Reconciling and improving formulations for thermodynamics and conservation principles in Earth System Models (ESMs)

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Reconciling and Improving Formulations for Thermodynamics and Conservation Principles in Earth System Models (ESMs)

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Key Points:

- Closing total energy budgets in Earth System Models without *ad hoc* fixers is a monumental task
- Largest errors are from missing processes/terms, thermodynamic inconsistencies and dynamical core
- Further research is needed on conservative discretizations, unified thermodynamics and missing processes

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Physics-Dynamics Coupling in Earth System Models (19w5153)

Organizers

Nicholas Kevlahan (McMaster University)

Peter Lauritzen (National Center for Atmospheric Research)

Banff International Research Station for Mathematical Innovation and Discovery

https://www.birs.ca/events/2019/5-day-workshops/19w5153

This is pretty rare:
Model development paper featured in Eos …

Eos

Eos = the science news magazine published by AGU

ABOUT SPECIAL REPORTS TOPICS PROJECTS NEWSLETTER SUBMIT TO *EOS*

Consistently Closing the Energy Budget in Earth System Models

Researchers review the challenges and prospects of Earth System Models that incorporate a consistent closed energy budget.

By **John Thuburn** 19 September 2022

https://eos.org/editor-highlights/consistently-closing-the-energy-budget-in-earth-system-models

Outline (of paper/talk)

Theoretical energetics/budgets (section 2)

Start with the dry hydrostatic primitive equations (HPE) and gradually increase the thermodynamic complexity by first adding water vapor and then condensates to the HPE. Special attention is given to the derivation of enthalpy terms (and associated reference states), latent heat terms and surface flux terms. For these models, a detailed explanation of the approximations made in large-scale models can be included rigorously. An in-depth discussion is included of surface fluxes and the complications arising due to falling precipitation and/or water entering the atmosphere using a single-component fluid approach.

Energy (existing & missing) budget terms of a climate model (section 3)

- Example: budget in NCAR's CAM (Community Atmosphere Model)/CESM (Community Earth System Model)

Energy budget errors (section 4)

- Numerical truncation energy errors in dynamical cores (adiabatic). (see Lauritzen and Williamson, 2019)
- Physics–dynamics coupling errors due to spatial and temporal discretization errors. (see Donahue & Caldwell, 2020, Lauritzen and Williamson, 2019)
- Thermodynamic inconsistency energy errors in physics:
  - **As an illustration we discuss a specific example in some detail: coupling the CLUBB cloud parameterization package with the CAM climate model.**
- Thermodynamic and vertical coordinate inconsistencies between dynamical core and parameterizations:
  * different vertical coordinates (see Lauritzen et al., in prep, for z-based MPAS coupling with p-based CAM)
  * different enthalpy definitions (e.g., FV3/SE coupled with CAM)
- Mass "clipping" errors and energy

Some remarks on how to define energy in a model

Total energy conserved by the governing equations of motion and associated thermodynamics we refer to as ***fluid equations of motion energy***: E_{feom}

However, the fluid equations of motion and the thermodynamics are usually approximated.

For example, the fluid equations of motion may make the

- hydrostatic assumption
  (neglects non-hydrostatic motion, breaking gravity waves and 3D turbulence)
- neglecting individual momentum equations for hydrometeors, and making the single temperature assumption, so that all components of moist air have the same temperature.

=> Total energy may be divided into E_{feom} and the energy associated with all motions and processes (such as radiation) not represented in the fluid equations of motion,

$$E_{atm} = E_{feom} + E_{other}$$

Some remarks on how to define energy in a model

In addition to this prior argument for the continuous equation of motion, there is an even more complex problem:

We must homogenize (i.e., average) processes smaller than about 50–100 km in operational climate models, and roughly 0.5–3 km for cutting edge convection-permitting global models:

$$E_{atm} = E^{(res)} + E^{(unres)}$$

Things now become complicated and less well understood. This topic, though immensely important, is not the main focus of this presentation!

Note: most models do NOT have a sub-grid-scale reservoir of energy!

See introductory discussion in Appendix A

Total energy equation

Assume:

- Primitive equations (hydrostatic, shallow atmosphere, ideal gas)
- Assume model top pressure is constant
- All components of moist air have the **same temperature** and move with the **same horizontal velocity**
- Assume that water entering the atmosphere (evaporation, snow drift, sea spray) has **same temperature** and **velocity** as water leaving the atmosphere (dew, liquid and frozen precipitation) **Just for notational simplicity!**

Then it can be shown that the following globally integrated total energy equation holds:

$$
\begin{aligned}
&\frac{\partial}{\partial t}\iiint \rho^{(d)}\Bigg\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+c_p^{(\ell)}\left(T-T_{00}\right)+h_{00}^{(ice)}\right] \\
&\qquad\qquad + m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\Bigg\}dA\,dz \\
&= \iint \Bigg\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\Bigg\}dA.
\end{aligned}
\tag{94}
$$

(ice reference enthalpy, $\widetilde{T}_s \equiv T_{atm,s} = T_{surf,s}$)

symbol	description	unit
$c_p^{(\ell)}$	heat capacity at constant pressure of species ℓ	$J/K/kg$
$F_{net}^{(\ell)}$	net flux of water species ℓ into the atmosphere	$kg/m^2/s$
$F_{net}^{(turb,rad)}$	Radiative and sensible/turbulent fluxes into atmosphere (90)	$J/m^2/s$
$h_{00}^{(\ell)}$	reference enthalpy for water form ℓ	J/kg
$m^{(\ell)}$	dry mixing ratio ($\equiv \rho^{(\ell)}/\rho^{(d)}$)	kg/kg
K	specific horizontal kinetic energy ($\equiv \frac{1}{2}\vec{v}^2$)	m^2/s^2
$L_{f,00}$.	latent heat of fusion	J/K
$L_{s,00}$.	latent heat of sublimation	J/K
$L_{v,00}$.	latent heat of vaportization	J/K
Φ_s	surface geopotential	m^2/s^2
ρ_d	dry air density	kg/m^3
T	temperature	K
$\widetilde{T}_s$	common temperature at surface	K
$\vec{v}$	horizontal velocity vector	m/s

Total energy equation

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- Assume th … entering the atmosphere (evaporation, snow drift, sea spray) has **same temperature** as water leaving the a … l frozen precipitation). **Just for notational simplicity!**

Here we assume that moist air is composed of

$$\mathcal{L}_{all} \equiv \{d, wv, cl, ci, rn, sw, gr\}$$

(in our geospace configuration dry air is species dependent) which can be divided into gases and condensates

$$\mathcal{L}_{cond} \equiv \{cl, ci, rn, sw, gr\}$$

It is convenient to also define the set of water species in air

$$\mathcal{L}_{H_2O} \equiv \{wv, cl, ci, rn, sw, gr\}$$

Total energy equation

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$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+c_p^{(\ell)}\left(T-T_{00}\right)+h_{00}^{(ice)}\right]\right.$$
$$\left.+\,m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\right\}dA\,dz$$
$$=\iint\left\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\right\}dA. \tag{94}$$

(ice reference enthalpy, $\widetilde{T}_s \equiv T_{atm,s} = T_{surf,s}$)

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Total energy equation

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- All components of moist air have the **same tempe** ... **tal velocity**
- Assume that water entering the atmosphere (evap ... **temperature** and **velocity** as water leaving the atmosphere (dew, li ... **tational simplicity!**

Then it can be shown that the following globally integrate

For notational simplicity the radiation and turbulent/sensible heat fluxes are combined into one term (radiation is really the integration of a divergence term throughout the atmosphere …)

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\Big\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+c_p^{(\ell)}\left(T-T_{00}\right)+h_{00}^{(ice)}\right] + m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\Big\}dA\,dz$$

$$=\iint\Big\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\tilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\tilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\Big\}dA. \quad (94)$$

(ice reference enthalpy, $\tilde{T}_s \equiv T_{atm,s} = T_{surf,s}$)

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Total energy equation

The total energy equation contains vast amounts of information and things get complicated fast … so let's start by discussing the different terms and how they manifest in a model …

Then it can be shown that th … globally integrated total energy equation holds:

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+c_p^{(\ell)}\left(T-T_{00}\right)+h_{00}^{(ice)}\right]\right.$$
$$\left.+\,m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\right\}dA\,dz$$
$$=\iint\left\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\right\}dA. \tag{94}$$

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Total energy equation: Dynamical core

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+c_p^{(\ell)}\left(T-T_{00}\right)+h_{00}^{(ice)}\right]\right.$$

$$\left.+\,m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\right\}dA\,dz$$

$$\xcancel{=\iint\left\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\right\}dA.} \tag{94}$$

(ice reference enthalpy, $\widetilde{T}_s \equiv T_{atm,s} = T_{surf,s}$)

No phase changes and globally integrated water species are conserved:

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left[\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left(-c_p^{(\ell)}T_{00}+h_{00}^{(ice)}\right)+m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\right]=0$$

So we end up with:

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}E_{feom}\,dA\,dz=\frac{\partial}{\partial t}\iiint \rho^{(d)}\left[\sum_{\ell\in\mathcal{L}_{all}} m^{(\ell)}\left(K+\Phi_s+c_p^{(\ell)}T\right)\right]dA\,dz=0.$$

(all enthalpy reference states, adiabatic dynamical core with inert water species)

Total energy equation: Dynamical core

Please note that the upper boundary condition affect energy formula. E.g.,
for constant volume and constant pressure model the energies conserved are different:

$$\frac{\partial}{\partial t}\iiint \left[K + c_v^{(d)}T + \Phi\right] \rho^{(d)} dA\, dz = 0, \;\; z_t \text{ constant}$$

$$\frac{\partial}{\partial t}\iiint \left[K + c_v^{(d)}T + \Phi\right] \rho^{(d)} dA\, dz + \frac{1}{g}\frac{\partial}{\partial t}\iint p_t \Phi_t dA = 0, \;\; p_t \text{ constant}$$

Pressure work at model top

The latter equation can be integrated by parts

$$\frac{\partial}{\partial t}\iiint \left[K + c_p^{(d)}T + \Phi_s\right] dA \frac{dp^{(d)}}{g} = 0, \;\; p_t \text{ constant}$$

So we end up with:

$$\frac{\partial}{\partial t}\iiint \rho^{(d)} E_{feom}\, dA\, dz = \frac{\partial}{\partial t}\iiint \rho^{(d)} \left[\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} \left(K + \Phi_s + c_p^{(\ell)} T\right)\right] dA\, dz = 0.$$

(all enthalpy reference states, adiabatic dynamical core with inert water species)

Total energy equation: Dynamical core

Please note that the upper boundary condition affect energy formula. E.g.,
for constant volume and constant pressure model the energies conserved are different:

$$\frac{\partial}{\partial t}\iiint\left[K+c_v^{(d)}T+\Phi\right]\rho^{(d)}dA\,dz=$$

$$c_v^{(d)}T+\Phi\Big]\,\rho^{(d)}dA\,dz+\frac{1}{g}\frac{\partial}{\partial}$$ nt

Integrand is local energy per unit mass!

The integrand is not local energy per unit mass!

Hence this equation only holds globally and NOT locally!

The latter equation can be integrated by parts

$$\frac{\partial}{\partial t}\iiint\left[K+c_p^{(d)}T+\Phi_s\right]dA\,\frac{dp^{(d)}}{g}=0,\;\;p_t\;\text{constant}$$

Also note that the energy flux is

$$\nabla\cdot\left\{\vec{v}\rho^{(d)}\left[K+c_p^{(d)}T+\Phi\right]\right\}$$

Even though the energy density is

$$\rho^{(d)}\left[K+c_v^{(d)}T+\Phi\right]$$

So we end up with:

$$\frac{\partial}{\partial t}\iiint\rho^{(d)}E_{feom}\,dA\,dz=\frac{\partial}{\partial t}\iiint\rho^{(d)}\left[\sum_{\ell\in\mathcal{L}_{all}}m^{(\ell)}\left(K+\Phi_s+c_p^{(\ell)}T\right)\right]dA\,dz=$$

(all enthalpy reference states, adiabatic dynamical core with inert water species)

Phase changes and falling precipitation

For the following discussions/observations the time-change of water species ℓ is separated into local phase changes and changes associated with water entering or leaving the column

$$\frac{\partial m^{(\ell)}}{\partial t} = \frac{\partial \check{m}^{(\ell)}}{\partial t} + \frac{\partial \hat{m}^{(\ell)}}{\partial t}$$

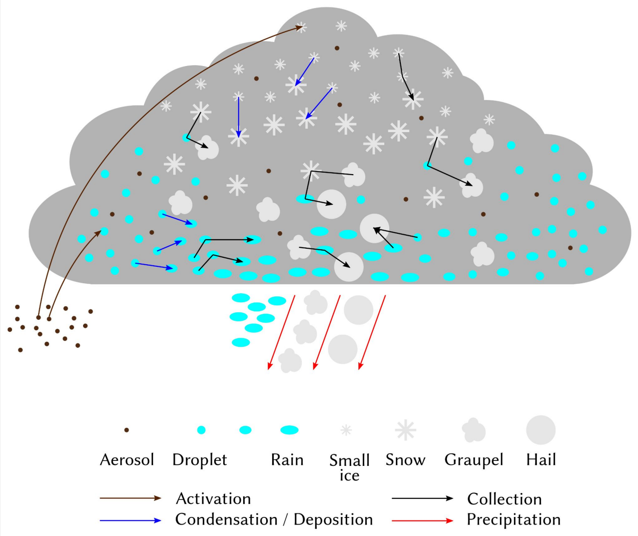


Figure: note that in the model we assume that the cloud moves with the single velocity and has single temperature; once falling precipitation is formed the process is no longer explicitly described by the fluid equations of motion energy

Let's start with phase changes!

Energy and phase changes

The temperature change associated with latent heat release should locally at each grid point satisfy

$$\frac{\partial}{\partial t}\left[\rho^{(d)}\sum_{\ell\in\mathcal{L}_{all}}\breve{m}^{(\ell)}c_p^{(\ell)}\left(T-T_{00}\right)+\rho^{(d)}\breve{m}^{(wv)}L_{s,00}+\rho^{(d)}\breve{m}^{(liq)}L_{f,00}\right]=0.$$

(ice reference; local phase changes; no falling precipitation or surface water source)

which leads to a closed energy budget:

$$\frac{\partial}{\partial t}\iiint\rho^{(d)}\Big\{K+\Phi_s+\underline{c_p^{(d)}T}+\sum_{\ell\in\mathcal{L}_{H_2O}}m^{(\ell)}\left[K+\Phi_s+\underline{c_p^{(\ell)}\left(T-T_{00}\right)}+h_{00}^{(ice)}\right]$$
$$\underline{+\,m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}}\Big\}dA\,dz$$
$$=\iint\Big\{\sum_{\ell\in\mathcal{L}_{H_2O}}F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\Big\}dA. \quad (94)$$

(ice reference enthalpy, $\widetilde{T}_s\equiv T_{atm,s}=T_{surf,s}$)

Note that total water is conserved during phase changes

-> so kinetic and geopotential terms do not change during phase changes!

Next: falling precip/evaporation

Energy (latent heat terms) and falling precipitation

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\widehat{m}^{(\ell)}\,dA = \iint F^{(\ell)}_{net}\,dA\,dz,$$

(falling precipitation and surface evaporation)

For falling precipitation / evaporation the latent heat terms on left and right-hand side exactly cancel:

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\Big\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+c_p^{(\ell)}\left(T-T_{00}\right)+h_{00}^{(ice)}\right]$$
$$+\underline{m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}}\Big\}dA\,dz$$
$$=\iint\Big\{\sum_{\ell\in\mathcal{L}_{H_2O}} F^{(\ell)}_{net}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+\underline{F^{(wv)}_{net}L_{s,00}+F^{(liq)}_{net}L_{f,00}}+F^{(turb,rad)}_{net}\Big\}dA. \quad (94)$$

(ice reference enthalpy, $\widetilde{T}_s \equiv T_{atm,s} = T_{surf,s}$)

Latent heat flux: When, for example, water evaporates from the ocean the atmosphere gains energy (and mass) which is compensated for by ocean cooling due to the latent heat flux. Hence, this process occurs without any net change in the total energy of the coupled system.

Energy (reference T & enthalpy) and falling precipitation

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\widehat{m}^{(\ell)}dA = \iint F_{net}^{(\ell)}dA\,dz,$$

(falling precipitation and surface evaporation)

For falling precipitation / evaporation the latent heat terms on left and right-hand side exactly cancel:

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\Big\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+c_p^{(\ell)}\left(T-T_{00}\right)+h_{00}^{(ice)}\right]$$
$$+m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\Big\}dA\,dz$$
$$=\iint\Big\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\Big\}dA. \quad (94)$$

(ice reference enthalpy, $\widetilde{T}_s \equiv T_{atm,s}=T_{surf,s}$)

Similarly for reference temperature terms (**physically the reference temperature does not matter!**)

Energy (enthalpy) and falling precipitation

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\widehat{m}^{(\ell)}\,d\,A = \iint F_{net}^{(\ell)}\,d\,A\,d\,z,$$

(falling precipitation and surface evaporation)

For falling precipitation / evaporation the enthalpy:

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\Big\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+\underline{c_p^{(\ell)}\left(T\right.}-T_{00}\big)+h_{00}^{(ice)}\right]$$
$$+m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\Big\}dA\,dz$$
$$=\iint\Big\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+\underline{c_p^{(\ell)}\big(\widetilde{T}_s}-T_{00}\big)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\Big\}dA. \quad (94)$$

(ice reference enthalpy, $\widetilde{T}_s \equiv T_{atm,s} = T_{surf,s}$)

The enthalpy terms should not necessarily cancel: WHY?

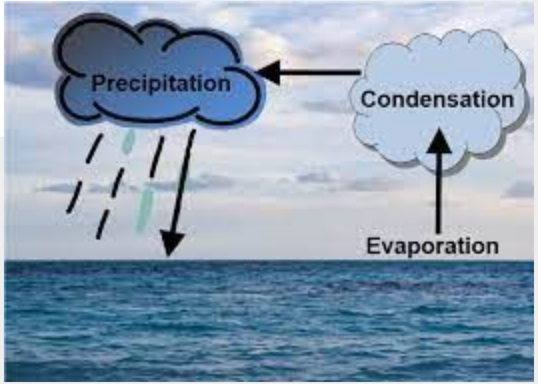
Enthalpy and falling precipitation

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\Big\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+c_p^{(\ell)}\left(T-T_{00}\right)+h_{00}^{(ice)}\right]$$

$$+\,m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\Big\}dA\,dz$$

$$=\iint\Big\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\Big\}dA.$$

$$(\text{ice reference enthalpy, } \widetilde{T}_s\equiv T_{atm,s}=T_{surf,s})$$



Note that the enthalpy terms should not necessarily cancel (as was the case for the latent heat terms):
The falling precipitation is formed away from surface and we do not rigorously represent processes as the water falls to the ground so the temperature with which precipitation hits the ground is ambiguous (in models). Similar argument for the kinetic energy terms associated with falling precipitation …

Kinetic energy

Assume:

- Primitive equations (hydrostatic, shallow atmosphere, ideal gas)
- Assume model top pressure is constant
- All components of moist air have the **same temperature** and move with the **same horizontal velocity**
- Assume that water entering the atmosphere (evaporation, snow drift, sea spray) has **same temperature** and **velocity** as water leaving the atmosphere (dew, liquid and frozen precipitation) **Just for notational simplicity!**

Then it can be shown that the following globally integrated total energy equation holds:

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\Big\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[\underline{K}+\Phi_s+\underline{c_p^{(\ell)}}\left(T-T_{00}\right)+h_{00}^{(ice)}\right] + m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\Big\}dA\,dz$$

$$=\iint\Big\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\Big\}dA. \quad (94)$$

(ice reference enthalpy, $\widetilde{T}_s \equiv T_{atm,s} = T_{surf,s}$)

symbol	description	unit
$c_p^{(\ell)}$	heat capacity at constant pressure of species ℓ	$J/K/kg$
$F_{net}^{(\ell)}$	net flux of water species ℓ into the atmosphere	$kg/m^2/s$
$F_{net}^{(turb,rad)}$	Radiative and sensible/turbulent fluxes into atmosphere (90)	$J/m^2/s$
$h_{00}^{(\ell)}$	reference enthalpy for water form ℓ	J/kg
$m^{(\ell)}$	dry mixing ratio ($\equiv \rho^{(\ell)}/\rho^{(d)}$)	kg/kg
K	specific horizontal kinetic energy ($\equiv \frac{1}{2}\vec{v}^2$)	m^2/s^2
$L_{f,00}$.	latent heat of fusion	J/K
$L_{s,00}$.	latent heat of sublimation	J/K
$L_{v,00}$.	latent heat of vaportization	J/K
Φ_s	surface geopotential	m^2/s^2
ρ_d	dry air density	kg/m^3
T	temperature	K
$\widetilde{T}_s$	common temperature at surface	K
$\vec{v}$	horizontal velocity vector	m/s

Kinetic energy

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Assume:

-
-
-
- and

licity!

The

Sources and sinks of momentum (e.g., gravity wave parameterization, boundary layer turbulence schemes or other drag parameterizations) affect kinetic energy, and enforcing total energy conservation in their presence is not straightforward due to its interaction with sub-grid-scales.

WARNING

We would like to point out that a “naive” closure of the energy budget by transferring kinetic energy change into heat is, in general, not physically correct

$$\left(\sum_{\ell\in\mathcal{L}_{all}} m^{(\ell)} c_p^{(\ell)}\right)\frac{\partial T}{\partial t} \neq -\left(\sum_{\ell\in\mathcal{L}_{all}} m^{(\ell)}\right)\frac{\partial K}{\partial t}$$

WHY?

	unit
	$J/K/kg$
	$kg/m^2/s$
sphere (90)	$J/m^2/s$
	J/kg
	kg/kg
	m^2/s^2
	J/K
	J/K
	J/K
	m^2/s^2
	kg/m^3
	K
	K
	m/s

Kinetic energy

Assume:

-
-
-
- and

licity!

The

Frictional terms due to the vertical mixing of horizontal momentum (represented as stress tensor):

$$\frac{D\vec{v}}{Dt} = -\vec{F}_h,$$

$$= -\frac{1}{\rho^{(all)}}\frac{\partial}{\partial z}\left(\vec{\tau}_z\right)$$

$$\int_{layer}\rho^{(all)}\frac{\partial K}{\partial t}dz = \int_{layer}\left[\underbrace{\frac{\partial}{\partial z}\left(\rho^{(all)}\nu^{(all)}\frac{\partial K}{\partial z}\right)}_{\text{Redistribution of }\rho^{(all)}K} - \underbrace{\rho^{(all)}\nu^{(all)}\left(\frac{\partial \vec{v}}{\partial z}\right)^2}_{\text{Shear production of TKE}}\right]dz. \quad (101)$$

which has the same form as molecular friction. In this special case, it can be argued that the last term in Equation 101 represents frictional/dissipative heating if $\nu^{(all)}$ is positive (note also that eddy diffusion with $\nu^{(all)} > 0$ fulfills the second law of thermodynamics [Schaefer-Rolffs & Becker, 2018]) and can be included in the (resolved-scale) thermodynamic equation for energy conservation (e.g., Bister & Emanuel, 1998):

$$\rho^{(d)}\left(\sum_{\ell\in\mathcal{L}_{all}} m^{(\ell)}c_p^{(\ell)}\right)\frac{\partial T}{\partial t} = \rho^{(d)}\left(\sum_{\ell\in\mathcal{L}_{all}} m^{(\ell)}\nu^{(\ell)}\right)\left(\frac{\partial \vec{v}}{\partial z}\right)^2. \quad (102)$$

$$\frac{\partial}{\partial t}$$

$$= \int$$

	unit
	$J/K/kg$
	$kg/m^2/s$
sphere (90)	$J/m^2/s$
	J/kg
	kg/kg
	m^2/s^2
	J/K
	J/K
	J/K
	m^2/s^2
	kg/m^3
	K
	K
	m/s

Total energy equation

Assume:

- Primitive equations (hydrostatic, shallow atmosphere, ideal gas)
- Assume model top pressure is constant
- All components of moist air have the **same temperature** and move with the **same horizontal velocity**
- Assume that water entering the atmosphere (evaporation, snow drift, sea spray) has **same temperature** and **velocity** as water leaving the atmosphere (dew, liquid and frozen precipitation) **Just for notational simplicity!**

Then it can be shown that the following globally integrated total energy equation holds:

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\Big\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+c_p^{(\ell)}\left(T-T_{00}\right)+h_{00}^{(ice)}\right] + m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\Big\}dA\,dz$$
$$=\iint\Big\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\Big\}dA. \tag{94}$$

(ice reference enthalpy, $\widetilde{T}_s \equiv T_{atm,s} = T_{surf,s}$)

symbol	description	unit
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K	specific horizontal kinetic energy ($\equiv \frac{1}{2}\vec{v}^2$)	m^2/s^2
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$L_{v,00}$.	latent heat of vaportization	J/K
Φ_s	surface geopotential	m^2/s^2
ρ_d	dry air density	kg/m^3
T	temperature	K
$\widetilde{T}_s$	common temperature at surface	K
$\vec{v}$	horizontal velocity vector	m/s

So far we have only discussed the continuous equations …

Total energy equation

Assume:

- Primitive equations (hydrostatic, shallow atmosphere, ideal gas)
- Assume model top pressure is constant
- All components of moist air have the **same temperature** and move with the **same horizontal velocity**
- Assume that water entering the atmosphere (evaporation, snow drift, sea spray) has **same temperature** and **velocity** as water leaving the atmosphere (dew, liquid and frozen precipitation) **Just for notational simplicity!**

Then it can be shown that the following globally integrated total energy equation holds:

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\Big\{\overline{K}+\overline{\Phi}_s+c_p^{(d)}\overline{T}+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[\overline{K}+\overline{\Phi}_s+c_p^{(\ell)}\left(\overline{T}-T_{00}\right)+h_{00}^{(ice)}\right]$$
$$+\overline{m}^{(wv)}L_{s,00}+\overline{m}^{(liq)}L_{f,00}\Big\}dA\,dz$$
$$=\iint\Big\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\overline{\Phi}_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+\overline{F}_{net}^{(wv)}L_{s,00}+\overline{F}_{net}^{(liq)}L_{f,00}+\overline{F}_{net}^{(turb,rad)}\Big\}dA. \quad (94)$$

(ice reference enthalpy, $\widetilde{T}_s \equiv \overline{T}_{atm,s} = \overline{T}_{surf,s}$)

$$E_{atm} = E^{(res)} + \cancel{E^{(unres)}}$$

Now also assume that the energy equation is valid for grid mean values in the model
(**QUESTIONABLE ASSUMPTION! No sub-grid reservoir of energy …**)

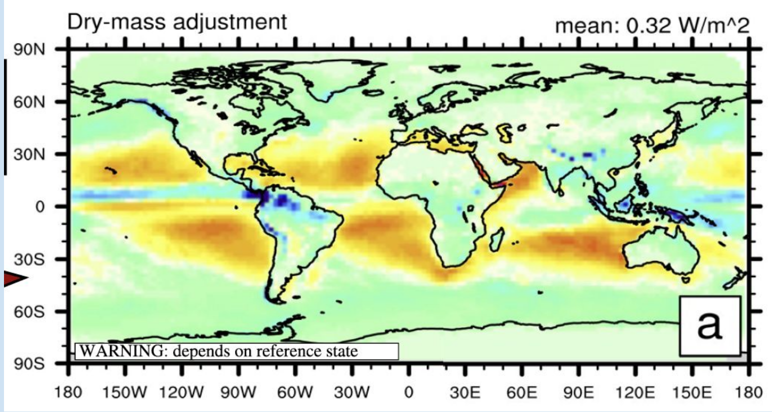
Total energy equation

In the following we will focus on the Physics (Parameterization) Vertically Integrated Energy Budget

Since there is no exchange of energy / transport between columns (column physics!), the energy equation should hold in each physics column

Note: The dynamical core redistributes energy but locally yields vanishing long-term time average of energy.

Hence we can look at column integrated energy budgets: I am going to show 1 year averages of the total energy budget imbalance or energy terms in each physics column.



Total energy equation

Assume:

- F
- A
- A ntal velocity
- A e temperature and

v otational simplicity!

Now to some assumptions typically made in global models!

Then it can be shown that the following globally integrated total energy equation holds:

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left\{\overline{K}+\overline{\Phi}_s+c_p^{(d)}\overline{T}+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[\overline{K}+\overline{\Phi}_s+c_p^{(\ell)}\left(\overline{T}-T_{00}\right)+h_{00}^{(ice)}\right]\right.$$
$$\left.+\overline{m}^{(wv)}L_{s,00}+\overline{m}^{(liq)}L_{f,00}\right\}dA\,dz$$
$$=\iint\left\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{\overline{K}}_s+\overline{\Phi}_s+c_p^{(\ell)}\left(\widetilde{\overline{T}}_s-T_{00}\right)+h_{00}^{(ice)}\right]+\overline{F}_{net}^{(wv)}L_{s,00}+\overline{F}_{net}^{(liq)}L_{f,00}+\overline{F}_{net}^{(turb,rad)}\right\}dA. \quad (94)$$

$$\text{(ice reference enthalpy, } \widetilde{\overline{T}}_s \equiv \overline{T}_{atm,s}=\overline{T}_{surf,s}\text{)}$$

$$E_{atm} = E^{(res)} + \cancel{E^{(unres)}}$$

Now also assume that the energy equation is valid for grid mean values in the model (QUESTIONABLE ASSUMPTION! No sub-grid reservoir of energy …)

Simplified total energy equation

This might be CAM specific:

Total water is assumed constant during physics updates!

Notation: $\overline{m}^{(H_2O)}_{t=t^n}$

Assume:
- Primitive e
- Assume m
- All compon ... tal velocity
- Assume th ... temperature and **velocity** a ... tational simplicity!

Then it can be sh ... egrated total energy equation holds:

Many models make these assumptions:

$\boxed{\mathcal{L}_{H_2O} =' wv'}$

$\boxed{c_p^{(\ell)} = c_p^{(d)}}$

Equivalent to assuming constant latent heats!

E.g. FV3 and NCAR-SE use variable latent heats and CAM physics not -> leads to 0.5W/m2 imbalance (Lauritzen and Williamson, 2019)

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left\{\overline{K}+\overline{\Phi}_s+c_p^{(d)}\overline{T}+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[\overline{K}+\overline{\Phi}_s+c_p^{(\ell)}\left(\overline{T}-T_{00}\right)+h_{00}^{(ice)}\right]\right.$$
$$\left.+\overline{m}^{(wv)}L_{s,00}+\overline{m}^{(liq)}L_{f,00}\right\}dA\,dz$$
$$=\iint\left\{\cancel{\sum_{\ell\in\mathcal{L}_{H_2O}} \overline{F}_{net}^{(\ell)}\left[\widetilde{\overline{K}}_s+\overline{\Phi}_s+c_p^{(\ell)}\left(\widetilde{\overline{T}}_s-T_{00}\right)+h_{00}^{(ice)}\right]}+\overline{F}_{net}^{(wv)}L_{s,00}+\overline{F}_{net}^{(liq)}L_{f,00}+\overline{F}_{net}^{(turb,rad)}\right\}dA. \quad (9$$

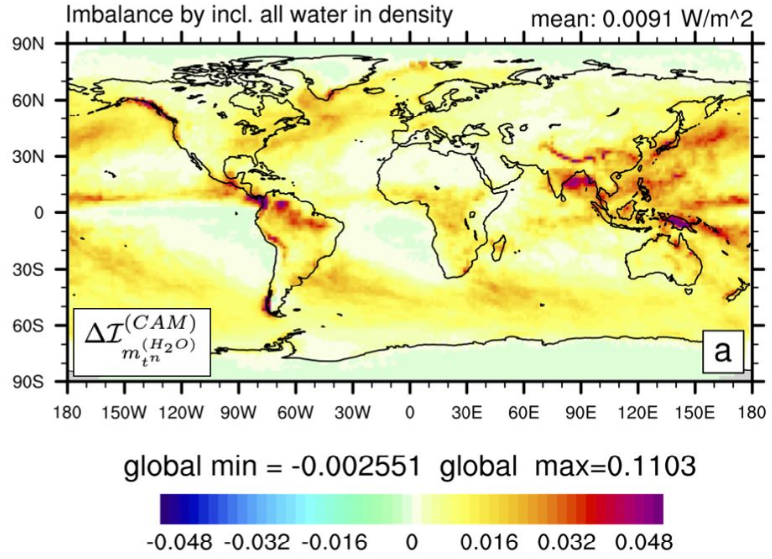
(ice reference enthalpy, $\widetilde{\overline{T}}_s \equiv \overline{T}_{atm,s} = \overline{T}_{surf,s}$)

Now also assume that the energy equation is valid for grid mean values in the model (**QUESTIONABLE ASSUMPTION! No sub-grid reservoir of energy …**)

Imbalance of incl. all forms of water in CAM's parameterization total energy equation:

$$\Delta\mathcal{I}^{(CAM)}_{m^{(H_2O)}_{tn}} =$$

$$\int\left[\rho^{(d)}\left(\sum_{\ell\in\mathcal{L}_{cond}}\overline{m}^{(\ell)}_{tn}\right)\right]\frac{\partial}{\partial t}\left(\overline{K}+\overline{\Phi}_s+c_p^{(d)}\overline{T}\right)dz$$



The total energy of suspended condensates is small … that said, error will grow with increased resolution …

Henceforth we assume that pressure/density incl. water although in CAM it only includes dry air and water vapor

FYI: in the process of adding all water to pressure/density in CAM

Figure: 1-year average column integrated total energy tendency for physics only.

CAM parameterization total energy equation

$$\frac{\partial}{\partial t}\int \overline{\rho}^{(d)}\left\{\left(1+\overline{m}_{t=t^n}^{(H_2O)}\right)\left[\overline{K}+\overline{\Phi}_s+c_p^{(d)}\left(\overline{T}-T_{00}\right)\right]+\overline{m}^{(wv)}L_{s,00}+\overline{m}^{(liq)}L_{f,00}\right\}dz$$

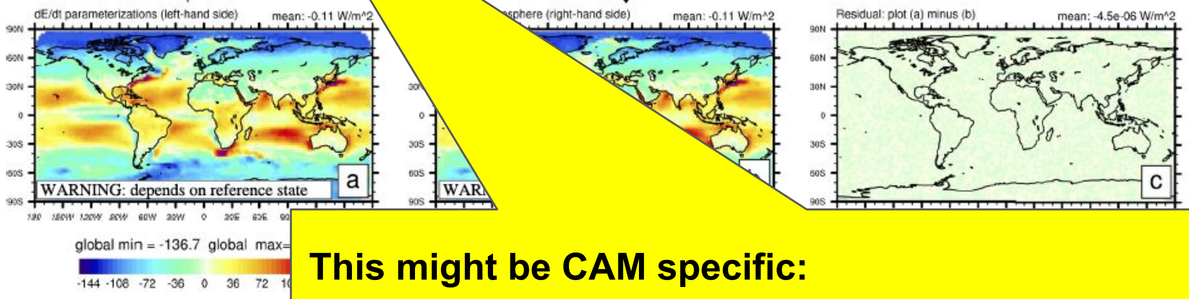
$$= -c_p^{(d)}T_{00}F_{net}^{(H_2O)}+\overline{F}_{net}^{(wv)}L_{s,00}+\overline{F}_{net}^{(liq)}L_{f,00}+\overline{F}_{net}^{(turb,rad)}$$

Each parameterization in CAM physics satisfied this equation (we have a check in the code!)

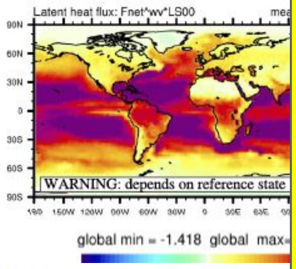
CAM parameterization total energy equation

$$\frac{\partial}{\partial t}\int \overline{\rho}^{(d)}\left\{\left(1+\overline{m}^{(H_2O)}_{t=t^n}\right)\left[\overline{K}+\overline{\Phi}_s+c_p^{(d)}\left(\overline{T}-T_{00}\right)\right]+\overline{m}^{(wv)}L_{s,00}+\overline{m}^{(liq)}L_{f,00}\right\}dz$$

$$-c_p^{(d)}T_{00}F^{(H_2O)}_{net}+\overline{F}^{(wv)}_{net}L_{s,00}+\overline{F}^{(liq)}_{net}L_{f,00}+\overline{F}^{(turb,rad)}_{net}$$



$$-c_p^{(d)}T_{00}F^{(H_2O)}_{net}$$



This might be CAM specific:

Total water is assumed constant during physics updates!

At the very end of physics total water is updated to reflect changes in total water due to falling precipitation and evaporation (called dry-mass adjustment in the code!);

The energy change associated with this is compensated for by a global energy fixer through a global temperature increment

Figure 2. One year average of vertically integ
Project (AMIP)-like simulation cycling over y
heat flux terms, Equation 111, (f) turbulent/sensible and radiative flux $F^{(turb,rad)}$. Note that plots (a, b, d, and e) depend on the specific reference state used in CAM (ice enthalpy reference state with $T_{00}=0$°C) whereas (c) does not. In the upper right corner of each plot is the global average of the term in question.

Updating water (pressure) in physics

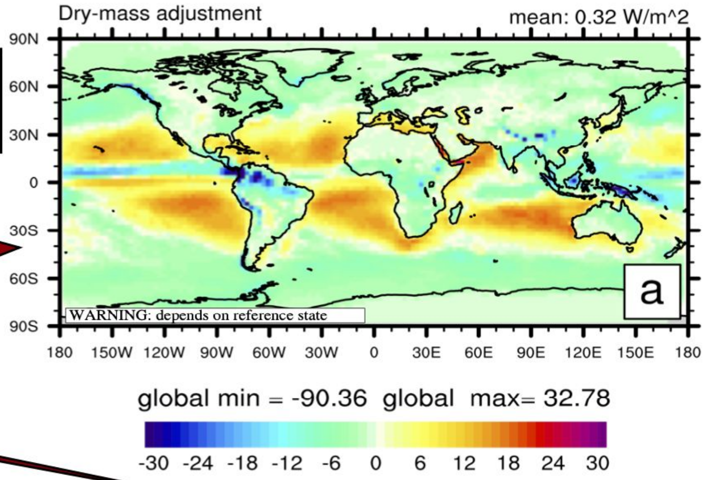
"spurious phase change term" due to CAM only incl. water vapor in total water

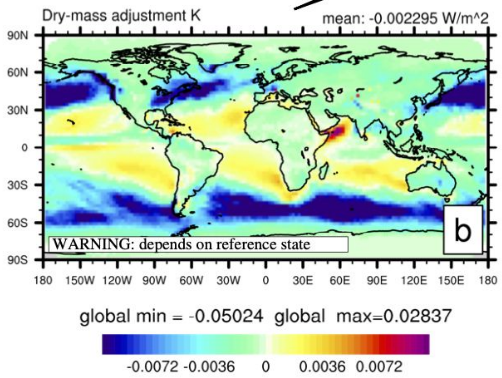
These 2 terms can not be separeted in our diagnostics!

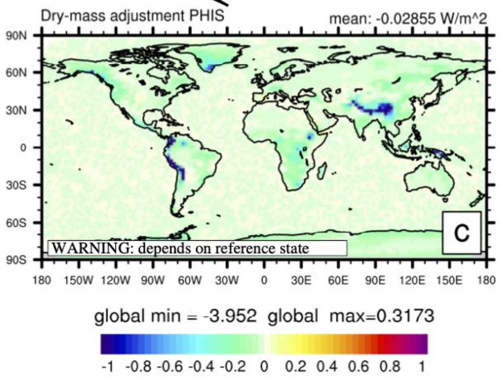
Total energy of falling precipitation and evaporation

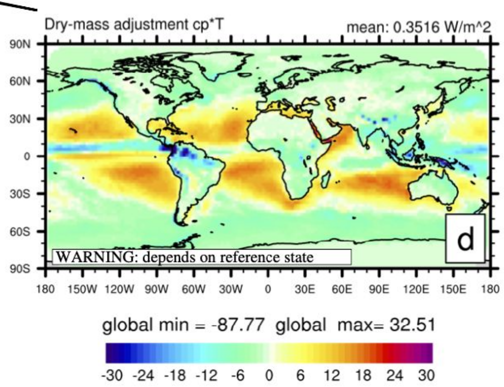
$$\Delta\mathcal{I}^{(CAM)}_{\partial m^{(wv)}/\partial t} = \Delta\check{\mathcal{I}}^{(CAM)}_{\partial m^{(wv)}/\partial t} + \Delta\widehat{\mathcal{I}}^{(CAM)}_{\partial m^{(wv)}/\partial t}$$

$$= \int \frac{\partial}{\partial t}\left[\rho^{(d)}\left(1+\overline{m}^{(wv)}\right)\right]\left(\overline{K}+\overline{\Phi}_s + c_p^{(d)}\overline{T}\right) dz$$









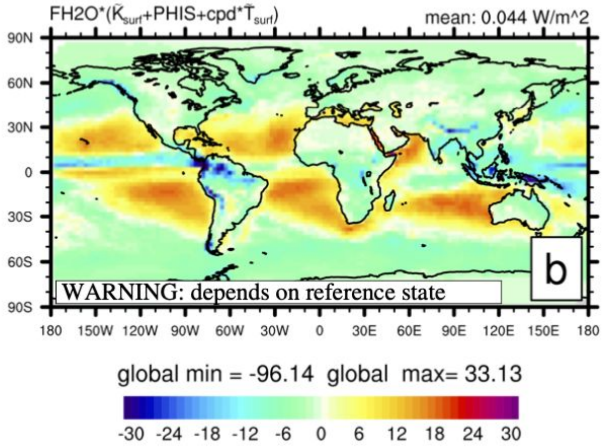

KEY
POINT

Let's have a look at the enthalpy, PHIS and kinetic energy flux terms that we neglected (using lowest model level T and K)

Modified CAM total energy equation incl. missing flux terms

$$\frac{\partial}{\partial t}\int \overline{\rho}^{(d)}\left\{\left(1+\overline{m}^{(H_2O)}\right)\left[\overline{K}+\overline{\Phi}_s+c_p^{(d)}\left(\overline{T}-T_{00}\right)\right]+\overline{m}^{(wv)}L_{s,00}+\overline{m}^{(liq)}L_{f,00}\right\}dz$$

$$-\Delta\widehat{\mathcal{I}}_{\partial m^{(H_2O)}/\partial t}-\Delta\mathcal{I}_{m_{tn}^{(H_2O)}}=\overline{F}_{net}^{(H_2O)}\left[c_p^{(d)}\left(\widetilde{\overline{T}}_s-T_{00}\right)+\widetilde{\overline{K}}_s+\overline{\Phi}_s\right]+\overline{F}_{net}^{(wv)}L_{s,00}+\overline{F}_{net}^{(liq)}L_{f,00}+\overline{F}_{net}^{(turb,rad)}$$



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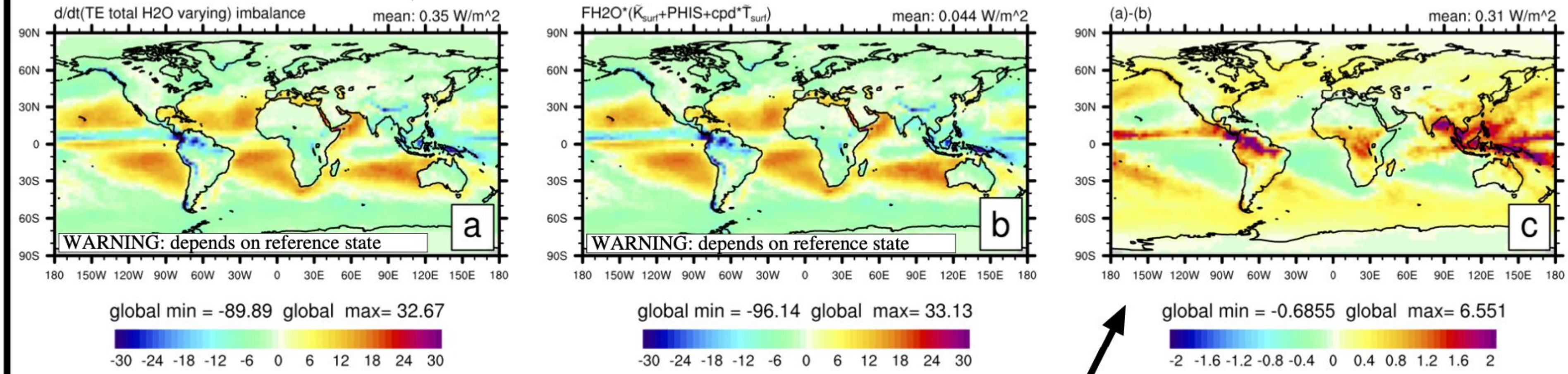


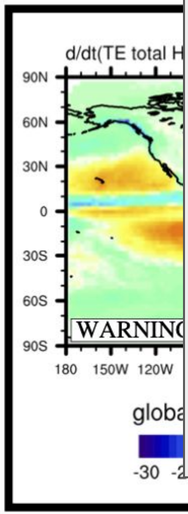
Challenging problems for implementing enthalpy flux in models:

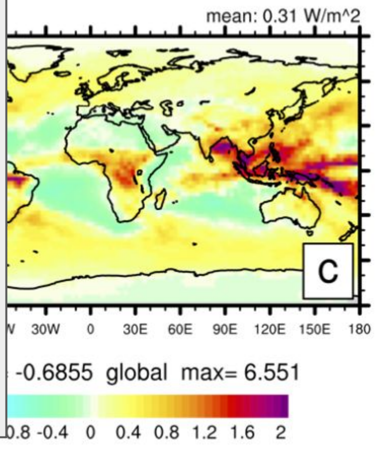
From an energy perspective it is problematic to consistently represent rain from the point at which it becomes falling precipitation: frictional dissipation (Pauluis et al, 2000),drag exerted by rain, $\widetilde{\overline{T}}_s$.

Note: it is possible to consistently incl. frictional dissipation of rain by using barycentric velocity framework
(see Appendix F in Lauritzen et al, 2022)

In a coupled climate model the enthalpy fluxes need to be passed to ocean, ice and land components (in the case of CESM the land component can NOT easily receive an enthalpy flux)

CAM assumes dry latent heats whereas MOM6 ocean model uses variable latent heats, i.e. CAM has to switch to variable latent heats to be consistent with MOM6!

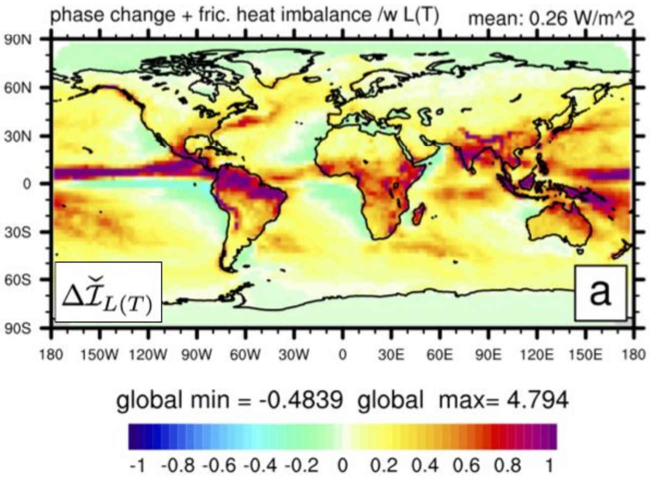




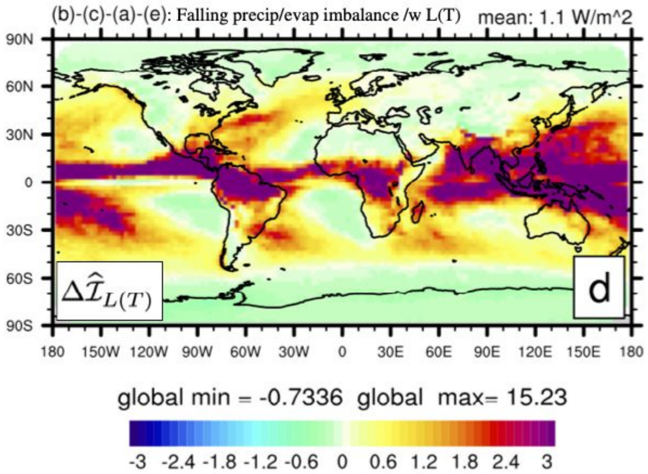
Modified (consistent) total energy equation assuming variable latent heats

$$\frac{\partial}{\partial t}\int \overline{\rho}^{(d)}\left\{\left(1+\overline{m}^{(H_2O)}\right)\left(\overline{K}+\overline{\Phi}_s\right)+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}}\overline{m}^{(\ell)}c_p^{(\ell)}\left(\overline{T}-T_{00}\right)+\overline{m}^{(wv)}L_{s,00}+\overline{m}^{(liq)}L_{f,00}\right\}dz$$

$$-\Delta\check{\mathcal{I}}_{L(T)}-\Delta\hat{\mathcal{I}}_{L(T)} = -\sum_{\ell\in\mathcal{L}_{H_2O}}\overline{F}_{net}^{(\ell)}\left[c_p^{(\ell)}\left(\tilde{\overline{T}}_s-T_{00}\right)+\tilde{\overline{K}}_s\right]+\overline{F}_{net}^{(wv)}L_{s,00}+\overline{F}_{net}^{(liq)}L_{f,00}+\overline{F}_{net}^{(turb,rad)}$$



(a) Imbalance for processes not involving falling precip. & evap.



(b) Imbalance for falling precip. & evap.

So far we have discussed missing terms in the energy budget …. Now to other errors …

Theoretical energetics/budgets (section 2)

Energy (existing & missing) budget terms of a climate model (section 3)

- See Oksana Guba's talk (for the purpose of this discussion E3SM and CAM are the same!)

Energy budget errors (section 4)

- Numerical truncation energy errors in dynamical cores (adiabatic). (see Lauritzen and Williamson, 2019)
- Physics–dynamics coupling errors due to spatial and temporal discretization errors. (see Donahue & Caldwell, 2020, Lauritzen and Williamson, 2019)
- Thermodynamic inconsistency energy errors in physics:
  ° **As an illustration we discuss a specific example in some detail: coupling the CLUBB cloud parameterization package with the CAM climate model.**
- Thermodynamic and vertical coordinate inconsistencies between dynamical core and parameterizations:
  * different vertical coordinates (see Lauritzen et al., in prep, for z-based MPAS coupling with p-based CAM)
  * different enthalpy definitions (e.g., FV3/SE coupled with CAM)
- Mass "clipping" errors and energy

Summary and future directions

Thermodynamic conserved variable inconsistency leading to total energy errors

An example: Coupling CLUBB with CAM (problem identified by Chris Golaz in 2010)

In detail, CLUBB transports an approximate form of the conserved moist potential temperature θ_l (see Tripoli & Cotton, 1981; Cotton et al., 2011), which is defined as

$$\theta_l \equiv T\Pi^{-1} - \frac{L_{v,00}}{c_p^{(d)}}\Pi^{-1}m^{(liq)}, \tag{151}$$

where Π is the Exner function, which is purely a function of pressure. CLUBB then returns to CAM the following tendency of θ_l,

$$\overline{\rho}^{(d)}\left(1+\overline{m}^{(wv)}\right)\frac{\partial\overline{\theta_l}}{\partial t}|_{\mathrm{CLUBB}} = -\frac{\partial}{\partial z}\left[\overline{\rho}^{(d)}\left(1+\overline{m}^{(wv)}\right)\overline{w'\theta_l'}\right], \tag{152}$$

(152) in terms of T and integrated in vertical

Assuming no surface fluxes and K changes in CLUBB

$$\int \frac{1}{\Pi_{t^n}}\left(c_p^{(d)}\Delta\overline{T} - L_{v,00}\Delta\overline{m}^{(liq)}\right)\overline{\rho}_{t^n}^{(d)}\left(1+\overline{m}_{t^n}^{(wv)}\right)dz = 0. \tag{155}$$

Thermodynamic conserved variable inconsistency leading to total energy errors

An example: Coupling CLUBB with CAM (problem identified by Chris Golaz in 2010)

CAM's conserved variable (only terms relevant to CLUBB retained and excl. kinetic energy and surface fluxes)

Host model

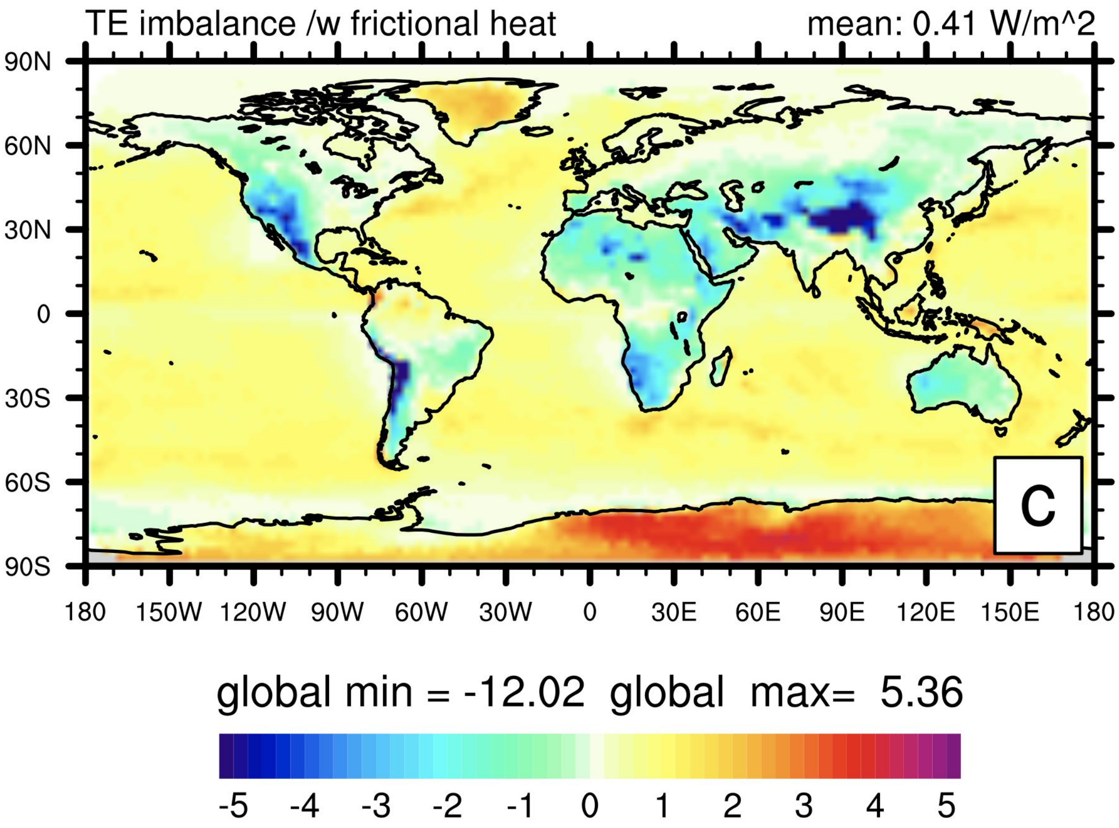
$$\int \left(c_p^{(d)} \Delta \overline{T} - L_{v,00} \Delta \overline{m}^{(liq)}\right) \overline{\rho}_{t^n}^{(d)} \left(1 + \overline{m}_{t^n}^{(wv)}\right) dz = 0. \qquad (158)$$

Parameterization

$$\int \frac{1}{\overline{\Pi}_{t^n}} \left(c_p^{(d)} \Delta \overline{T} - L_{v,00} \Delta \overline{m}^{(liq)}\right) \overline{\rho}_{t^n}^{(d)} \left(1 + \overline{m}_{t^n}^{(wv)}\right) dz = 0. \qquad (155)$$

CLUBB's conserved variable

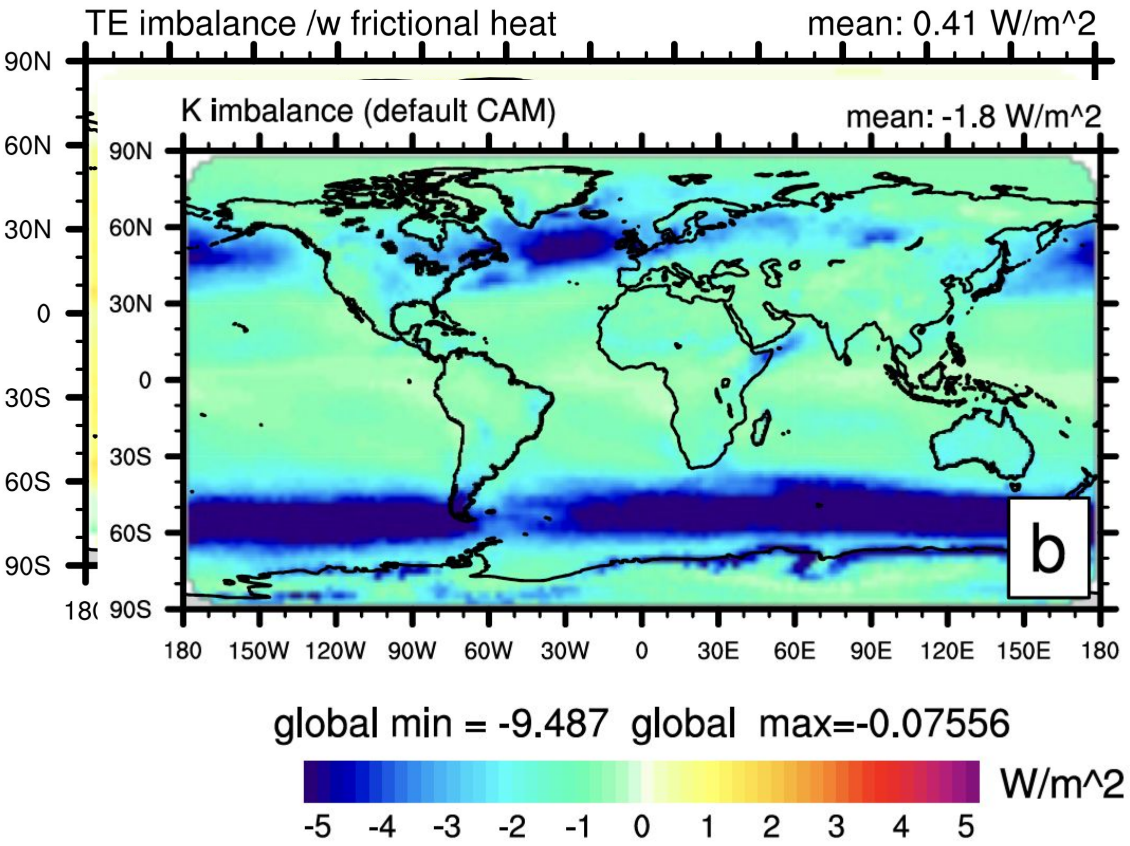
1-year column averaged imbalance using CAM (CESM)



To make CAM physics with CLUBB pass the energy budget checks in CAM, the implementers chose to add a temperature increment in each column to compensate for thermodynamic/energy inconsistency!

(similarly for kinetic energy)

1-year column averaged imbalance using CAM (CESM)



To make CAM physics with CLUBB pass the energy budget checks in CAM, the implementers choose to add a temperature increment in each column to compensate for thermodynamic/energy inconsistency!

(similarly for kinetic energy)

Summary of energy errors

- **Dynamical core total** energy errors can be large (~0.1–1 W/m 2) at large-scale model resolutions (~1°; Lauritzen & Williamson, 2019). Errors are expected to decrease as horizontal/vertical resolution increases
- **Temporal physics–dynamics coupling** errors (where tendencies are added throughout the dynamical core time loop) can be large (~0.4 W/m 2, Lauritzen & Williamson, 2019)
- Physics–dynamical errors due to the fact that the energy associated with the continuous equations of the dynamical core is different than the energy of the physics (e.g., due to **different a-principio approximations**) can be large when the dynamical core uses variable latent heats and physics does not (~0.5 W/m 2, Lauritzen & Williamson, 2019).
- **Enthalpy** tendencies associated with falling precipitation and water entering the atmosphere are large (~0.3 W/m 2) when using constant latent heats and even larger (~1.1 W/m 2) when using variable latent heats. Locally, the errors can be orders of magnitude larger. This error, in general, is not expected to decrease with increased resolution. In fact, larger water contents at higher resolution may make the issue worse.
- **Kinetic and potential** energies associated with falling precipitation (and evaporation or other forms of water entering the atmosphere) are small (~<0.02 W/m 2).
- Errors associated with not including **all forms of water in pressure/mass** are small (~<0.01 W/m 2). Local errors could increase with increased resolution as the water content locally is larger at higher resolutions.
- **Thermodynamic inconsistency errors in parameterizations**: These imbalances are, of course, specific to the inconsistency in question. For example, we showed that the inconsistency between CLUBB using moist potential temperature θ_l as its conserved variable and CAM using enthalpy leads to ~0.4 W/m 2 imbalance.

Future directions

Nearer term directions:

- Incl. enthalpy flux in coupled climate models (some challenges remain!)
- Move to variable latent heats in physics (many dycores already use variable latent heats)
- Carefully study/understand assumptions in individual parameterizations and host models

A warning to CCPP: we have to be careful importing new physics schemes into host models without carefully examining thermodynamic/energetic consistency

Longer term direction:

Global models are moving away from shallow atmosphere, hydrostatic formulations. As this transition is made, it becomes increasingly tedious and error-prone to ensure that fundamental physical principles are satisfied.

- Rather than working at the level of the equations of motion, a more powerful approach is to instead work with a **geometric mechanics formulation**
- Structure-Preserving Discretizations
- Thermodynamic Potentials

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Thermodynamic inconsistency in sensible heat flux in CAM-CLUBB

Neglect kinetic energy (i.e. assume for the moment that CLUBB does not alter winds), neglect radiation and assume that there are no phase changes. Then CAM's energy equation reduces to:

$$\frac{\partial}{\partial t} \int \left\{ \overline{\rho}^{(d)} \left[1 + \overline{m}_{t=t^n}^{(wv)} \right] c_p^{(d)} \overline{T} \right\} dz = \overline{F}_{net}^{(turb)}. \tag{159}$$

In contrast, CLUBB conserves

$$\frac{\partial}{\partial t} \int \left\{ \overline{\rho}^{(d)} \left[1 + \overline{m}_{t=t^n}^{(wv)} \right] \theta_\ell \right\} dz = \overline{\rho}^{(d)} \left[1 + \overline{m}_{t=t^n}^{(wv)} \right] \overline{w'\theta_l'} \Big|_{\text{surface}} \equiv \overline{\mathcal{F}}_{net}^{(turb)}. \tag{160}$$

That is, CLUBB conserves a potential temperature variable rather than temperature. In the absence of phase changes, (160) becomes

$$\frac{\partial}{\partial t} \int \left\{ \overline{\rho}^{(d)} \left[1 + \overline{m}_{t=t^n}^{(wv)} \right] \frac{T}{\Pi} \right\} dz = \overline{\mathcal{F}}_{net}^{(turb)}. \tag{161}$$

Sensible heat flux should be scaled with Exner - was not done in CAM (fixed now!)

$$\boxed{\overline{\mathcal{F}}_{net}^{(turb)} = \frac{\overline{F}_{net}^{(turb)}}{c_p^{(d)} \Pi_s}.}$$

Thermodynamic inconsistency in sensible heat flux in CAM-CLUBB