Frequently used acronyms in this talk:

CESM = Community Earth System Model
CAM = Community Atmosphere Model
TE = Total Energy

Consistently Closing the Energy Budget in Earth System Models

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JAMES Journal of Advances in Modeling Earth Systems®

Reconciling and Improving Formulations for Thermodynamics and Conservation Principles in Earth System Models (ESMs)

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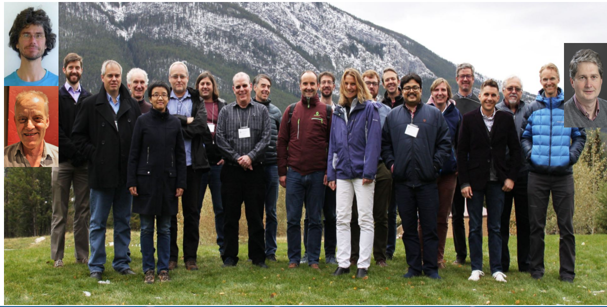
Featured as Editor's Highlight in Eos:
https://eos.org/editor-highlights/consistently-closing-the-energy-budget-in-earth-system-models

The paper was the result of a BIRS workshop held in 2019

Paper link: https://agupubs.onlinelibrary.wiley.com/doi/full/10.1029/2022MS003117
(warning: 83 pages; 166 equations excluding equations in the Appendices)

Awarded UCAR/NCAR 2023 Outstanding paper award



Overview

Total energy (TE) equation for large scale atmosphere model

Ways in which Earth System Models do NOT close their total energy budgets:

- TE tendencies associated with falling precipitation and evaporation
- coupling atmosphere with surface (ocean-atmosphere only!)
- dynamical core energy dissipation
- physics-dynamics coupling: (i) temporal, (ii) spatial and/or (iii) energy formula discrepancy
- Discrepancy between conserved quantity in parameterization and host model

Component level: Closed energy budget?

$$\frac{\partial}{\partial t}\iiint \rho\left(E_{atm}\right) dV = -\oiint \mathcal{F}_{atm}^{(top)} d\sigma + \oiint \mathcal{F}_{atm}^{(bottom)} d\sigma,$$

… and similarly for other components, e.g., ocean

$$\frac{\partial}{\partial t}\iiint \rho^{(all)}\left(E_{ocn}\right) dV = -\oiint \mathcal{F}_{ocn}^{(top)} dA + \oiint \mathcal{F}_{ocn}^{(bottom)} dA$$

where the fluxes across components should match

$$\mathcal{F}_{atm}^{(bottom)} = \mathcal{F}_{ocn}^{(top)}$$

How to define energy?

$$E_{atm} = E_{feom} + E_{other}.$$

(feom=fluid equations of motion)

The total energy conserved by the governing equations of motion and associated thermodynamics is referred to as the **fluid equations of motion energy**.

The fluid equations of motion and associated thermodynamics are approximated:

- Neglecting non-hydrostatic motion, breaking gravity waves and 3D turbulence
- Neglecting individual momentum equations for hydrometeors, and making single temperature (T) assumption

How to define energy?

$$E_{atm} = E^{(res)} + E^{(unres)}$$

Even more complex problem. It is not possible to run models at the small scales necessary to resolve all processes.

We must therefore homogenize (i.e., average) processes smaller than about 50–100 km in operational climate models, and roughly 0.5–3 km for cutting edge convection-permitting global models

-> **Energy will always have both a resolved and an unresolved component**

In general, we have a good idea of how averaging and subgrid modeling works for fluid turbulence (LES closures such as Smagorinsky 1963, Germano 1992, etc.)

However, subgrid models for, e.g., thermodynamics are problematic?

$$\overline{U^{(all)}} := \overline{c_v^{(all)} T} \approx \overline{c}_v^{(all)} \overline{T}, \qquad \text{(A1)}$$

where the generalized specific heat $c_v^{(all)}$ is the continuous formula replaced with dynamical core prognostic state values (i.e., from the resolved fluid dynamics)

$$\overline{c}_v^{(all)} = \frac{\sum_{\ell \in \mathcal{L}_{all}} c_v^{(\ell)} \overline{m}^{(\ell)}}{\sum_{\ell \in \mathcal{L}_{all}} \overline{m}^{(\ell)}}. \qquad \text{(A2)}$$

How to define energy?

$$E_{atm} = E^{(res)} + E^{(unres)}$$

In this talk I will assume that there is no sub-grid-scale reservoir of energy and simply assume that the conserved energy is that of the resolved scale

In addition to thi … ere is an
even more comp … ales
necessary to res …

We must therefo … ut 50–100 km in operational climate models, and roughly 0.5–3 km for cutting edge convection-permitting global models

-> **Energy will always have both a resolved and an unresolved component**

Dry hydrostatic primitive equations

(feom=fluid equations of motion)

$$E_{feom} = K_h + I + \Phi$$

- K_h is horizontal kinetic energy
- In a shallow-atmosphere geometry, $\Phi = gz$ with g the constant acceleration of gravity.
- For an ideal perfect gas:

$$I = c_v^{(d)} T, \qquad \text{(dry air = ideal perfect gas)}$$

Dry hydrostatic primitive equations

Assuming constant pressure at model top the hydrostatic primitive equations of motion conserve:

(feom=fluid equations of motion)

$$\iiint \rho^{(d)} E_{feom} dV = \iiint \rho^{(d)} \Bigg(K_h + \underbrace{I + \frac{p^{(d)}}{\rho^{(d)}}}_{\text{specific enthalpy}} + \Phi_s \Bigg) dA\, dz$$

(Kasahara, 1974)

$$I + p^{(d)}/\rho^{(d)} = c_p^{(d)} T, \qquad \text{(ideal perfect gas)}$$

Dry hydrostatic primitive

Caution: Since its mass-weighted integral coincides with total energy, it is tempting to regard

as total energy per unit mass. **This is incorrect!** In the derivation it has been assumed that

- Pressure is constant at model lid
- Integration by parts used

$$\iiint \rho^{(d)} E_{feom} dV = \iiint \rho^{(d)} \Big(K_h + \underbrace{I + \frac{p^{(d)}}{\rho^{(d)}}}_{\text{specific enthalpy}} + \Phi_s \Big) dA\, dz,$$

(Kasahara, 1974)

$$I + p^{(d)}/\rho^{(d)} = c_p^{(d)} T, \qquad \text{(ideal perfect gas)}$$

TE for moist primitive equations

where K_h ... eal perfect gas)

Assuming ... gy
(pressure-

$$\mathcal{L}_{all} \equiv \{\text{`}d, \text{`}wv\text{`}, \text{`}cl\text{`}, \text{`}ci\text{`}, \text{`}rn\text{`}, \text{`}sw\text{`}\}$$

$$\mathcal{L}_{H_2O} \equiv \{\text{`}wv\text{`}, \text{`}cl\text{`}, \text{`}ci\text{`}, \text{`}rn\text{`}, \text{`}sn\text{`}\}$$

Assumptions:

1. All constituents have the same temperature (T)
2. All constituents move with the same barycentric velocity
3. Ideal perfect gas

The perfect gas law is often rewritten in the form

$$p = \rho R^{(d)} T_v,$$

with virtual temperature

$$T_v = T \left(\ldots \right), \quad m^{(\ell)} \equiv \frac{\rho^{(\ell)}}{\rho^{(d)}}$$

where $\epsilon \equiv \frac{R^{(d)}}{R^{(wv)}}$ is the ratio between the dry and water vapor gas constants

$\iiint$ ... z,

(Kas

... (ideal perfect gas)

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Specific Enthalpy of moist air

The specific enthalpy of an air constituent ℓ can be written on the form

$$h^{(\ell)} = h_{00}^{(\ell)} + c_p^{(\ell)}(T - T_{00}),$$

$$\rho I + p = \sum_{\ell \in \mathcal{L}_{all}} \rho^{(\ell)} \left[h_{00}^{(\ell)} + c_p^{(\ell)}(T - T_{00}) \right]$$

$$\mathcal{L}_{all} \, \{`d, `wv`, `cl`, `ci`, `rn`, `sw`\}$$

$$\rho I + p \;=\; \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right) + \rho^{(wv)} \left(h_{00}^{(wv)} + c_p^{(wv)} (T - T_{00}) \right) + \rho^{(liq)} \left(h_{00}^{(liq)} + c_p^{(liq)} (T - T_{00}) \right) + \rho^{(ice)} \left(h_{00}^{(ice)} + c_p^{(ice)} (T - T_{00}) \right),$$

$$c_p^{(ice)} = c_p^{(sn)} = c_p^{(ci)} \text{ and } c_p^{(liq)} = c_p^{(rn)} = c_p^{(cl)}$$

Specific Enthalpy of moist air

The specific enthalpy of an air constituent ℓ can be written on the form

$$h^{(\ell)} = h_{00}^{(\ell)} + c_p^{(\ell)}(T - T_{00}),$$

Only enthalpy differences are of physical relevance

-> rewrite equations using Kirchoff's equations for latent heat ...

$$\rho I + p = \sum_{\ell \in \mathcal{L}_{all}} \rho^{(\ell)} \left[h_{00}^{(\ell)} + c_p^{(\ell)}(T - T_{00}) \right]$$

$$\mathcal{L}_{all} \{ \text{‘}d, \text{‘}wv\text{‘}, \text{‘}cl\text{‘}, \text{‘}ci\text{‘}, \text{‘}rn\text{‘}, \text{‘}sw\text{‘} \}$$

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right) + \rho^{(wv)} \left(h_{00}^{(wv)} + c_p^{(wv)} (T - T_{00}) \right) + \rho^{(liq)} \left(h_{00}^{(liq)} + c_p^{(liq)} (T - T_{00}) \right) + \rho^{(ice)} \left(h_{00}^{(ice)} + c_p^{(ice)} (T - T_{00}) \right),$$

$$c_p^{(ice)} = c_p^{(sn)} = c_p^{(ci)} \text{ and } c_p^{(liq)} = c_p^{(rn)} = c_p^{(cl)}$$

Specific Enthalpy of moist air

The latent heat formulas for vaporization (liquid → water vapor):

$$L_v(T) = L_{v,00} + \left(c_p^{(wv)} - c_p^{(liq)}\right)(T - T_{00}), \text{ where } L_{v,00} \equiv h_{00}^{(wv)} - h_{00}^{(liq)} \tag{44}$$

The latent heat formulas for sublimation (solid → water vapor):

$$L_s(T) = L_{s,00} + \left(c_p^{(wv)} - c_p^{(ice)}\right)(T - T_{00}), \text{ where } L_{s,00} \equiv h_{00}^{(wv)} - h_{00}^{(ice)}, \tag{45}$$

The latent heat formulas for fusion (solid → liquid):

$$L_i(T) = L_{i,00} + \left(c_p^{(liq)} - c_p^{(ice)}\right)(T - T_{00}), \text{ where } L_{i,00} \equiv h_{00}^{(liq)} - h_{00}^{(ice)}, \tag{46}$$

(Emanuel, 1994, see, e.g., p. 114-5). Note that the latent heat of fusion, $L_i(T)$ may also be written in terms of latent heat of vaporization and sublimation

$$L_i(T) = L_s(T) - L_v(T). \tag{47}$$

Specific enthalpy of moist air: Reference state: 'wv', 'liq', 'ice'

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right) + \rho^{(H_2O)} \left(h_{00}^{(wv)} + c_p^{(wv)} (T - T_{00}) \right) - \rho^{(liq)} L_v(T) - \rho^{(ice)} L_s(T).$$

(water vapor reference state)

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right) + \rho^{(H_2O)} \left(h_{00}^{(liq)} + c_p^{(liq)} (T - T_{00}) \right) + \rho^{(wv)} L_v(T) - \rho^{(ice)} L_i(T)$$

(liquid reference state)

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right) + \rho^{(H_2O)} \left(h_{00}^{(ice)} + c_p^{(ice)} (T - T_{00}) \right) + \rho^{(wv)} L_s(T) + \rho^{(liq)} L_i(T),$$

(ice reference state)

Specific enthalpy of moist air:
Reference state: ‘wv’, ’liq’, ’ice’

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right) + \rho^{(H_2O)} \left(h_{00}^{(wv)} + c_p^{(wv)} (T - T_{00}) \right) - \rho^{(liq)} L_v(T) - \rho^{(ice)} L_s(T).$$

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(liquid reference state)

$$\rho I + p = \rho^{(d)} c_p^{(d)} T + \rho^{(d)} \left(h_{00}^{(d)} - c_p^{(d)} T_{00} \right) + \rho^{(H_2O)} \left(h_{00}^{(ice)} + c_p^{(ice)} (T - T_{00}) \right) + \rho^{(wv)} L_s(T) + \rho^{(liq)} L_i(T),$$

(ice reference state)

When taking the global integral and time-derivative of enthalpy, crossed out terms are constant if dry air mass is constant!

Total energy equation

symbol	description	unit
$c_p^{(\ell)}$	heat capacity at constant pressure of species ℓ	$J/K/kg$
$F_{net}^{(\ell)}$	net flux of water species ℓ into the atmosphere	$kg/m^2/s$
$F_{net}^{(turb,rad)}$	Radiative and sensible/turbulent fluxes into atmosphere (90)	$J/m^2/s$
$h_{00}^{(\ell)}$	reference enthalpy for water form ℓ	J/kg
$m^{(\ell)}$	dry mixing ratio ($\equiv \rho^{(\ell)}/\rho^{(d)}$)	kg/kg
K	specific horizontal kinetic energy ($\equiv \frac{1}{2}\vec{v}^2$)	m^2/s^2
$L_{f,00}$.	latent heat of fusion	J/K
$L_{s,00}$.	latent heat of sublimation	J/K
$L_{v,00}$.	latent heat of vaportization	J/K
Φ_s	surface geopotential	m^2/s^2
ρ_d	dry air density	kg/m^3
T	temperature	K
$\tilde{T}_s$	common temperature at surface	K
$\vec{v}$	horizontal velocity vector	m/s

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\Bigg\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+c_p^{(\ell)}\left(T-T_{00}\right)+h_{00}^{(ice)}\right]$$
$$+\,m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\Bigg\}dA\,dz$$
$$=\iint\Bigg\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\tilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\tilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\Bigg\}dA.$$

(ice reference enthalpy, $\tilde{T}_s \equiv T_{atm,s} = T_{surf,s}$)

Ambiguous how to specify temperature of falling precipitation (more obvious with evaporation)!

Total energy equation

symbol	description	unit
$c_p^{(\ell)}$	heat capacity at constant pressure of species ℓ	$J/K/kg$
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T	temperature	K
$\widetilde{T}_s$	common temperature at surface	K
$\vec{v}$	horizontal velocity vector	m/s

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+c_p^{(\ell)}\left(T-T_{00}\right)+h_{00}^{(ice)}\right]\right.$$

$$\left.+\,m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\right\}dA\,dz$$

$$=\iint\left\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\right\}dA.$$

$$\text{(ice reference enthalpy, } \widetilde{T}_s\equiv T_{atm,s}=T_{surf,s})$$

The radiative fluxes represent a bulk source/sink of atmospheric energy, not a surface (or ToA) boundary term; only put here for notational simplicity

Approximated TE equation

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+c_p^{(\ell)}\left(T-T_{00}\right)+h_{00}^{(ice)}\right]\right.$$

$$\left.+\,m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\right\}dA\,dz$$

$\mathcal{L}_{H_2O} =' wv'$

$c_p^{(\ell)} = c_p^{(d)}$

$$=\iint\left\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\right\}dA.$$

Many models make these assumptions:

$$\text{(ice reference enthalpy, } \widetilde{T}_s \equiv T_{atm,s} = T_{surf,s}\text{)}$$

In CAM physics this equation is satisfied/(enforced) in each physics column when pressure is held constant!

Approxima

$$\int \left[\rho^{(d)} \left(\sum_{\ell \in \mathcal{L}_{cond}} \overline{m}_{tn}^{(\ell)} \right) \right] \frac{\partial}{\partial t} \left(\overline{K} + \overline{\Phi}_s + c_p^{(d)} \overline{T} \right) dz$$

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ K + \Phi_s + c_p^{(d)} T + \sum_{\ell \in \mathcal{L}_{H_2O}} m^{(}$$

$$\mathcal{L}_{H_2O} =' wv'$$

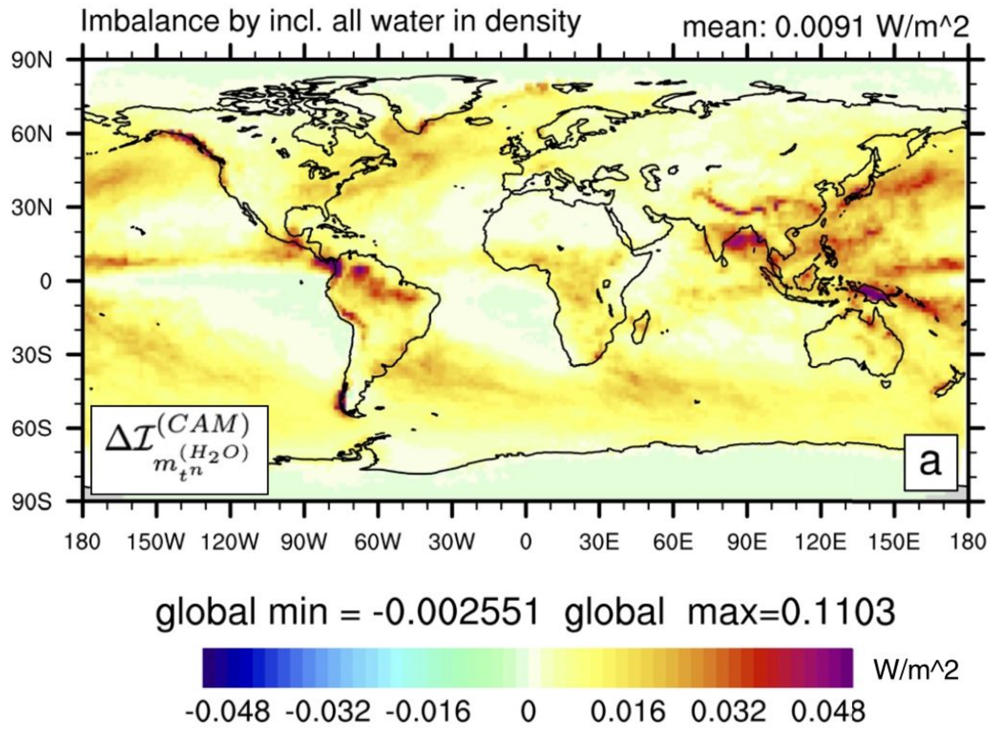
$$+ m^{(wv)} L_{s,00}$$

$$= \iint \Big(\quad F_{net}^{(\ell)} \left[\widetilde{K}_s + \Phi_s + c_p^{(\ell)} \left(\widetilde{T}_s - \right. \right.$$

$A.$

Many models make these assumptions:

(ice reference enthal



is

d)

eld

constant?

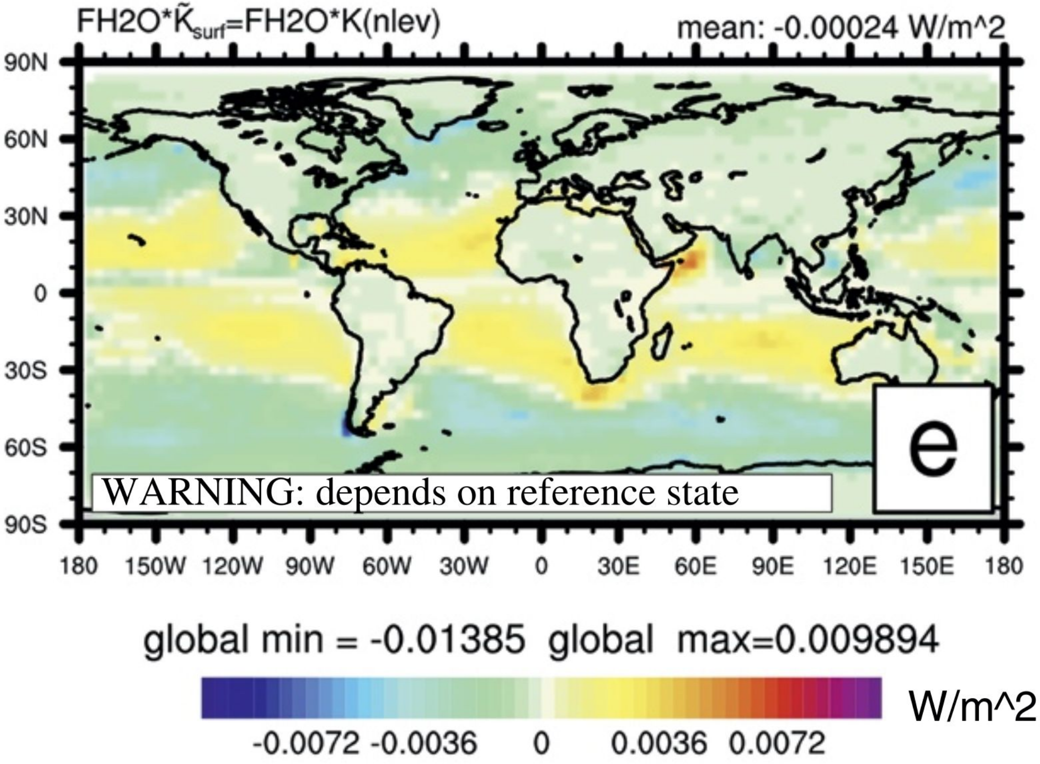
Approximated TE equation

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left\{K + \ldots c_p^{(d)}T + \sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)} \ldots + m^{(wv)}L_{s,00} + \ldots\right.$$

$\mathcal{L}_{H_2O} =' wv'$

$$= \iint \left\{ \sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\tilde{K}_s + \Phi_s + c_p^{(\ell)}\left(\tilde{T}_s - T_0 \ldots\right.\right.\right.$$

At very high vertical resolution and using a no-slip boundary condition the winds should be zero at the surface making this terms zero



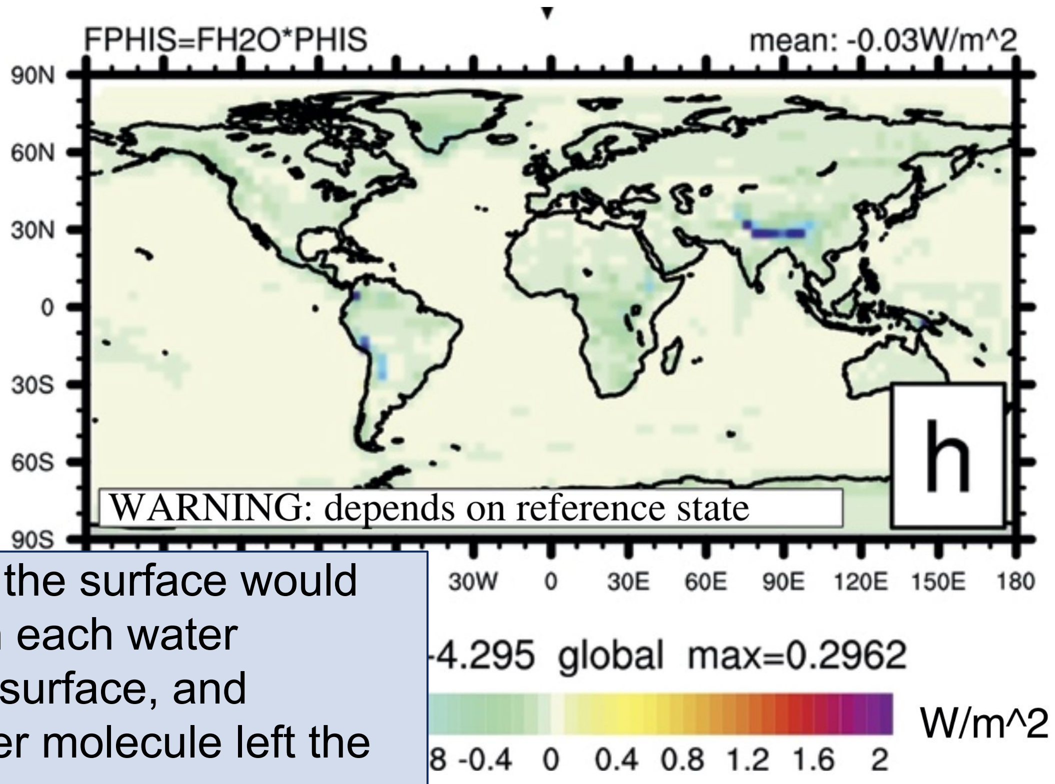
Lauritzen et al. (2022)

Approximated TE equation

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left\{K+\Phi_s+ \quad + \sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)} \quad m^{(wv)} L_{s,00} + \right.$$

$\mathcal{L}_{H_2O} =' wv'$

$$= \iint \left\{ \sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s - T_0\right.\right.\right.$$



Ideally the potential energy flux through the surface would require one to track the altitude at which each water molecule in the air evaporated from the surface, and subtract the geopotential when that water molecule left the atmosphere as precipitation.

Approximated TE equation

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left\{K + \Phi_s + c_p^{(d)}T + \sum_{\ell \in \mathcal{L}_{H_2O}} m^{(\ell)}\left[K + \right.\right.$$

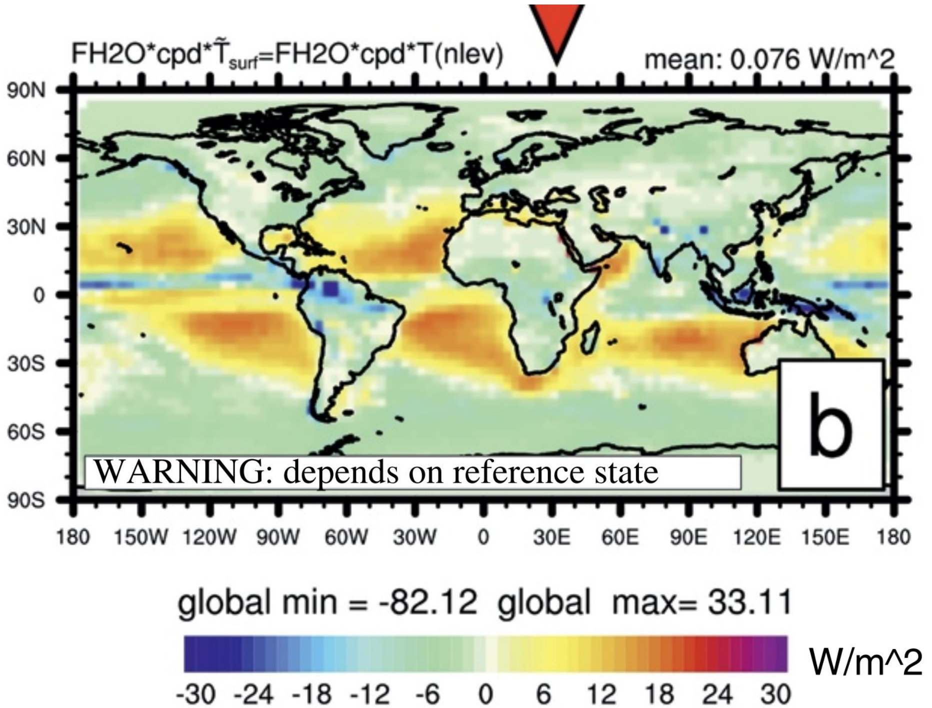
$\mathcal{L}_{H_2O} =' wv'$

$$+ m^{(wv)}L_{s,00} + m^{(liq}$$

$$= \iint \left\{ \sum_{\ell \in \mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\tilde{K}_s + \Phi_s + c_p^{(\ell)}\left(\tilde{T}_s - T_{00}\right) + \right.\right.$$

Many models make these assumptions:

(ice reference e[...]py, $\tilde{T}_s$ =



Approximated TE equation

$$\frac{\partial}{\partial t}\iiint \rho^{(d)}\left\{K+\Phi_s+c_p^{(d)}T+\sum_{\ell\in\mathcal{L}_{H_2O}} m^{(\ell)}\left[K+\Phi_s+c_p^{(\ell)}\left(T-T_{00}\right)+h_{00}^{(ice)}\right]\right.$$

$$\left.+\,m^{(wv)}L_{s,00}+m^{(liq)}L_{f,00}\right\}dA\,dz$$

$\mathcal{L}_{H_2O} =' wv'$

$c_p^{(\ell)} = c_p^{(d)}$

$$=\iint\left\{\sum_{\ell\in\mathcal{L}_{H_2O}} F_{net}^{(\ell)}\left[\widetilde{K}_s+\Phi_s+c_p^{(\ell)}\left(\widetilde{T}_s-T_{00}\right)+h_{00}^{(ice)}\right]+F_{net}^{(wv)}L_{s,00}+F_{net}^{(liq)}L_{f,00}+F_{net}^{(turb,rad)}\right\}dA.$$

Many models make these assumptions:

(ice reference enthalpy, $\widetilde{T}_s \equiv T_{atm,s}$

At end of CAM physics pressure is updated to reflect precipitation/evaporation; TE tendency is restored with global energy fixer …

Energy tendency associated with updating pressure matches neglected boundary flux terms well (at least locally)

$$\frac{\partial}{\partial t}\int \overline{\rho}^{(d)}\left\{\left(1+\overline{m}^{(H_2O)}\right)\left[\overline{K}+\overline{\Phi}_s+c_p^{(d)}\left(\overline{T}-T_{00}\right)\right]+\overline{m}^{(wv)}L_{s,00}+\overline{m}^{(liq)}L_{f,00}\right\}dz$$

$$-\Delta\widehat{\mathcal{I}}_{\partial m^{(H_2O)}/\partial t}-\Delta\mathcal{I}_{m_{tn}^{(H_2O)}} = \overline{F}_{net}^{(H_2O)}\left[c_p^{(d)}\left(\widetilde{\overline{T}}_s-T_{00}\right)+\widetilde{\overline{K}}_s+\overline{\Phi}_s\right]+\overline{F}_{net}^{(wv)}L_{s,00}+\overline{F}_{net}^{(liq)}L_{f,00}+\overline{F}_{net}^{(turb,rad)}$$

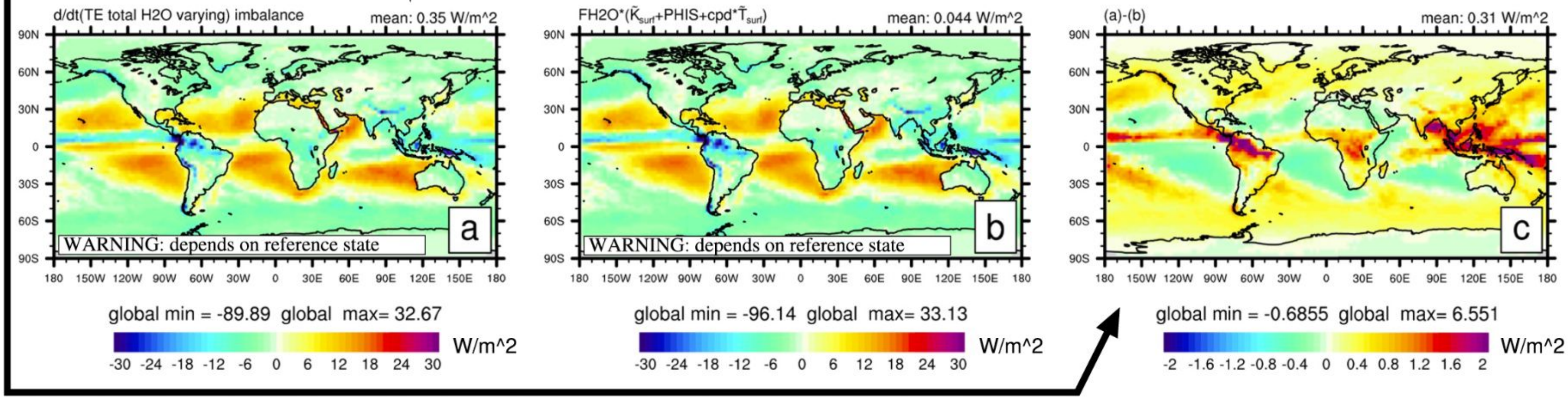
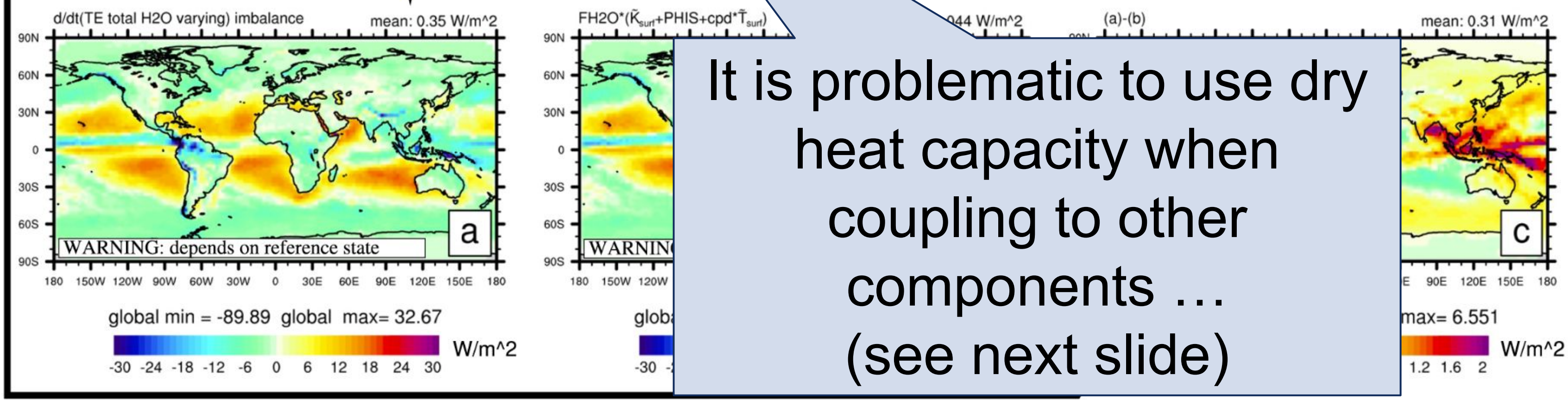


Figure 6. Modified (consistent) CAM total energy equation terms in W/m^2: (a) Imbalance introduced by "dry-mass adjustment" using all forms of water in the kinetic, geopotential and enthalpy terms, (b) missing flux terms, and (c) is the difference between (a and b). Note that the imbalance is locally much reduced when using the modified total energy equation. Also, the imbalance does not depend on the reference state (as should always be the case).

Energy tendency associated with updating pressure matches neglected boundary flux terms well (at least locally)

$$\frac{\partial}{\partial t}\int \overline{\rho}^{(d)}\left\{\left(1+\overline{m}^{(H_2O)}\right)\left[\overline{K}+\overline{\Phi}_s+c_p^{(d)}\left(\overline{T}-T_{00}\right)\right]+\overline{m}^{(wv)}L_{s,00}+\overline{m}^{(liq)}L_{f,00}\right\}dz$$

$$-\Delta\widehat{\mathcal{I}}_{\partial m^{(H_2O)}/\partial t}-\Delta\mathcal{I}_{m_{tn}^{(H_2O)}} = \overline{F}_{net}^{(H_2O)}\left[c_p^{(d)}\left(\widetilde{T}-T_{00}\right)+\widetilde{K}_s+\overline{\Phi}_s\right]+\overline{F}_{net}^{(wv)}L_{s,00}+\overline{F}_{net}^{(liq)}L_{f,00}+\overline{F}_{net}^{(turb,rad)}$$



It is problematic to use dry heat capacity when coupling to other components …
(see next slide)

Figure 6. Modified (consistent) CAM total energy equation terms in W/m²: (a) Imbalance introduced by "dry-mass adjustment" using all forms of water in the kinetic, geopotential and enthalpy terms, (b) missing flux terms, and (c) is the difference between (a and b). Note that the imbalance is locally much reduced when using the modified total energy equation. Also, the imbalance does not depend on the reference state (as should always be the case).

Enthalpy flux terms and coupling with MOM6 (= CESM3 ocean model)

Ocean (liquid reference state + constant latent heats)

$$F^{(h)}_{net} \approx F^{(H_2O)}_{net} \left[c^{(liq)}_p \left(\tilde{T}_s - T_{00}\right) + h^{(liq)}_{00}\right] + F^{(wv)}_{net} L_v \left(\tilde{T}_s\right) - F^{(ice)}_{net} L_f \left(\tilde{T}_s\right)$$

Atmosphere (ice reference state + dry heat capacity + constant latent heat)

$$F^{(h)}_{net} \approx \overline{F}^{(H_2O)}_{net} \left[c^{(d)}_p \left(\tilde{\overline{T}}_s - T_{00}\right) + \overline{F}^{(wv)}_{net} L_{s,00} + \overline{F}^{(liq)}_{net} L_{f,00}\right.$$

Inconsistent … I don't see how this can be made consistent!

https://agupubs.onlinelibrary.wiley.com/doi/epdf/10.1029/2022MS003117

Current CESM3: MOM6 passes its enthalpy flux to atmosphere through global fixer in the coupler and atmosphere fixes its enthalpy flux using global energy fixer.

Loosely speaking: each components does it’s own thing and fixes its own thing independently of each other ...

$$F_{net}^{(h)} \approx F_{net}^{(H_2O)} \left[c_p^{(liq)} \left(\tilde{T}_s - T_{00}\right) + h_{00}^{(liq)}\right] + F_{net}^{(wv)} L_v \left(\tilde{T}_s\right) - F_{net}^{(ice)} L_f \left(\tilde{T}_s\right)$$

Atmosphere (ice reference state + dry heat capacity + constant latent heat)

$$F_{net}^{(h)} \approx \overline{F}_{net}^{(H_2O)} \left[c_p^{(d)} \left(\tilde{\overline{T}}_s - T_{00}\right) + \overline{F}_{net}^{(wv)} L_{s,00} + \overline{F}_{net}^{(liq)} L_{f,00}\right.$$

Inconsistent … I don’t see how this can be made consistent!

https://agupubs.onlinelibrary.wiley.com/doi/epdf/10.1029/2022MS003117

Solution (easier said than done though!):

Both components use variable latent heats (and be very careful with different reference states)

$$\approx \sum_{\ell \in \mathcal{L}_{H_2O}} F_{net}^{(\ell)} \left[c_p^{(\ell)} \left(\tilde{T}_s - T_{00}\right) + h_{00}^{(liq)}\right] + F_{net}^{(wv)} L_{v,00} - F_{net}^{(ice)} L_{f,00}$$

MOM6

$$\left(\text{liquid reference state}, \tilde{T}_s \equiv T_{atm,s} = T_{surf,s}\right)$$

$$\approx \sum_{\ell \in \mathcal{L}_{H_2O}} F_{net}^{(\ell)} \left[c_p^{(\ell)} \left(\tilde{T}_s - T_{00}\right) + h_{00}^{(ice)}\right] + F_{net}^{(wv)} L_{s,00} + F_{net}^{(liq)} L_{f,00},$$

CAM7

$$\left(\text{ice reference state}, \tilde{T}_s \equiv T_{atm,s} = T_{surf,s}\right)$$

https://agupubs.onlinelibrary.wiley.com/doi/epdf/10.1029/2022MS003117

Changing to variable latent heats is difficult in the atmosphere

Heating from phase changes must be updated to be T dependent (variable latent heats)… which can not be done after the fact in CAM!

Also vertical mixing under variable latent heats is not straightforward!

Temporal/spatial coupling challenges: in CESM3 we have chosen to compute enthalpy fluxes in the atmosphere and pass to other components so that all components see the same enthalpy flux (converted to the relevant reference state)

I'd be happy to discuss my interim solution with anyone who is interested and get feedback!

https://agupubs.onlinelibrary.wiley.com/doi/epdf/10.1029/2022MS003117

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Dynamical core total energy dissipation

1-year averaged total energy tendencies from spectral-element dynamical core (horizontal resolution ~100km)

```
  dE/dt dycore                                       -0.1850 W/M^2

Adiabatic dynamics can be divided into quasi-horizontal and vertical remapping:


  dE/dt floating dynamics             (dAD-dBD)   -0.1285 W/M^2
  dE/dt vertical remapping            (dAR-dAD)   -0.0565 W/M^2

Breakdown of floating dynamics:

  dE/dt hypervis del4                 (dCH-dBH)   -0.6720 W/M^2
  dE/dt hypervis frictional heating (dAH-dCH)      0.6211 W/M^2
  dE/dt hypervis del4 total           (dAH-dBH)   -0.0509 W/M^2
  dE/dt hypervis sponge del2          (dAS-dBS)   -0.0193 W/M^2
  dE/dt explicit diffusion total                  -0.0702 W/M^2

  dE/dt residual (time-truncation errors,...)    -0.0584 W/M^2
```

"dE/dt dycore" for FV3 and MPAS (when run in CESM) is ~1W/m^2

Dynamical core total energy dissipation

1-year averaged total energy tendencies from spectral-e

```
  dE/dt dycore                                 -0.1850 W/M^2

Adiabatic dynamics can be divided into quasi-horizontal and verti


  dE/dt floating dynamics             (dAD-dBD)   -0.1285 W/M^2
  dE/dt vertical remapping            (dAR-dAD)   -0.0565 W/M^2

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  dE/dt explicit diffusion total                  -0.0702 W/M^2

  dE/dt residual (time-truncation errors,...)     -0.0584 W/M^2
```
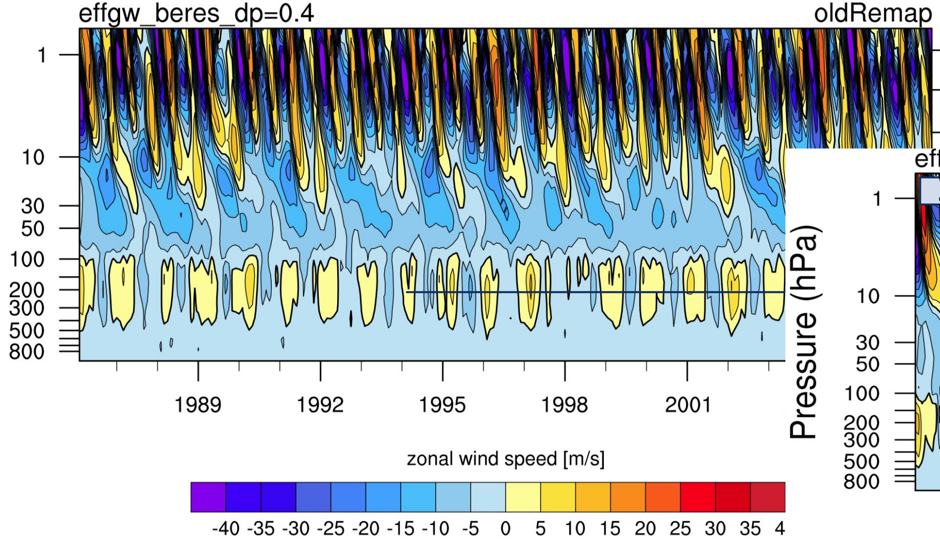
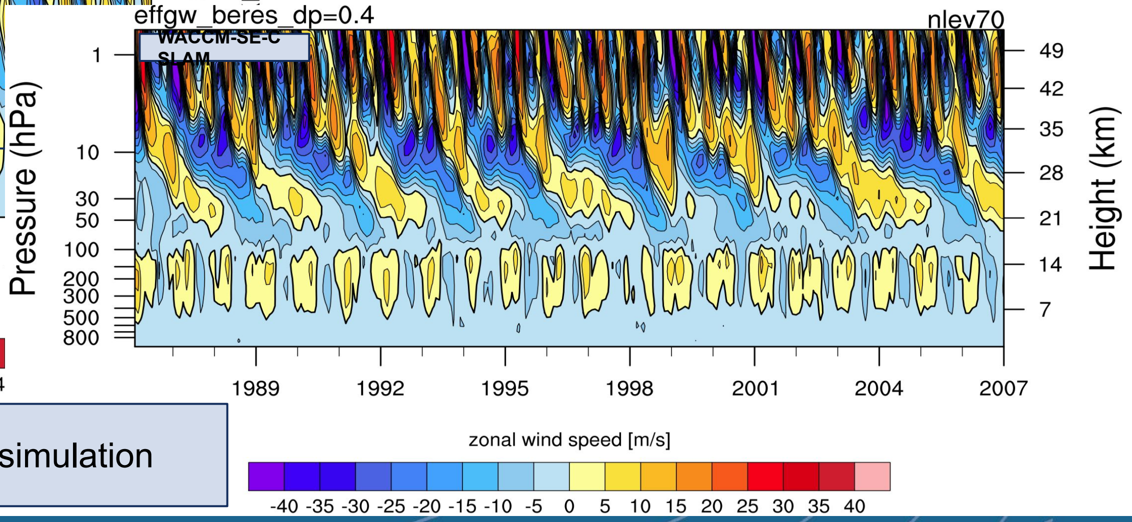

Switching vertical remapping method to 3rd order splines (algorithm taken from FV3) from piecewise parabolic method (PPM) reduced dE/dt by approximately 2x (used to be ~-0.12W/m2)

Lauritzen and Williamson (2019)

Aside: QBO and vertical remapping algorithm

Time–height plot of monthly-mean, zonal-mean equatorial zonal wind: (left) PPM (right) splines





Still need higher vertical resolution for good QBO simulation

Dynamical core total energy dissipation

1-year averaged total energy tendencies from spectral-e

```
  dE/dt dycore                                    -0.1850 W/M^2

Adiabatic dynamics can be divided into quasi-horizontal and verti


  dE/dt floating dynamics             (dAD-dBD)   -0.1285 W/M^2
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  dE/dt explicit diffusion total                  -0.0702 W/M^2

  dE/dt residual (time-truncation errors,...)     -0.0584 W/M^2
```

We use “frictional heating”, i.e. kinetic energy change resulting from hyperviscosity operators added locally back as heating

This is **not** supported by theory … (next slide)

Lauritzen and Williamson (2019)

Explicit diffusion operators and energy

Consider artificial Laplacian diffusion of momentum added to the momentum equations

$$\frac{\partial \vec{v}}{\partial t} = \cdots + \nu_2 \nabla_h^2 \vec{v},$$

The associated kinetic energy equation is

$$\frac{\partial K}{\partial t} = \vec{v} \cdot \frac{\partial \vec{v}}{\partial t} = \cdots + \underbrace{\nu_2 \nabla_h^2 (K)}_{\text{Diffusion of } K} - \underbrace{\nu_2 \left(\nabla_h \vec{v}\right)^2}_{\text{Dissipation of } K},$$

Redistributes ***K*** (hence the global integral of that term is zero)

Dissipation of ***K*** (always negative); for closed energy budget must be added as heating (Becker, 2003)

Lauritzen et al. (2022)

Explicit diffusion operators and energy

Consider a[…] equations

For higher-order operators (e.g., ∇_h^4) it is less obvious how to assign a physical meaning to the terms and separate them into diffusive and dissipative parts …

The associ[…]

$$\frac{\partial K}{\partial t} = \vec{v} \cdot \frac{\partial \vec{v}}{\partial t} = \cdots + \underbrace{\nu_2 \nabla_h^2 (K)}_{\text{Diffusion of } K} - \underbrace{\nu_2 \left(\nabla_h \vec{v}\right)^2}_{\text{Dissipation of } K},$$

Redistributes ***K*** (hence the global integral of that term is zero)

Dissipation of ***K*** (always negative); for closed energy budget must be added as heating (Becker, 2003) "naive" closure of the energy budget by transferring kinetic energy change into heat is, in general, not physically correct (although done in CAM)

$$\left(\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} c_p^{(\ell)}\right) \frac{\partial T}{\partial t} \neq -\left(\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)}\right) \frac{\partial K}{\partial t}$$

$$\frac{\partial K}{\partial t} = \vec{v} \cdot \frac{\partial \vec{v}}{\partial t} = \cdots + \underbrace{\nu_2 \nabla_h^2 (K)}_{\text{Diffusion of } K} - \underbrace{\nu_2 \left(\nabla_h \vec{v}\right)^2}_{\text{Dissipation of } K},$$

Redistributes ***K*** (hence the global integral of that term is zero)

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Lauritzen et al. (2022)

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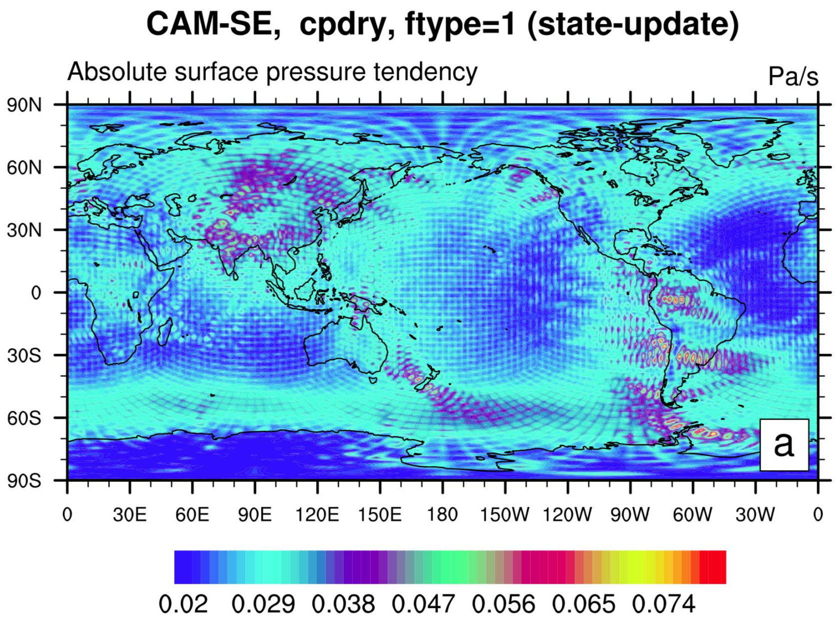
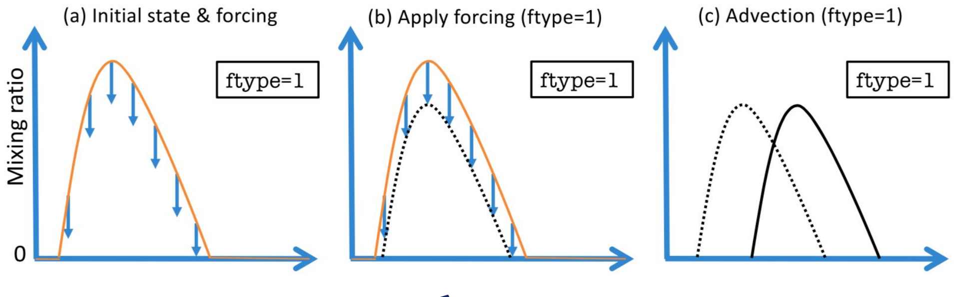
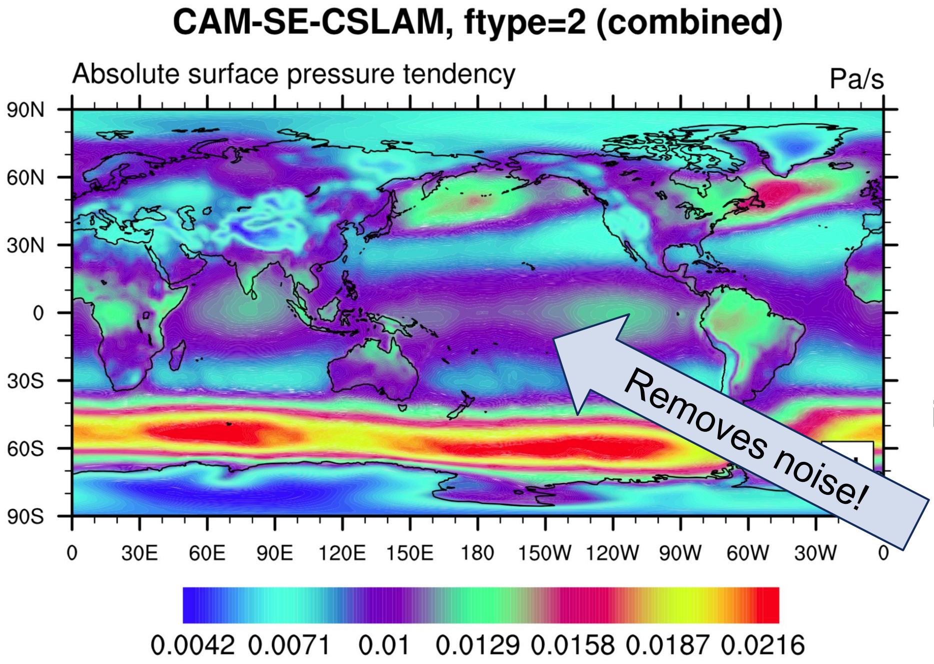
Physics-dynamics coupling





Note: no spurious temporal physics-dynamics errors but noise in simulations …
(see left Figure)

Physics-dynamics coupling: temporal errors



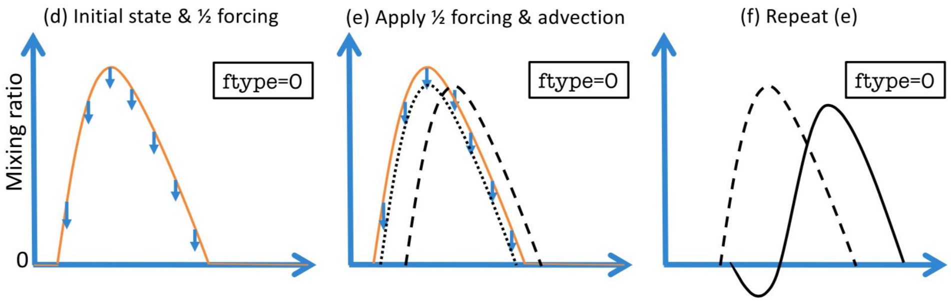


Figure illustrates “dribbling” tendencies throughout dycore integration (not necessarily mass-conservative)

Hybrid approach (ftype=2):

- state-update tracers (inherently mass-conservative)
- “dribble” u,v, T tendencies

dE/dt =~ 0.05W/m2; if we “dribble” mass-weighted tendencies then dE/dt = ~ 0.02W/m2

Lauritzen and Williamson (2019)

Separate physics, transport and dynamics grid

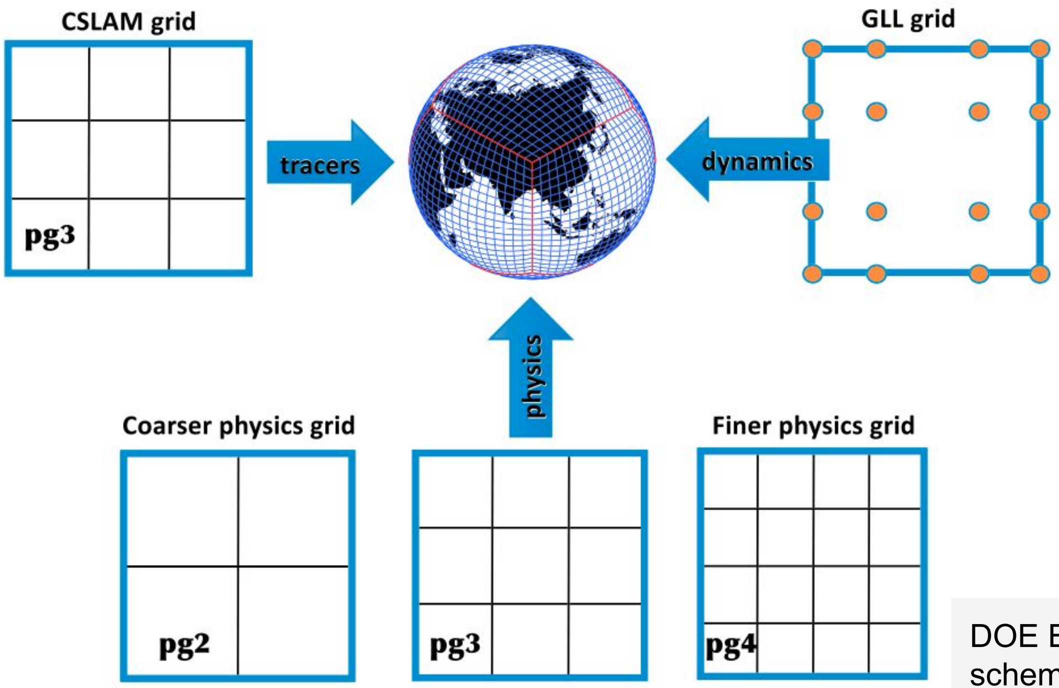


Figure 1. An overview of the different grids in CAM-SE-CSLAM.

For CESM3 we use pg3 grid for CAM physics!

Separating grids is not trivial - mapping between grids must be done carefully!
https://agupubs.onlinelibrary.wiley.com/doi/full/10.1029/2019ms001684

Transport scheme:
Conservative Semi-LAgrangian Multi-tracer scheme
(consistent coupling with spectral-elements dycore described here https://journals.ametsoc.org/view/journals/mwre/145/3/mwr-d-16-0258.1.xml)

Note: Dry-mass vertical coordinate makes CSLAM-SE dycore coupling more consistent!

DOE E3SM is using similar approach (but transport scheme faster and supports variable resolution grids)

UK Met Office is exploring separation of grids as well

Physics-dynamics coupling: mapping errors

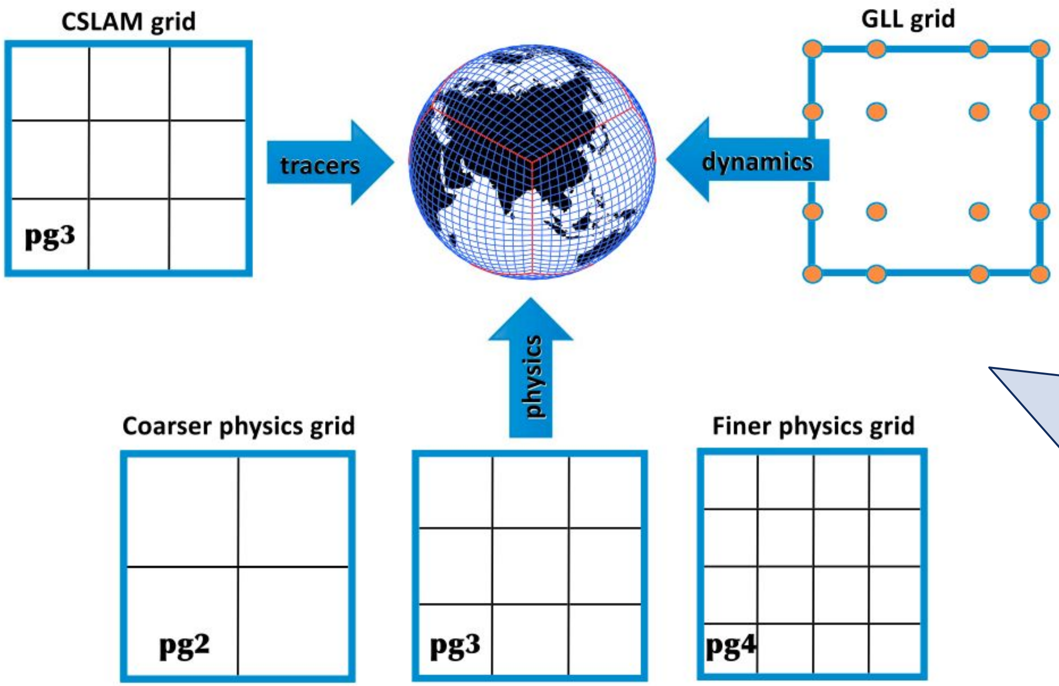


Figure 1. An overview of the different grids in CAM-SE-CSLAM.

Physics tendencies for u,v and T must be mapped to GLL grid:

dE/dt = .0025 ~ W/m2

(using 3rd-order Lagrange interpolation)

Physics-dynamics coupling: energy formula discrepancy errors

(a) Constant volume versus constant pressure

Constant volume and constant pressure models/components conserve different energies:

MPAS

$$\frac{\partial}{\partial t}\iiint\left[K + c_v^{(d)}T + \Phi\right]\rho^{(d)}\,dA\,dz = 0,\ z_t \text{ constant}, \quad (35)$$

$$\frac{\partial}{\partial t}\iiint\left[K + c_v^{(d)}T + \Phi\right]\rho^{(d)}\,dA\,dz + \underbrace{\frac{1}{g}\frac{\partial}{\partial t}\iint p_t\Phi_t\,dA}_{\text{work done by } p \text{ at top}} = 0,\ p_t \text{ constant} \quad (36)$$

SE, FV3

(dry atmosphere)

Note: only difference between hydrostatic and non-hydrostatic energy is K (2D or 3D)!

Physics-dynamics coupling: energy formula discrepancy errors

(b) Thermodynamic active water species discrepancy: (i) mass and/or (ii) enthalpy

Dynamical cores used for high resolution include condensate loading, however, most physics packages (I know of) do not:

$$\frac{\partial}{\partial t}\iiint \rho^{(d)} E_{feom}\, dA\, dz = \frac{\partial}{\partial t}\iiint \rho^{(d)} \left[\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} \left(K + \Phi_s + c_p^{(d)} T\right)\right] dA\, dz$$

FV3 and SE use generalized cp

$$\mathcal{L}_{all} \equiv \{d, wv, cl, ci, rn, sw, gr\}$$

Dynamical core with condensate loading!

$$\sum_{\ell \in \{'d', 'wv'\}}$$

Physics package

National Center for Atmospheric Research is a major facility sponsored by the NSF under Cooperative Agreement No. 1852977

Physics-dynamics coupling: energy formula discrepancy errors

How do we enforce energetic consistency in CAM?

- Change **global energy fixer** to use dynamical core total energy formula

  This is implemented using a dycore specific variable that is passed to energy subroutine:

$$
\texttt{cp\_or\_cv\_dycore} = \begin{cases} c_p^{(d)} & , \text{ FV} \\ \frac{\sum_{\ell \in \mathcal{L}_{all}} m^{(\ell)} c_p^{(\ell)}}{\sum_{\mathcal{L}_{all}} m^{(\ell)}} & , \text{ SE} \\ \frac{R^*}{R^{(d)}} c_v^{(d)} & , \text{ MPAS} \end{cases}
$$

- Before dT/dt from physics is passed to the dycore: scaled for energetic consistency:

  dT/dt -> (cp(d)/cp_or_cv_dycore)* dT/dt

  Equivalent to adding heating under the assumptions of the dycore!

See also Eldred et al., (2022) https://doi.org/10.1002/qj.4353

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Thermodynamic conserved variable inconsistency leading to total energy errors

An example: Coupling CLUBB with CAM (problem identified by Chris Golaz in 2010)

CAM's conserved variable (only terms relevant to CLUBB retained and excl. kinetic energy and surface fluxes)

Host model

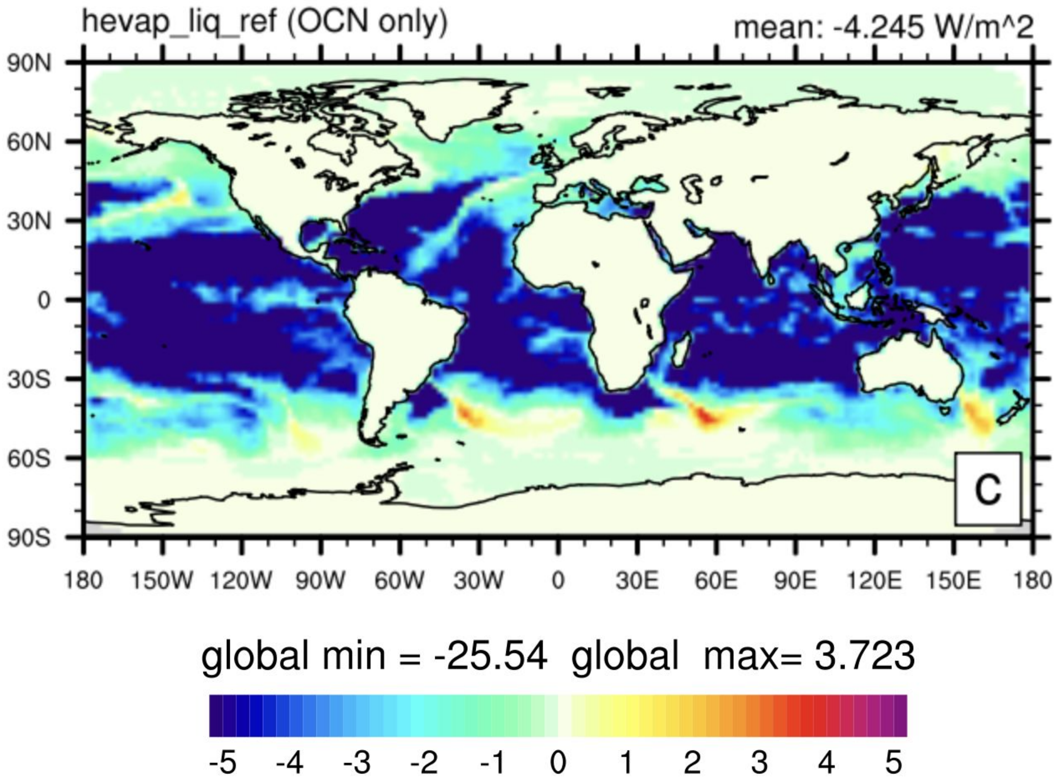
$$\int \left(c_p^{(d)} \Delta \overline{T} - L_{v,00} \Delta \overline{m}^{(liq)}\right) \overline{\rho}_{t^n}^{(d)} \left(1 + \overline{m}_{t^n}^{(wv)}\right) dz = 0. \quad (158)$$

Parameterization

$$\int \frac{1}{\overline{\Pi}_{t^n}} \left(c_p^{(d)} \Delta \overline{T} - L_{v,00} \Delta \overline{m}^{(liq)}\right) \overline{\rho}_{t^n}^{(d)} \left(1 + \overline{m}_{t^n}^{(wv)}\right) dz = 0. \quad (155)$$

CLUBB's conserved variable

1-year column averaged imbalance in CAM (CESM)



To make CAM physics with CLUBB pass the energy budget checks in CAM, the implementers chose to add a temperature increment in each column to compensate for thermodynamic/energy inconsistency!

(similarly for kinetic energy)

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Recommendations for future directions and priorities

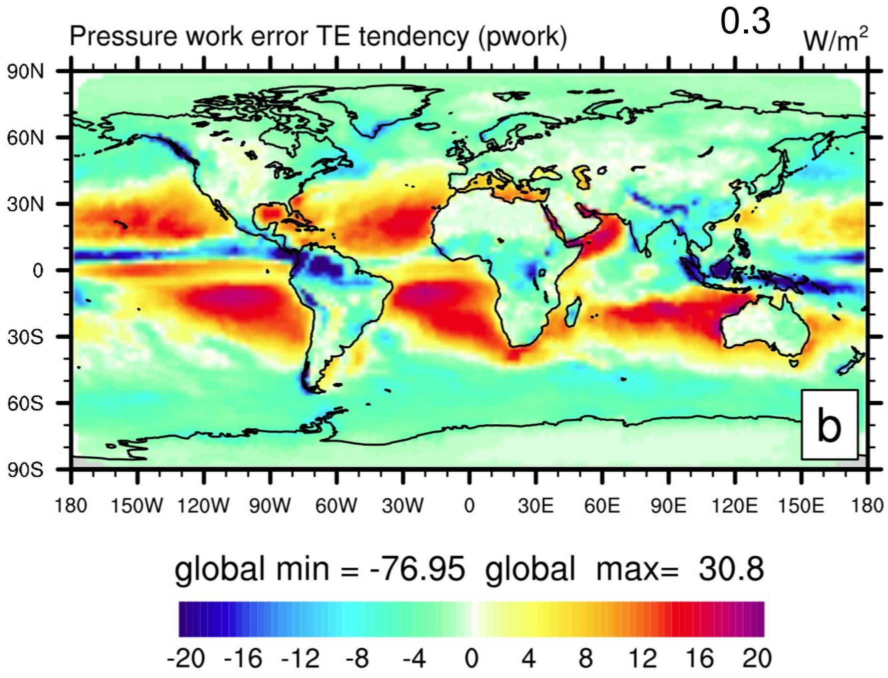
Inclusion of Neglected Physical Processes
Incorporating processes such as frictional heating caused by falling precipitation and surface heating/cooling from precipitation.

Consistent Thermodynamic Treatment
Using more self-consistent thermodynamic methods (thermodynamic potentials).

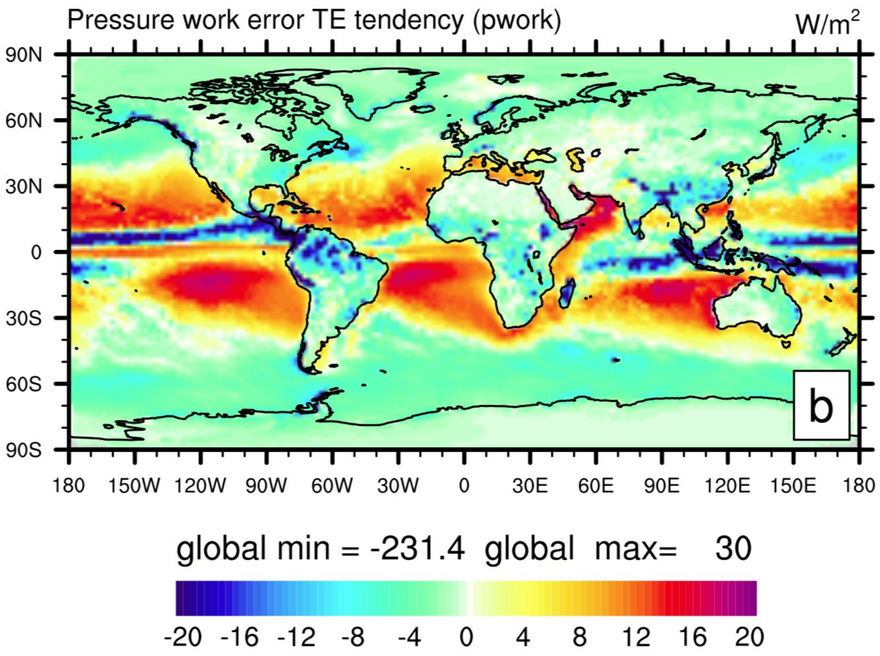
Energy-Conserving Numerical Methods
Employing/deriving numerical methods that inherently conserve energy and/or careful accounting of kinetic energy loss by the dynamical core

Increasing horizontal resolution does not reduce TE imbalance!

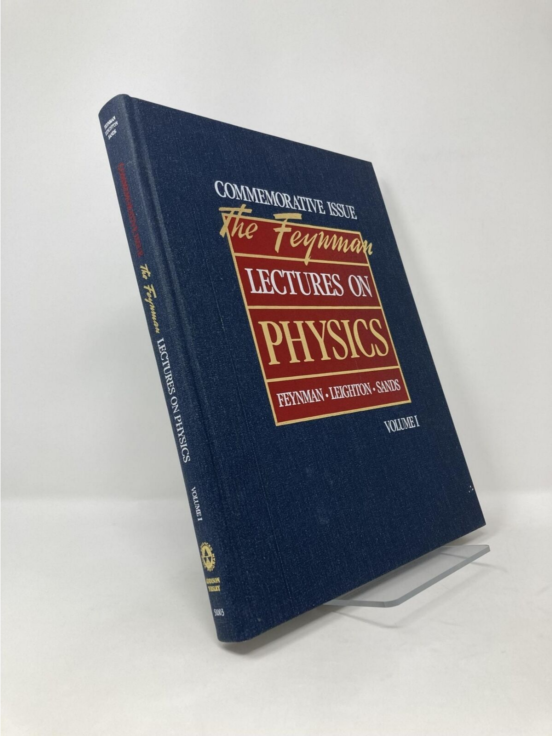
1 degree CAM6, CESM2.2



¼ degree CAM6, CESM2.2



Plots show one year averages using cam6_2_017 using CAM-SE-CSLAM; ¼ simulation courtesy of Adam Herrington



Richard Feynmann remarked that (Feynman et al., 1989),

> The subject of thermodynamics is complicated because there are so many different ways of describing the same thing …with respect to the internal energy $U^{(all)}$, we might say that it depends on the temperature and volume, if those are the variables we have chosen - but we might also say that it depends on the temperature and pressure, or the pressure and volume, and so on.

Dynamical core total energy dissipation

Dynamical cores typically employ numerical filters (either implicitly or explicitly) to control waves at or near the grid-scale:

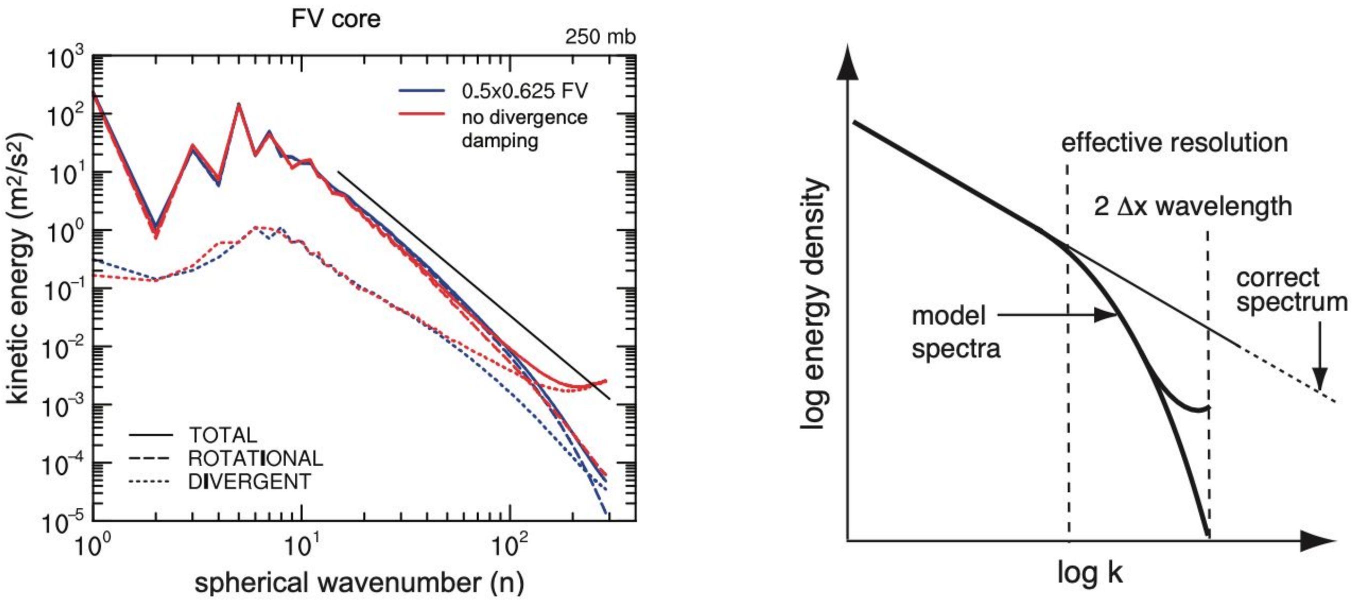


Figure: (left) Solid black line shows k^{-3} slope (courtesy of D.L. Williamson). (right) Schematic of 'effective resolution' (Figure from Skamarock (2011)).