Reconciling and improving formulations for thermodynamics and conservation principles in Earth System Models (ESMs)

Peter Hjort Lauritzen, pel@ucar.edu

http://www.cgd.ucar.edu/cms/pel

Atmospheric Modeling and Prediction (AMP) Section
Climate and Global Dynamics (CGD) Laboratory
National Center for Atmospheric Research (NCAR)

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Reconciling and improving formulations for thermodynamics and conservation principles in Earth System Models (ESMs)

P.H. Lauritzen[1], N.K.-R. Kevlahan[2], T. Toniazzo[3,14], C. Eldred[4], T. Dubos[5], A. Gassmann[6], V.E. Larson[7], C. Jablonowski[8], O. Guba[4], B. Shipway[9], B.E. Harrop[10], F. Lemarié[11], R. Tailleux[12], A.R. Herrington[1], W. Large[1], P.J. Rasch[10], A.S. Donahue[13], H. Wan[10], A. Conley[1], J.T. Bacmeister[1]

Physics-Dynamics Coupling in Earth System Models (19w5153)

Organizers

Nicholas Kevlahan (McMaster University)

Peter Lauritzen (National Center for Atmospheric Research)

Banff International Research Station
for Mathematical Innovation and Discovery

https://www.birs.ca/events/2019/5-day-workshops/19w5153

Outline (of manuscript)

Theoretical energetics/budgets (section 2)

Start with the dry hydrostatic primitive equations (HPE) and gradually increase the thermodynamic complexity by first adding water vapor and then condensates to the HPE. Special attention is given to the derivation of enthalpy terms (and associated reference states), latent heat terms and surface flux terms. For these models, a detailed explanation of the approximations made in large-scale models can be included rigorously. An in-depth discussion is included of surface fluxes and the complications arising due to falling precipitation and/or water entering the atmosphere using a single-component fluid approach.

Energy (existing & missing) budget terms of a climate model (section 3)

- See also Oksana Guba's talk (for the purpose of this discussion E3SM and CAM are the same!)

Energy budget errors (section 4)

- Numerical truncation energy errors in dynamical cores (adiabatic). (see Lauritzen and Williamson, 2019)
- Physics–dynamics coupling errors due to spatial and temporal discretization errors. (see Donahue & Caldwell, 2020, Lauritzen and Williamson, 2019)
- Thermodynamic inconsistency energy errors in physics:
  - **As an illustration we discuss a specific example in some detail: coupling the CLUBB cloud parameterization package with the CAM climate model.**
- Thermodynamic and vertical coordinate inconsistencies between dynamical core and parameterizations:
  * different vertical coordinates (see Lauritzen et al., in prep, for z-based MPAS coupling with p-based CAM)
  * different enthalpy definitions (e.g., FV3/SE coupled with CAM)
- Mass "clipping" errors and energy

Total energy equation

Assume:

Lauritzen et al. (2022, submitted)

- Primitive equations (hydrostatic, shallow atmosphere, ideal gas)
- Assume model top pressure is constant
- All components of moist air have the **same temperature** and move with the **same horizontal velocity**
- Assume that water entering the atmosphere (evaporation, snow drift, sea spray) has **same temperature** as water leaving the atmosphere (dew, liquid and frozen precipitation) **Just for notational simplicity!**

Then it can be shown that the following globally integrated total energy equation holds:

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+c_p^{(\ell)}\left(T-T_{00}\right)+h_{00}^{(ice)}\right]\right.$$
$$\left.+\,m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\right\}dA\,dz$$
$$=\iint\left\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\right\}dA. \quad (94)$$

(ice reference enthalpy, $\widetilde{T}_s \equiv T_{atm,s} = T_{surf,s}$)

symbol	description	unit
$c_p^{(\ell)}$	heat capacity at constant pressure of species ℓ	$J/K/kg$
$F_{net}^{(\ell)}$	net flux of water species ℓ into the atmosphere	$kg/m^2/s$
$F_{net}^{(turb,rad)}$	Radiative and sensible/turbulent fluxes into atmosphere (90)	$J/m^2/s$
$h_{00}^{(\ell)}$	reference enthalpy for water form ℓ	J/kg
$m^{(\ell)}$	dry mixing ratio ($\equiv \rho^{(\ell)}/\rho^{(d)}$)	kg/kg
K	specific horizontal kinetic energy ($\equiv \frac{1}{2}\vec{v}^2$)	m^2/s^2
$L_{f,00}.$	latent heat of fusion	J/K
$L_{s,00}.$	latent heat of sublimation	J/K
$L_{v,00}.$	latent heat of vaportization	J/K
Φ_s	surface geopotential	m^2/s^2
ρ_d	dry air density	kg/m^3
T	temperature	K
$\widetilde{T}_s$	common temperature at surface	K
$\vec{v}$	horizontal velocity vector	m/s

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$$=\iint\left\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{\overline{K}}_s+\overline{\Phi}_s+c_p^{(\ell)}\left(\widetilde{\overline{T}}_s-T_{00}\right)+h_{00}^{(ice)}\right]+\overline{F}_{net}^{(wv)}L_{s,00}+\overline{F}_{net}^{(liq)}L_{f,00}+\overline{F}_{net}^{(turb,rad)}\right\}dA. \quad (94)$$

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Now also assume that the energy equation is valid for grid mean values in the model (**QUESTIONABLE ASSUMPTION! No sub-grid reservoir of energy …**)

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Then it can be shown that the following globally integrated total energy equation holds:

Equivalent to assuming constant latent heats!

Many models make these assumptions:

$$\boxed{\mathcal{L}_{H_2O} =' wv'}$$

$$\boxed{c_p^{(\ell)} = c_p^{(d)}}$$

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left\{\overline{K}+\overline{\Phi}_s+c_p^{(d)}\overline{T}+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[\overline{K}+\overline{\Phi}_s+c_p^{(\ell)}\left(\overline{T}-T_{00}\right)+h_{00}^{(ice)}\right] + \overline{m}^{(wv)}L_{s,00}+\overline{m}^{(liq)}L_{f,00}\right\}dA\,dz$$

$$= \iint\left\{\cancel{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\overline{\widetilde{K}}_s+\overline{\Phi}_s+c_p^{(\ell)}\left(\overline{\widetilde{T}}_s-T_{00}\right)+h_{00}^{(ice)}\right]}+\overline{F}_{net}^{(wv)}L_{s,00}+\overline{F}_{net}^{(liq)}L_{f,00}+\overline{F}_{net}^{(turb,rad)}\right\}dA. \quad (9$$

(ice reference enthalpy, $\overline{\widetilde{T}}_s \equiv \overline{T}_{atm,s} = \overline{T}_{surf,s}$)

E.g. FV3 and NCAR-SE use variable latent heats and CAM physics not -> leads to 0.5W/m2 imbalance (Lauritzen and Williamson, 2019)

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$$\boxed{\mathcal{L}_{H_2O} =' wv'} \qquad + m$$

$$= \cancel{\iint\left\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\overline{\Phi}_s+\right.\right.}$$

(ice refere

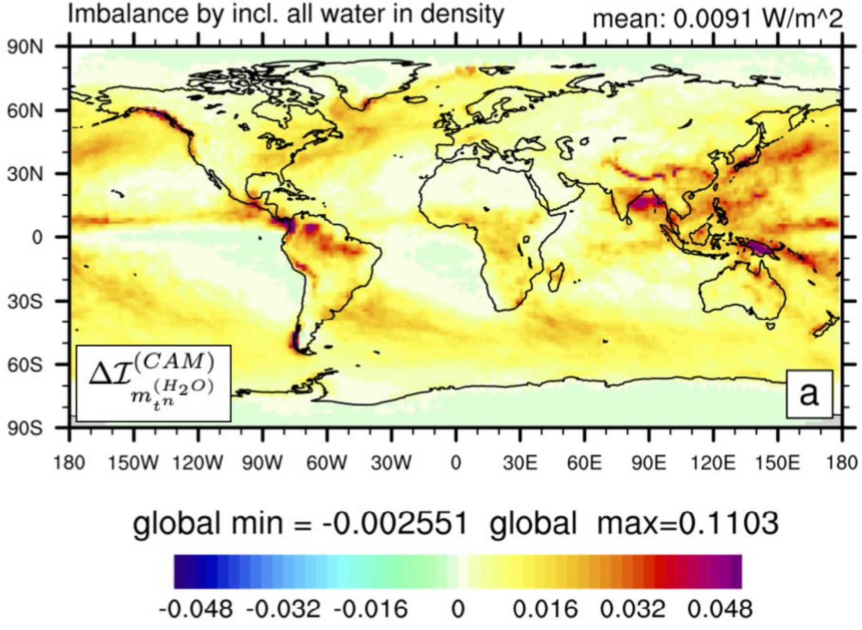
Now also assume that the energy equation i **E ASSUMPTION!**

No sub-grid reservoir of energy …)

Imbalance of incl. all forms of water in CAM's parameterization total energy equation:

$$\Delta\mathcal{I}^{(CAM)}_{m_{tn}^{(H_2O)}} =$$

$$\int\left[\rho^{(d)}\left(\sum_{\ell\in\mathcal{L}_{cond}} \overline{m}_{tn}^{(\ell)}\right)\right]\frac{\partial}{\partial t}\left(\overline{K}+\overline{\Phi}_s+c_p^{(d)}\overline{T}\right)dz$$



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Updating water (pressure) in physics

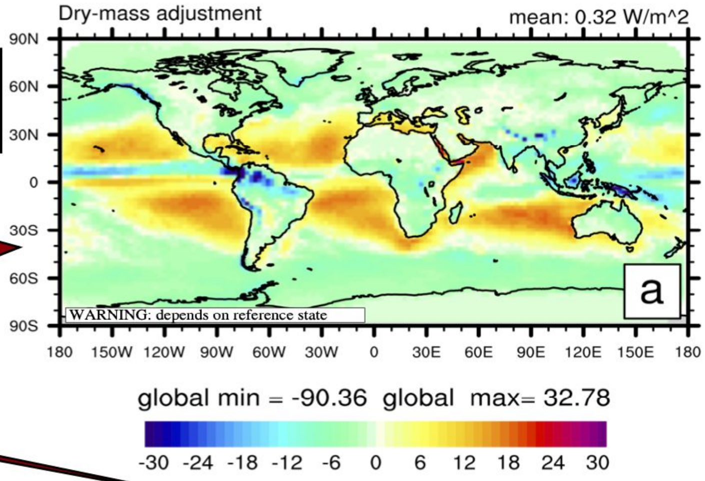
"spurious phase change term" due to CAM only incl. water vapor in total water

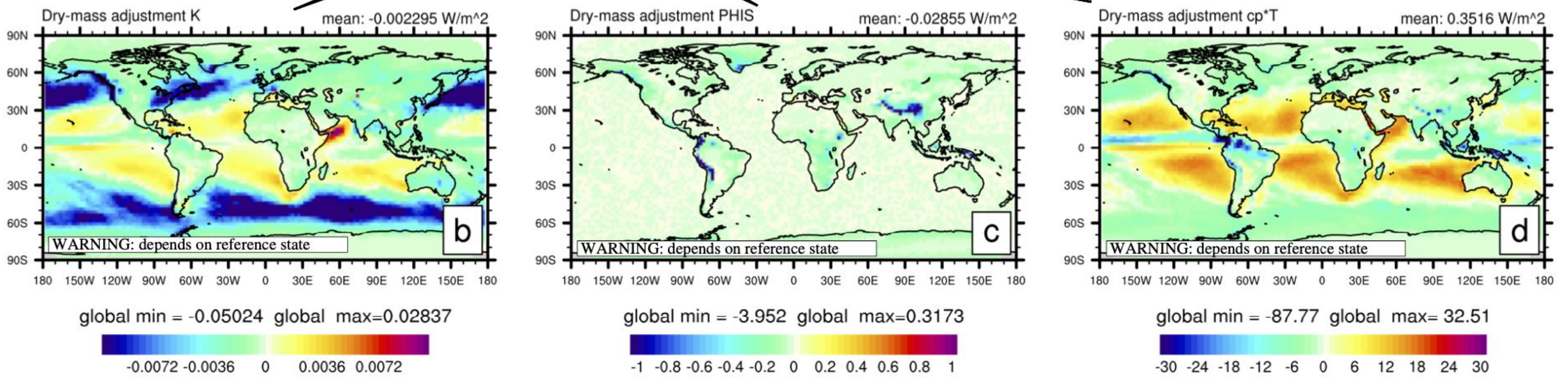
These 2 terms can not be separeted in our diagnostics!

Total energy of falling precipitation and evaporation

$$\Delta\mathcal{I}^{(CAM)}_{\partial m^{(wv)}/\partial t} = \Delta\check{\mathcal{I}}^{(CAM)}_{\partial m^{(wv)}/\partial t} + \Delta\widehat{\mathcal{I}}^{(CAM)}_{\partial m^{(wv)}/\partial t}$$

$$= \int \frac{\partial}{\partial t}\left[\rho^{(d)}\left(1+\overline{m}^{(wv)}\right)\right]\left(\overline{K}+\overline{\Phi}_s+c_p^{(d)}\overline{T}\right)dz$$





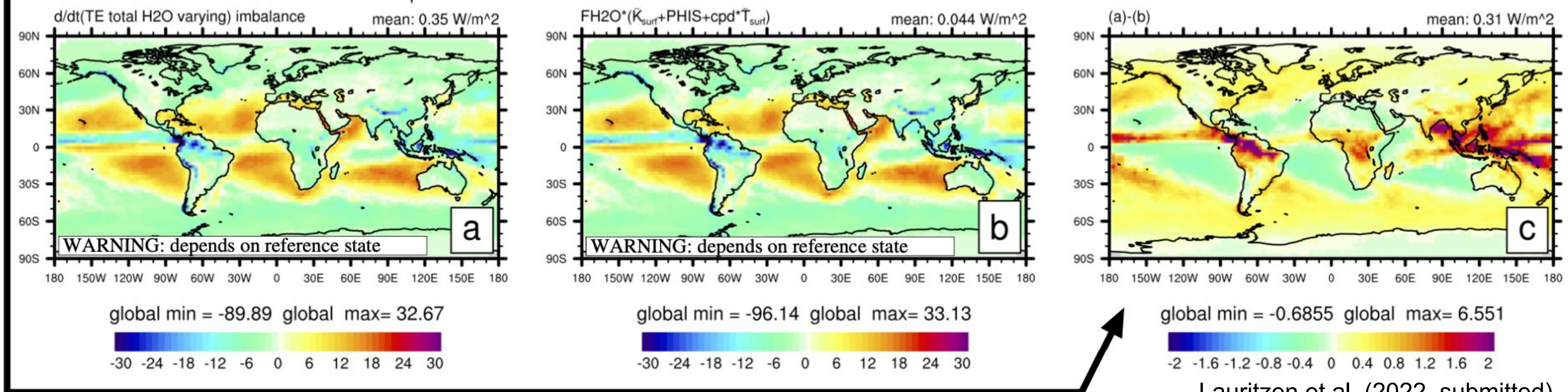
Lauritzen et al. (2022, submitted)

From energy perspective it is problematic to consistently represent rain from the point at which it becomes falling precipitation: frictional dissipation (Pauluis et al, 2000), T_s, drag exerted by rain. **Note**: possible to consistently incl. frictional dissipation of rain by using barycentric velocity framework (see Appendix F in Lauritzen et al, 2022, submitted)

Modified CAM total energy equation incl. missing flux terms

$$\frac{\partial}{\partial t}\int \overline{\rho}^{(d)}\left\{\left(1+\overline{m}^{(H_2O)}\right)\left[\overline{K}+\overline{\Phi}_s+c_p^{(d)}\left(\overline{T}-T_{00}\right)\right]+\overline{m}^{(wv)}L_{s,00}+\overline{m}^{(liq)}L_{f,00}\right\}dz$$

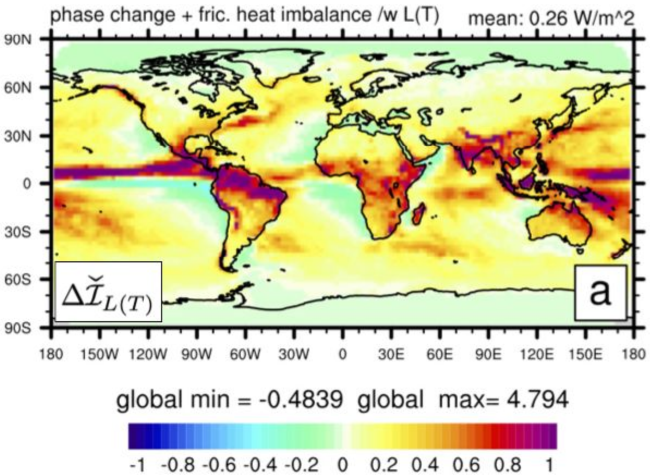
$$-\Delta\widehat{\mathcal{I}}_{\partial m^{(H_2O)}/\partial t}-\Delta\mathcal{I}_{m_{tn}^{(H_2O)}}=\overline{F}_{net}^{(H_2O)}\left[c_p^{(d)}\left(\widetilde{\overline{T}}_s-T_{00}\right)+\widetilde{\overline{K}}_s+\overline{\Phi}_s\right]+\overline{F}_{net}^{(wv)}L_{s,00}+\overline{F}_{net}^{(liq)}L_{f,00}+\overline{F}_{net}^{(turb,rad)}$$



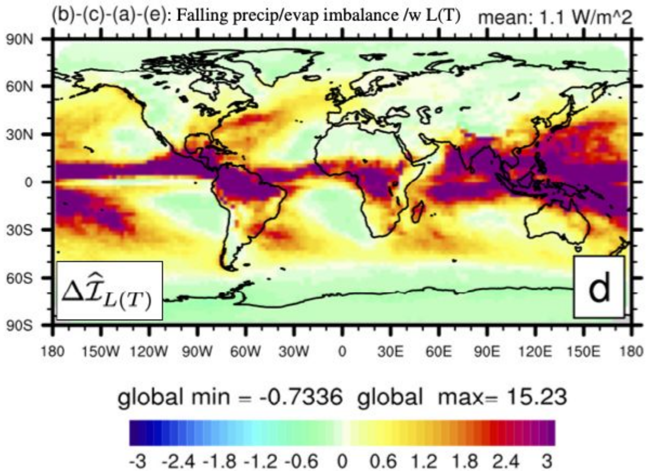
Lauritzen et al. (2022, submitted)

Modified (consistent) total energy equation assuming variable latent heats

$$
\begin{aligned}
&\frac{\partial}{\partial t}\int \overline{\rho}^{(d)}\left\{\underline{\left(1+\overline{m}^{(H_2O)}\right)\left(\overline{K}+\overline{\Phi}_s\right)+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}}\overline{m}^{(\ell)}c_p^{(\ell)}\left(\overline{T}-T_{00}\right)}+\overline{m}^{(wv)}L_{s,00}+\overline{m}^{(liq)}L_{f,00}\right\}dz \\
&-\Delta\check{\mathcal{I}}_{L(T)}-\Delta\widehat{\mathcal{I}}_{L(T)} = -\sum_{\ell\in\mathcal{L}_{H_2O}}\underline{\overline{F}_{net}^{(\ell)}\left[c_p^{(\ell)}\left(\overline{\widetilde{T}}_s-T_{00}\right)+\overline{\widetilde{K}}_s\right]}+\overline{F}_{net}^{(wv)}L_{s,00}+\overline{F}_{net}^{(liq)}L_{f,00}+\overline{F}_{net}^{(turb,rad)}
\end{aligned}
$$



(a) Imbalance for processes not involving falling precip. & evap.



(b) Imbalance for falling precip. & evap.

Lauritzen et al. (2022, submitted)

Outline (of manuscript)

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Summary and future directions

Thermodynamic conserved variable inconsistency leading to total energy errors

An example: Coupling CLUBB with CAM (problem identified by Chris Golaz in 2010)

In detail, CLUBB transports an approximate form of the conserved moist potential temperature θ_l (see Tripoli & Cotton, 1981; Cotton et al., 2011), which is defined as

$$\theta_l \equiv T\Pi^{-1} - \frac{L_{v,00}}{c_p^{(d)}}\Pi^{-1} m^{(liq)}, \tag{151}$$

where Π is the Exner function, which is purely a function of pressure. CLUBB then returns to CAM the following tendency of θ_l,

$$\overline{\rho}^{(d)}\left(1 + \overline{m}^{(wv)}\right) \frac{\partial \overline{\theta_l}}{\partial t}\Big|_{\mathrm{CLUBB}} = -\frac{\partial}{\partial z}\left[\overline{\rho}^{(d)}\left(1 + \overline{m}^{(wv)}\right) \overline{w'\theta_l'}\right], \tag{152}$$

(152) in terms of T

Assuming no surface fluxes and K changes in CLUBB

$$\int \frac{1}{\Pi_{t^n}} \left(c_p^{(d)} \Delta \overline{T} - L_{v,00} \Delta \overline{m}^{(liq)}\right) \overline{\rho}_{t^n}^{(d)} \left(1 + \overline{m}_{t^n}^{(wv)}\right) dz = 0. \tag{155}$$

Lauritzen et al. (2022, submitted)

Thermodynamic conserved variable inconsistency leading to total energy errors

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$$\int \left(c_p^{(d)} \Delta \overline{T} - L_{v,00} \Delta \overline{m}^{(liq)}\right) \overline{\rho}_{t^n}^{(d)} \left(1 + \overline{m}_{t^n}^{(wv)}\right) dz = 0. \qquad (158)$$

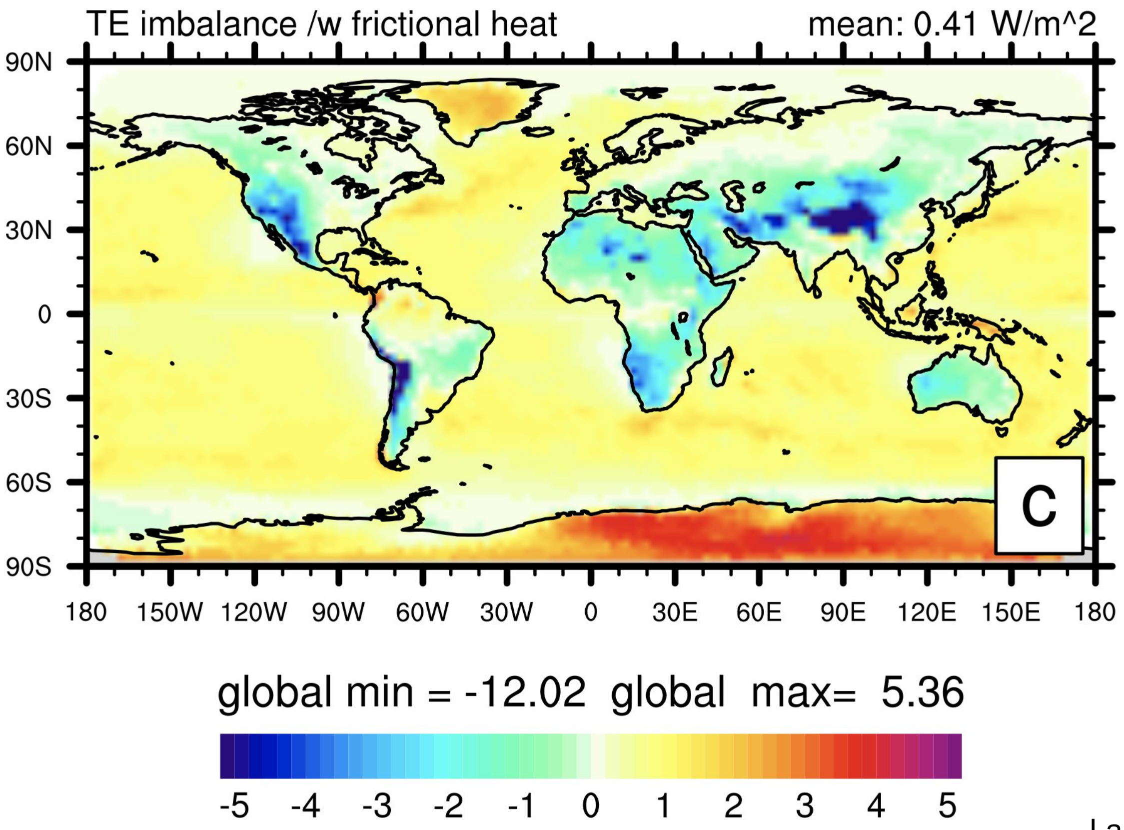
CAM's conserved variable

$$\int \frac{1}{\Pi_{t^n}} \left(c_p^{(d)} \Delta \overline{T} - L_{v,00} \Delta \overline{m}^{(liq)}\right) \overline{\rho}_{t^n}^{(d)} \left(1 + \overline{m}_{t^n}^{(wv)}\right) dz = 0. \qquad (155)$$

CLUBB's conserved variable

Lauritzen et al. (2022, submitted)

1-year column averaged imbalance using CAM (CESM)



Lauritzen et al. (2022, submitted)

Thermodynamic inconsistency in sensible heat flux in CAM-CLUBB

Neglect kinetic energy (i.e. assume for the moment that CLUBB does not alter winds), neglect radiation and assume that there are no phase changes. Then CAM's energy equation reduces to:

$$\frac{\partial}{\partial t}\int\left\{\overline{\rho}^{(d)}\left[1+\overline{m}_{t=t^n}^{(wv)}\right]c_p^{(d)}\overline{T}\right\}dz = \overline{F}_{net}^{(turb)}. \tag{159}$$

In contrast, CLUBB conserves

$$\frac{\partial}{\partial t}\int\left\{\overline{\rho}^{(d)}\left[1+\overline{m}_{t=t^n}^{(wv)}\right]\theta_\ell\right\}dz = \overline{\rho}^{(d)}\left[1+\overline{m}_{t=t^n}^{(wv)}\right]\overline{w'\theta_l'}\Big|_{\text{surface}} \equiv \overline{\mathcal{F}}_{net}^{(turb)}. \tag{160}$$

That is, CLUBB conserves a potential temperature variable rather than temperature. In the absence of phase changes, (160) becomes

$$\frac{\partial}{\partial t}\int\left\{\overline{\rho}^{(d)}\left[1+\overline{m}_{t=t^n}^{(wv)}\right]\frac{T}{\Pi}\right\}dz = \overline{\mathcal{F}}_{net}^{(turb)}. \tag{161}$$

Sensible heat flux should be scaled with Exner - currently not done in CAM (changing soon though!)

$$\boxed{\overline{\mathcal{F}}_{net}^{(turb)} = \frac{\overline{F}_{net}^{(turb)}}{c_p^{(d)}\Pi_s}.}$$

Lauritzen et al. (2022, submitted)

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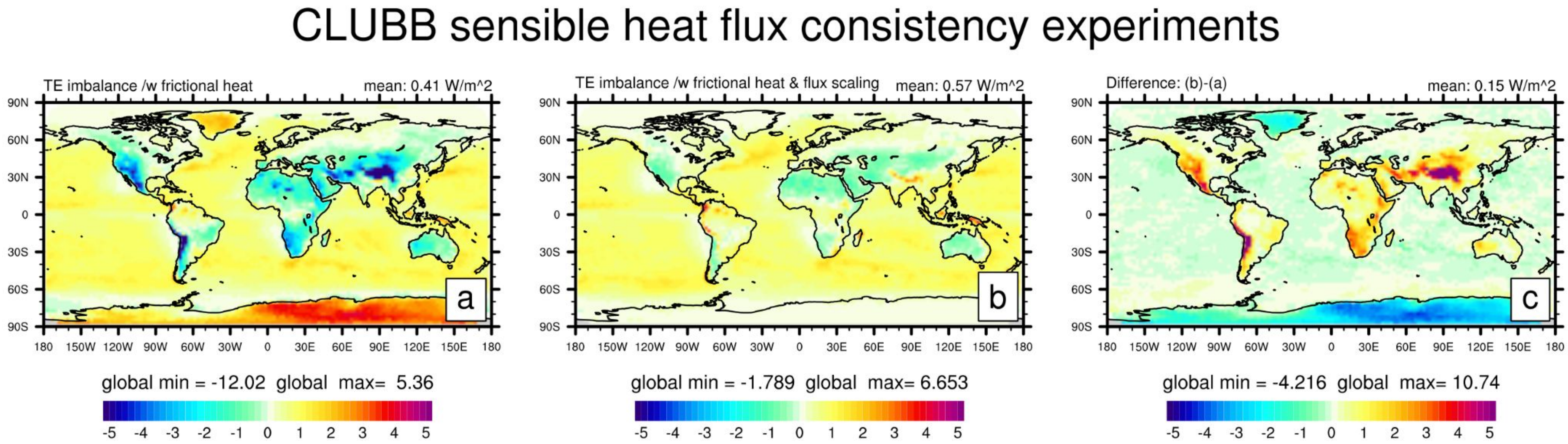

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