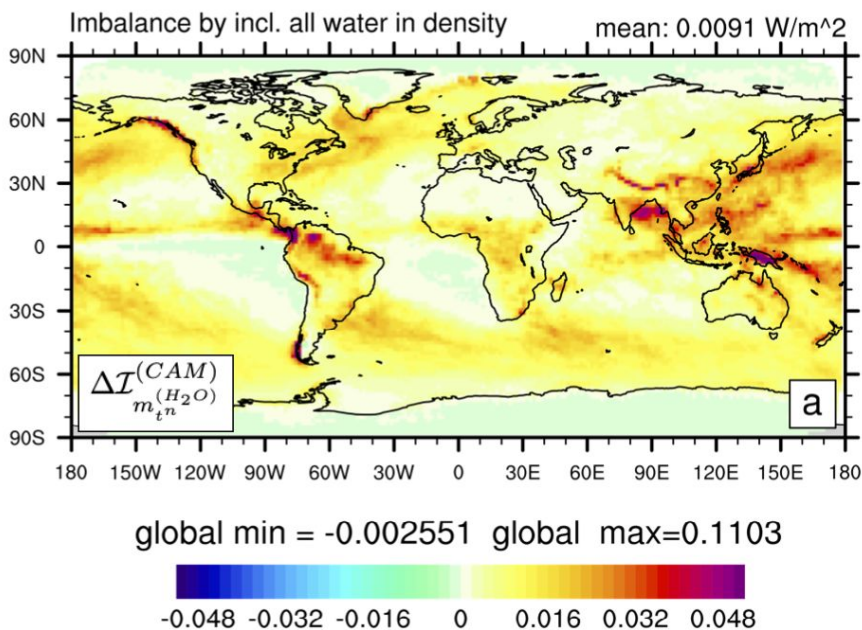
$$= \iint \left\{ \sum_{\ell \in \mathcal{L}_{cond}} F_{net}^{(\ell)} \left[ \tilde{\bar{K}}_s + \bar{\Phi}_s + \right. \right.$$

(ice referen

Now also assume that the energy equation is **ASSUMPTION!**

$$\Delta \mathcal{I}_{m_{t^n}^{(H_2O)}}^{(CAM)} =$$

$$\int \left[ \rho^{(d)} \left( \sum_{\ell \in \mathcal{L}_{cond}} \bar{m}_{t^n}^{(\ell)} \right) \right] \frac{\partial}{\partial t} \left( \bar{K} + \bar{\Phi}_s + c_p^{(d)} \bar{T} \right) dz$$



velocity  
perature as water  
**RATE!**

**E**



# Total energy equation



Assume:

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Then it can be shown that the following globally integrated total energy equation holds:

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$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \bar{K} + \bar{\Phi}_s + c_p^{(d)} \bar{T} + \sum_{\ell \in \mathcal{L}_{H_2O}} m^{(\ell)} \left[ \bar{K} + \bar{\Phi}_s + c_p^{(\ell)} (\bar{T} - T_{00}) + h_{00}^{(ice)} \right] + m^{(wv)} L_{s,00} + m^{(liq)} L_{f,00} \right\} dA dz$$

$$= \iiint \left\{ \sum_{\ell \in \mathcal{L}_{H_2O}} F_{net}^{(\ell)} \left[ \tilde{\bar{K}} + \bar{\Phi}_s + c_p^{(\ell)} (\tilde{\bar{T}} - T_{00}) + h_{00}^{(ice)} \right] + F_{net}^{(wv)} L_{s,00} + F_{net}^{(liq)} L_{f,00} + F_{net}^{(turb,rad)} \right\} dA.$$

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(ice reference enthalpy,  $\tilde{T}_s \equiv T_{atm,s} = T_{surf,s}$ )

Now also assume that the energy equation is valid for grid mean values in the model (**QUESTIONABLE ASSUMPTION!**)





# Updating water (pressure) in physics



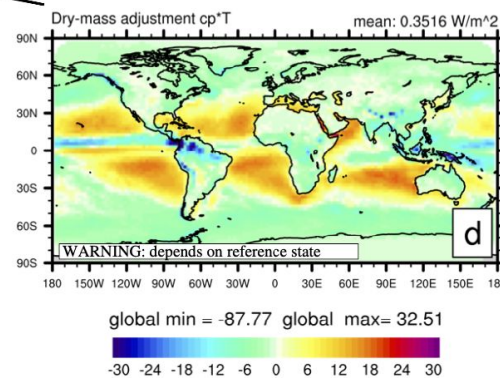
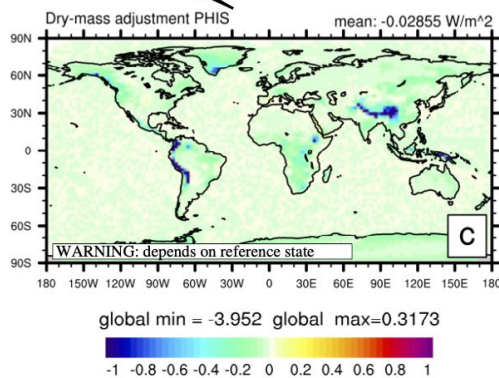
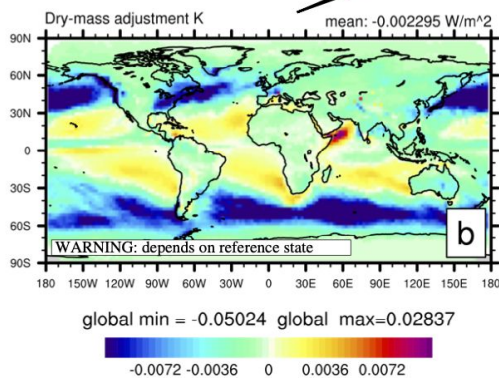
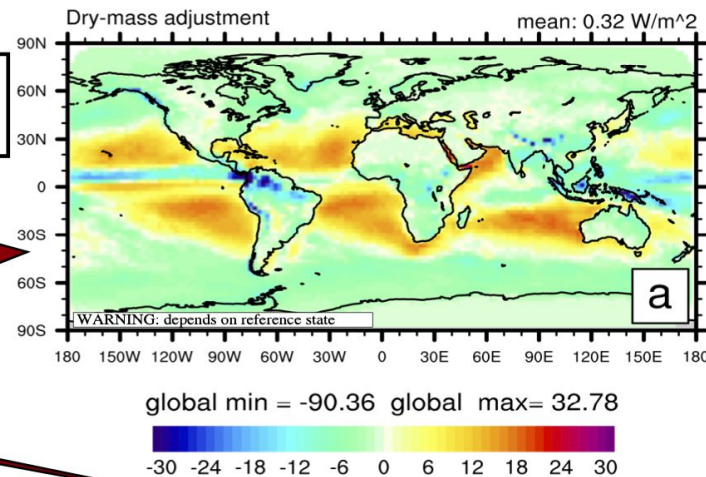
"spurious phase change term" due to CAM only incl. water vapor in total water

These 2 terms can not be separated in our diagnostics!

Total energy of falling precipitation and evaporation

$$\Delta \mathcal{I}_{\partial m^{(wv)}/\partial t}^{(CAM)} = \Delta \tilde{\mathcal{I}}_{\partial m^{(wv)}/\partial t}^{(CAM)} + \Delta \hat{\mathcal{I}}_{\partial m^{(wv)}/\partial t}^{(CAM)}$$

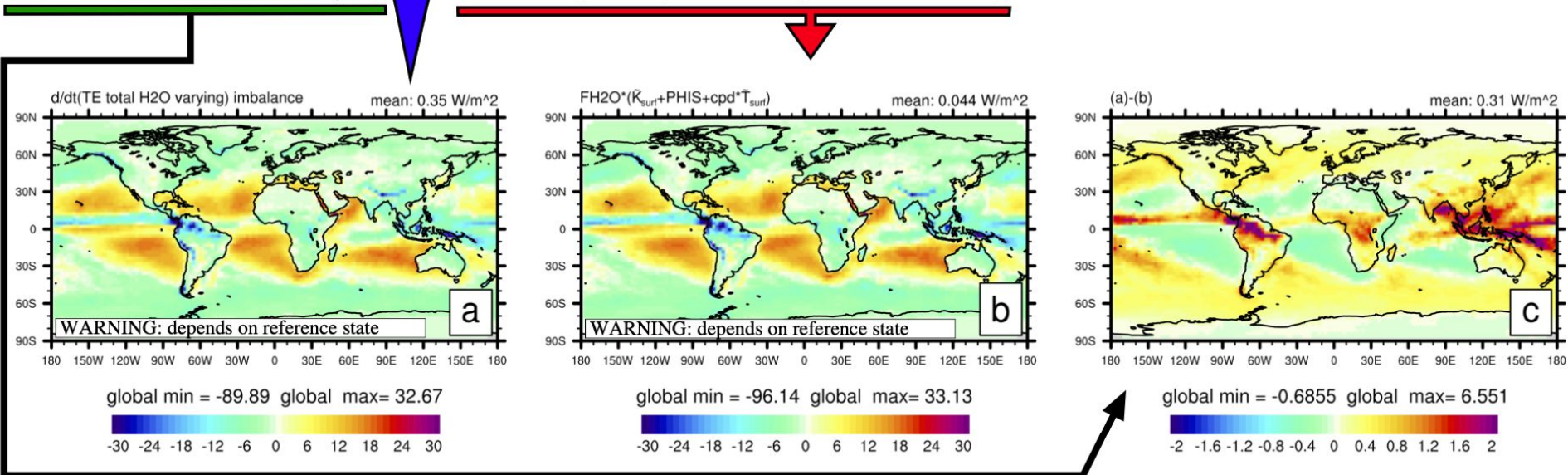
$$= \int \frac{\partial}{\partial t} \left[ \rho^{(d)} \left( 1 + \bar{m}^{(wv)} \right) \right] \left( \bar{K} + \bar{\Phi}_s + c_p^{(d)} \bar{T} \right) dz$$



# Modified CAM total energy equation incl. missing flux terms

$$\frac{\partial}{\partial t} \int \bar{\rho}^{(d)} \left\{ \left( 1 + \bar{m}^{(H_2O)} \right) \left[ \bar{K} + \bar{\Phi}_s + c_p^{(d)} (\bar{T} - T_{00}) \right] + \bar{m}^{(wv)} L_{s,00} + \bar{m}^{(liq)} L_{f,00} \right\} dz$$

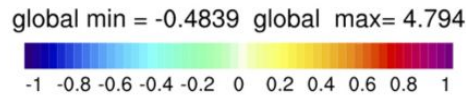
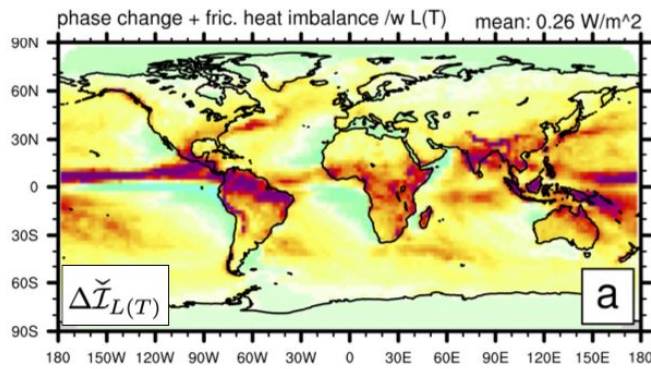
$$-\Delta \hat{\mathcal{I}}_{\partial m^{(H_2O)}/\partial t} - \Delta \mathcal{I}_{m_t^{(H_2O)}} = \bar{F}_{net}^{(H_2O)} \left[ c_p^{(d)} (\tilde{T}_s - T_{00}) + \tilde{K}_s + \bar{\Phi}_s \right] + \bar{F}_{net}^{(wv)} L_{s,00} + \bar{F}_{net}^{(liq)} L_{f,00} + \bar{F}_{net}^{(turb,rad)}$$



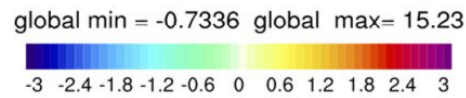
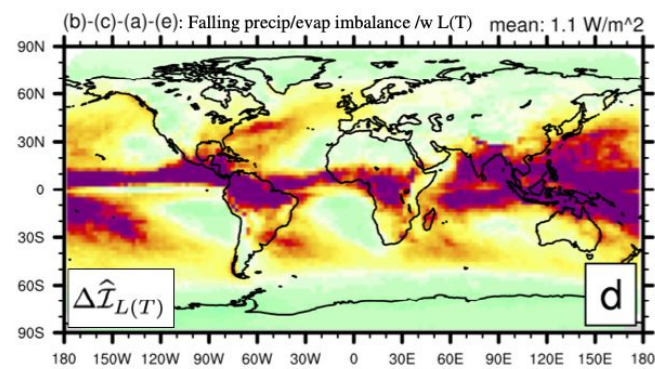
# Modified (consistent) total energy equation assuming variable latent heats

$$\frac{\partial}{\partial t} \int \bar{\rho}^{(d)} \left\{ \underbrace{\left(1 + \bar{m}^{(H_2O)}\right) \left(\bar{K} + \bar{\Phi}_s\right) + c_p^{(d)} T + \sum_{\ell \in \mathcal{L}_{H_2O}} \bar{m}^{(\ell)} c_p^{(\ell)} \left(\bar{T} - T_{00}\right) + \bar{m}^{(wv)} L_{s,00} + \bar{m}^{(liq)} L_{f,00}}_{\text{...}} \right\} dz$$

$$-\Delta \tilde{\mathcal{I}}_{L(T)} - \Delta \hat{\mathcal{I}}_{L(T)} = - \sum_{\ell \in \mathcal{L}_{H_2O}} \bar{F}_{net}^{(\ell)} \left[ c_p^{(\ell)} \left(\tilde{T}_s - T_{00}\right) + \tilde{K}_s \right] + \bar{F}_{net}^{(wv)} L_{s,00} + \bar{F}_{net}^{(liq)} L_{f,00} + \bar{F}_{net}^{(turb,rad)}$$



(a) Imbalance for processes not involving falling precip. & evap.



(b) Imbalance for falling precip. & evap.





# Concluding remarks



- **Most global models do NOT rigorously account for processes associated with falling precipitation and evaporation in terms of kinetic, potential and internal energy**  
-> incl. boundary fluxes (in particular, enthalpy flux) improves energy budget massively!  
(other processes: frictional heating of falling precipitation, horizontal drag of precipitation, ...)
- **Being rigorous in terms of monitoring energy conservation forces modelers to consider thermodynamic consistency between different parameterizations as well as dynamical core!**  
(inconsistency between CAM and CLUBB discussed in Lauritzen et al. (2022, in prep))
- **For the enthalpy fluxes to be consistent with modern ocean models (e.g. MOM6), atmosphere models must use variable latent heats**





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