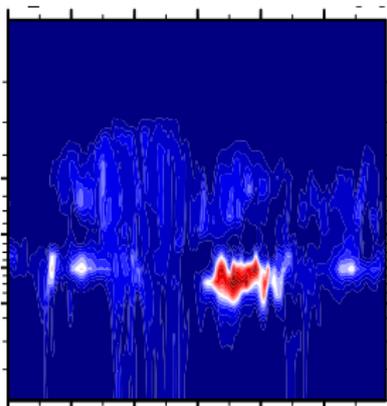


# Resolved and sub-grid-scale transport in CAM5-FV

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NCAR Earth System Laboratory (NESL)  
National Center for Atmospheric Research (NCAR)



ACD group meeting

# Motivation

Sometime in May Steven Massie and William Randel came into my visitor office in ACD and asked me: 'How do tracers get moved around in CAM?'

This Monday Laura Pan said: 'I have a lot of questions regarding how convection is represented in models like CAM/WACCM?'



The question I am going to address:

If you add a tracer to CAM-FV (with CAM5 physics), how is it 'moved around' both grid-scale and sub-grid-scale?

# CAM5 process 'flow chart'

## Physical processes on tracers

- 'Resolved' scale transport (Lin and Rood, 1996)
- Deep convective transport (Zhang and McFarlane, 1995; Neale et al., 2008)
- Shallow convective transport (Park and Bretherton, 2009)
- Turbulent transport (Park and Bretherton, 2009)
- Scavenging through wet deposition (only for aerosols not trace gases)
- Chemistry (for reactive tracers)

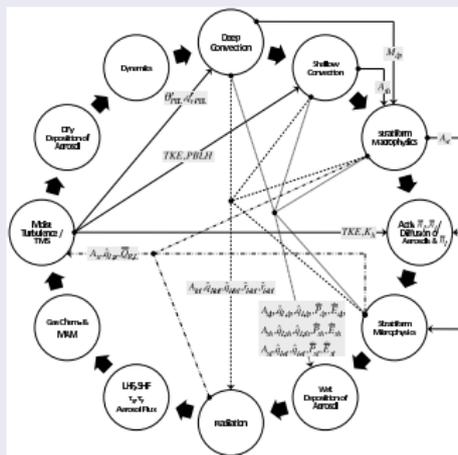


Figure from Park et al. (2012) - process split advancement of tendencies

- CAM-FV uses a Lagrangian ('floating') vertical coordinate  $\xi$  so that

$$\frac{d\xi}{dt} = 0,$$

i.e. vertical surfaces are material surfaces (no flow across them).

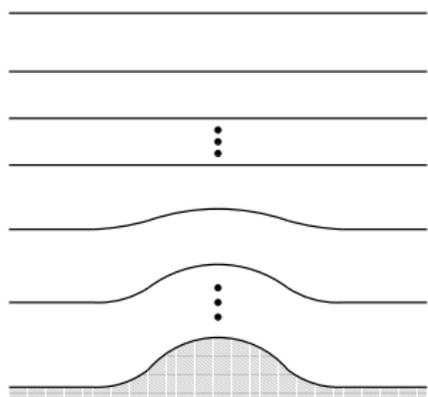


Figure shows 'usual' hybrid  $\sigma - p$  vertical coordinate  $\eta(p_s, p)$  (where  $p_s$  is surface pressure):

- $\eta(p_s, p)$  is a monotonic function of  $p$ .
- $\eta(p_s, p_s) = 1$
- $\eta(p_s, 0) = 0$
- $\eta(p_s, p_{top}) = \eta_{top}$ .

Boundary conditions are:

- $\frac{d\eta(p_s, p_s)}{dt} = 0$
- $\frac{d\eta(p_s, p_{top})}{dt} = \omega(p_{top}) = 0$

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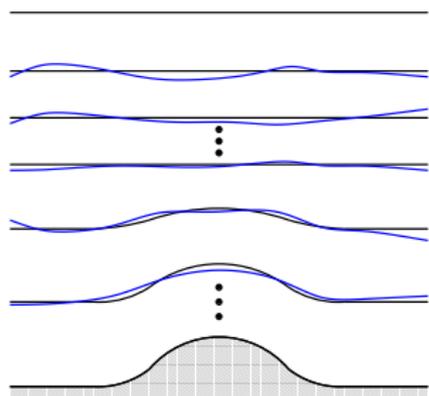


Figure:

- Set  $\xi = \eta$  at time  $t_{start}$  (black lines).
- For  $t > t_{start}$  the vertical levels deform as they move with the flow (blue lines).
- To avoid excessive deformation of the vertical levels (non-uniform vertical resolution) the prognostic variables defined in the Lagrangian layers  $\xi$  are periodically remapped (= conservative interpolation) back to the Eulerian reference coordinates  $\eta$  (more on this later).

# Adiabatic frictionless equations of motion using Lagrangian vertical coordinates

Assuming a Lagrangian vertical coordinate the hydrostatic equations of motion integrated over a layer can be written as

$$\begin{aligned} \text{mass air:} & \quad \frac{\partial(\delta p)}{\partial t} = -\nabla_h \cdot (\vec{v}_h \delta p), \\ \text{mass tracers:} & \quad \frac{\partial(\delta p q)}{\partial t} = -\nabla_h \cdot (\vec{v}_h q \delta p), \\ \text{horizontal momentum:} & \quad \frac{\partial \vec{v}_h}{\partial t} = -(\zeta + f) \vec{k} \times \vec{v}_h - \nabla_h \kappa - \nabla_p \Phi, \\ \text{thermodynamic:} & \quad \frac{\partial(\delta p \Theta)}{\partial t} = -\nabla_h \cdot (\vec{v}_h \delta p \Theta) \end{aligned}$$

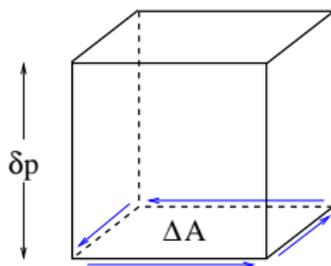
where  $\delta p$  is the layer thickness,  $\vec{v}_h$  is horizontal wind,  $q$  tracer mixing ratio,  $\zeta$  vorticity,  $f$  Coriolis,  $\kappa$  kinetic energy,  $\Theta$  potential temperature. The momentum equations are written in vector invariant form.

# Adiabatic frictionless equations of motion using Lagrangian vertical coordinates

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The equations of motion are discretized using an Eulerian finite-volume approach.



Integrate the flux-form continuity equation horizontally over a control volume:

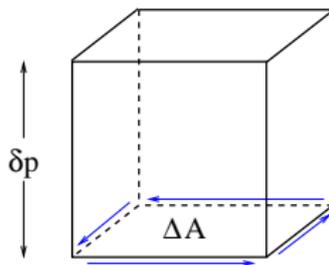
$$\frac{\partial}{\partial t} \iint_A \delta p \, dA = - \iint_A \nabla_h (\vec{v}_h \delta p) \, dA, \quad (1)$$

where  $A$  is the horizontal extent of the control volume. Using Gauss's divergence theorem for the right-hand side of (1) we get:

$$\frac{\partial}{\partial t} \iint_A \delta p \, dA = - \oint_{\partial A} \delta p \vec{v} \cdot \vec{n} \, dA, \quad (2)$$

where  $\partial A$  is the boundary of  $A$  and  $\vec{n}$  is outward pointing normal unit vector of  $\partial A$ .

# Finite-volume discretization of continuity equation



Integrate the flux-form continuity equation horizontally over a control volume:

$$\frac{\partial}{\partial t} \iint_A \delta p \, dA = - \iint_A \nabla_h (\vec{v}_h \delta p) \, dA, \quad (1)$$

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$$\frac{\partial}{\partial t} \iint_A \delta p \, dA = - \oint_{\partial A} \delta p \vec{v} \cdot \vec{n} \, dA, \quad (2)$$

Right-hand side of (2) represents the instantaneous flux of mass through the vertical faces of the control volume.

$$\frac{\partial}{\partial t} \iint_A \delta p \, dA = - \oint_{\partial A} \delta p \vec{v} \cdot \vec{n} \, dA. \quad (3)$$

Discretize (3) in space

$$\Delta A \frac{\partial \bar{\delta p}}{\partial t} = - \sum_{f=1}^4 [\langle \delta p \vec{v} \rangle \cdot \vec{n} \Delta \ell]_f, \quad (4)$$

where

- $\bar{\delta p}$  = horizontal mean value of  $\delta p$
- $\vec{n}_f$  = unit vector normal to the  $f$ th cell face pointing outward
- $\Delta \ell_f$  is the length of the face in question
- $\vec{v}_f$  = instantaneous values of  $\vec{v}$  at the cell face  $f$
- brackets represent averages in either  $\lambda$  or  $\theta$  direction over the cell face.

$$\frac{\partial}{\partial t} \iint_A \delta \rho \, dA = - \oint_{\partial A} \delta \rho \vec{v} \cdot \vec{n} \, dA. \quad (3)$$

Discretize (3) in space

$$\Delta A \frac{\partial \overline{\delta \rho}}{\partial t} = - \sum_{f=1}^4 [ \langle \delta \rho \vec{v} \rangle \cdot \vec{n} \Delta \ell ]_f, \quad (4)$$

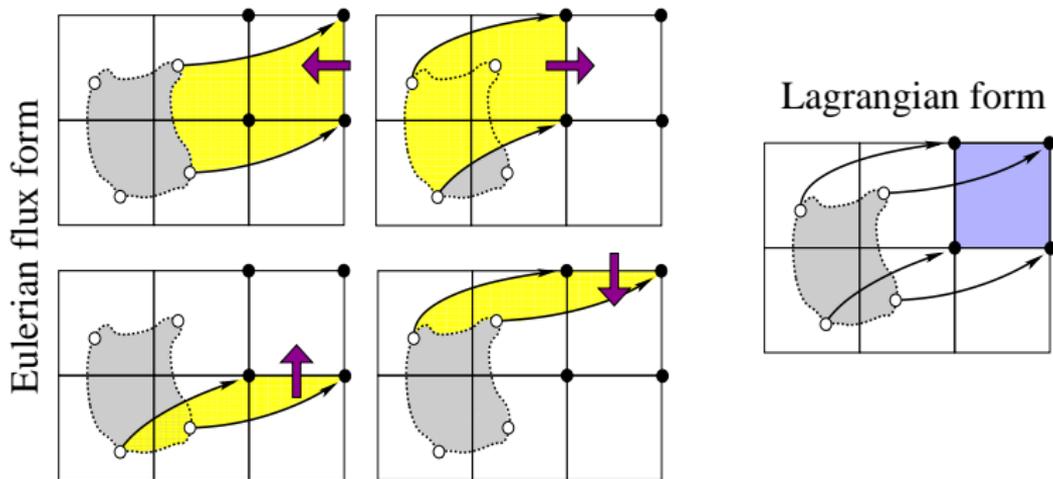
and integrate (4) over the time-step  $\Delta t_{dyn}$

$$\Delta A \overline{\delta \rho}^{n+1} = \Delta A \overline{\delta \rho}^n - \Delta t_{dyn} \sum_{f=1}^4 \left[ \overline{\langle \delta \rho \vec{v} \rangle \cdot \vec{n} \Delta \ell} \right]_f, \quad (5)$$

where  $n$  is the time-level index and the double-bar refers to the time average over  $\Delta t_{dyn}$ .

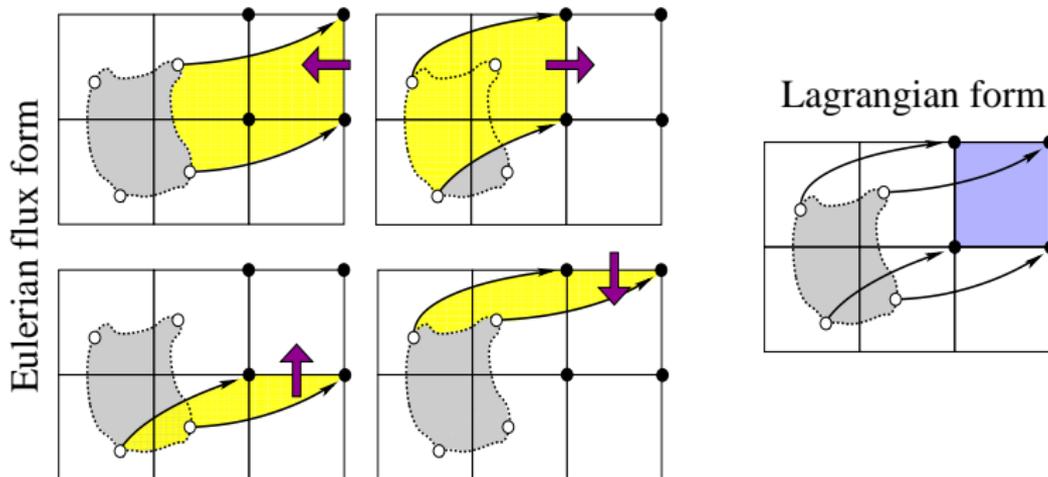
Each term in the sum on the right-hand side of (6) represents the mass transported through one of the four vertical control volume faces into the cell during one time-step (graphical illustration on next page).

# Finite-volume discretization of continuity equation: Tracking mass



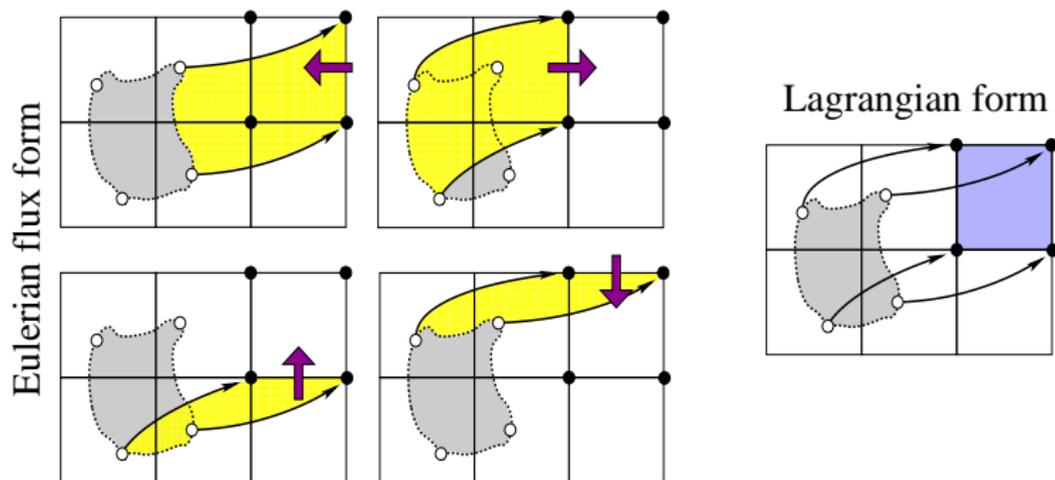
The yellow areas are 'swept' through the control volume faces during one time-step. The grey area is the corresponding Lagrangian area (area moving with the flow with no flow through its boundaries that ends up at the Eulerian control volume after one time-step). Black arrows show parcel trajectories.

Equivalence between Eulerian flux-form and Lagrangian form!



Until now everything has been exact. How do we approximate the fluxes numerically?

- In CAM-FV the Lin and Rood (1996) scheme is used which is a dimensionally split scheme (that is, rather than estimating the boundaries of the yellow areas and integrate over them, fluxes are estimated by successive applications of one-dimensional operators in each coordinate direction).



Until now everything has been exact. How do we approximate the fluxes numerically?

- (before showing equations for Lin and Rood (1996) scheme) What is the effective Lagrangian area associated with the Lin and Rood (1996) scheme?

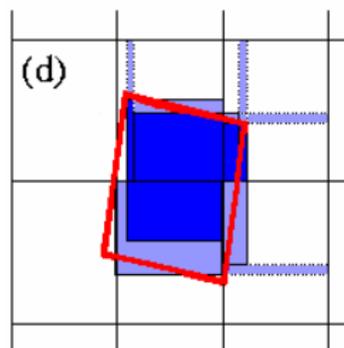


Figure: Red lines define boundary of exact Lagrangian cell for a special case with deformational, rotational and divergent wind field. Blue colors is Lagrangian cell associated with the Lin and Rood (1996) scheme. Dark blue shading weights integrated mass with 1 and light blue shading weights integrated mass with 1/2. See Machenhauer et al. (2009) for details.

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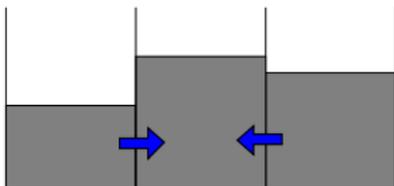
$$\overline{\delta p}^{n+1} = \overline{\delta p}^n + F^\lambda \left[ \frac{1}{2} \left( \overline{\delta p}^n + f^\theta(\overline{\delta p}^n) \right) \right] + F^\theta \left[ \frac{1}{2} \left( \overline{\delta p}^n + f^\lambda(\overline{\delta p}^n) \right) \right],$$

where

$F^{\lambda,\theta}$  = flux divergence in  $\lambda$  or  $\theta$  coordinate direction

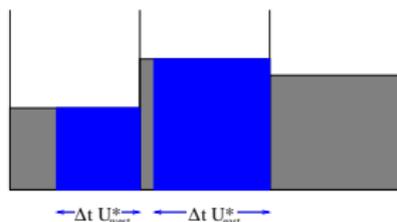
$f^{\lambda,\theta}$  = advective update in  $\lambda$  or  $\theta$  coordinate direction

$$\overline{\delta p}^{n+1} = \overline{\delta p}^n + F^\lambda \left[ \frac{1}{2} \left( \overline{\delta p}^n + f^\theta(\overline{\delta p}^n) \right) \right] + F^\theta \left[ \frac{1}{2} \left( \overline{\delta p}^n + f^\lambda(\overline{\delta p}^n) \right) \right],$$



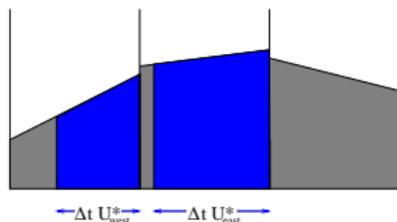
- Figure: Graphical illustration of flux-divergence operator  $F^\lambda$ . Shaded areas show cell average values for the cell we wish to make a forecast for and the two adjacent cells.

$$\overline{\delta p}^{n+1} = \overline{\delta p}^n + F^\lambda \left[ \frac{1}{2} \left( \overline{\delta p}^n + f^\theta (\overline{\delta p}^n) \right) \right] + F^\theta \left[ \frac{1}{2} \left( \overline{\delta p}^n + f^\lambda (\overline{\delta p}^n) \right) \right],$$



- $u_{east/west}^*$  are the time-averaged winds on each face (more on how these are obtained later).
- $F^\lambda$  is proportional to the difference between mass 'swept' through east and west cell face.
- $f^\lambda = F^\lambda + \overline{\overline{\delta p}} \Delta t_{dyn} D$ , where  $D$  is divergence.
- On Figure we assume constant sub-grid-cell reconstructions for the fluxes.

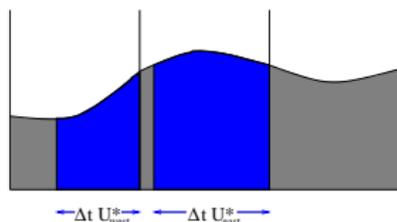
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Higher-order approximation to the fluxes:

- Piecewise linear sub-grid-scale reconstruction (van Leer, 1977): Fit a linear function using neighboring grid-cell average values with mass-conservation as a constraint (i.e. area under linear function = cell average.).

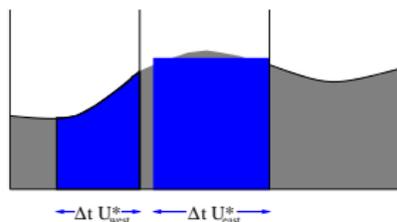
$$\overline{\delta p}^{n+1} = \overline{\delta p}^n + F^\lambda \left[ \frac{1}{2} \left( \overline{\delta p}^n + f^\theta(\overline{\delta p}^n) \right) \right] + F^\theta \left[ \frac{1}{2} \left( \overline{\delta p}^n + f^\lambda(\overline{\delta p}^n) \right) \right],$$



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- Piecewise parabolic sub-grid-scale reconstruction (Colella and Woodward, 1984): Fit parabola using neighboring grid-cell average values with mass-conservation as a constraint. Note: Reconstruction is  $C^0$  across cell edges.

$$\overline{\delta p}^{n+1} = \overline{\delta p}^n + F^\lambda \left[ \frac{1}{2} \left( \overline{\delta p}^n + f^\theta(\overline{\delta p}^n) \right) \right] + F^\theta \left[ \frac{1}{2} \left( \overline{\delta p}^n + f^\lambda(\overline{\delta p}^n) \right) \right],$$



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- Piecewise parabolic sub-grid-scale reconstruction (Colella and Woodward, 1984): Fit parabola using neighboring grid-cell average values with mass-conservation as a constraint. Note: Reconstruction is continuous at cell edges.
- Reconstruction function may 'over'- or 'undershoot' which may lead to unphysical and/or oscillatory solutions. Use limiters to render reconstruction function shape-preserving.

$$\overline{\delta p}^{n+1} = \overline{\delta p}^n + F^\lambda \left[ \frac{1}{2} \left( \overline{\delta p}^n + f^\theta(\overline{\delta p}^n) \right) \right] + F^\theta \left[ \frac{1}{2} \left( \overline{\delta p}^n + f^\lambda(\overline{\delta p}^n) \right) \right],$$

## Advantages:

- Inherently mass conservative (note: conservation does not necessarily imply accuracy!).
- Formulated in terms of one-dimensional operators.
- Preserves a constant for a non-divergent flow field (if the finite-difference approximation to divergence is zero).
- Preserves linear correlations between trace species (if shape-preserving filters are not applied)
- Has shape-preserving options. **Note:** Since the Lin and Rood (1996) is dimensionally split and the shape-preserving filters are applied along the coordinate axis tiny over-/under-shoot may be present in the traverse direction.

# Free-stream preserving 'super-cycling' of tracers with respect to air $\rho$

Simply solving the tracer continuity equation for  $\overline{q\delta\rho}^{n+1}$  using  $\Delta t_{trac}$  will lead to inconsistencies. Why?

Continuity equation for air  $\delta\rho$

$$\frac{\partial\delta\rho}{\partial t} + \nabla \cdot (\delta\rho \vec{v}_h) = 0, \quad (6)$$

and a tracer with mixing ratio  $q$

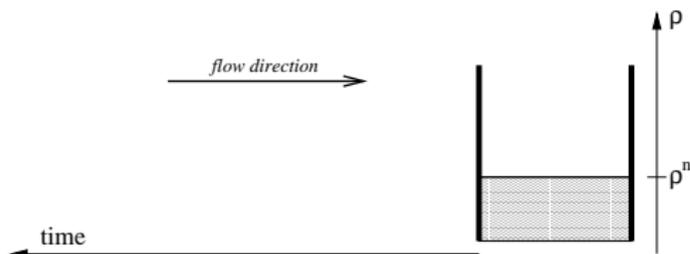
$$\frac{\partial(\delta\rho q)}{\partial t} + \nabla \cdot (\delta\rho q \vec{v}_h) = 0, \quad (7)$$

For  $q = 1$  equation (7) reduces to (6). If this is satisfied in the numerical discretizations, the scheme is 'free-stream' preserving.

Solving (7) with  $q = 1$  using  $\Delta t_{trac}$  will NOT produce the same solution as solving (6) `nsp1trac` times using  $\Delta t_{dyn}$ !

# Graphical illustration of 'free stream' preserving transport of tracers

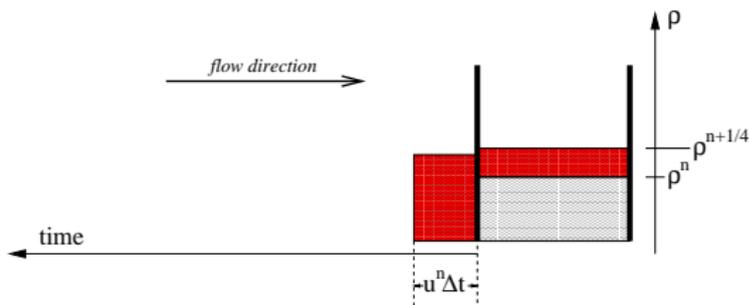
Assume no flux through east cell wall.



- Solve continuity equation for air  $\rho = \delta\rho$  together with momentum and thermodynamics equations.

# Graphical illustration of 'free stream' preserving transport of tracers

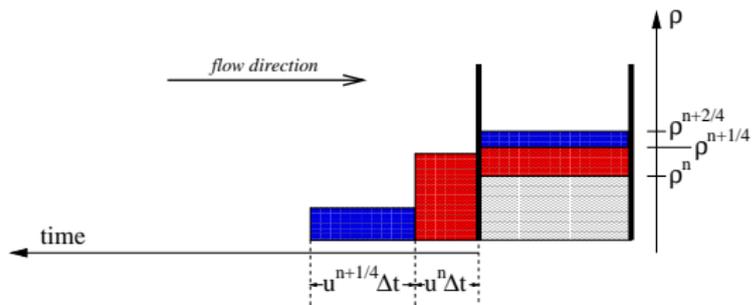
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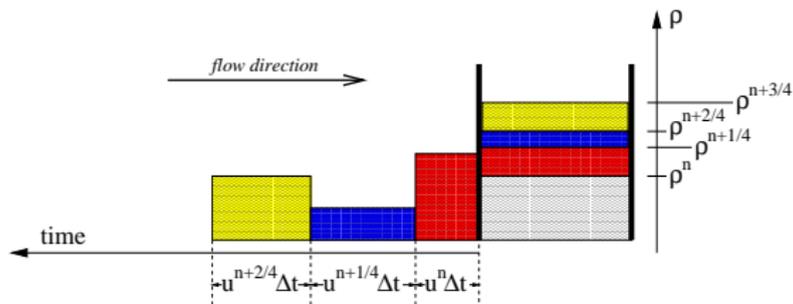
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- Solve continuity equation for air  $\rho = \delta\rho$  together with momentum and thermodynamics equations.
- Repeat *ksplit* times

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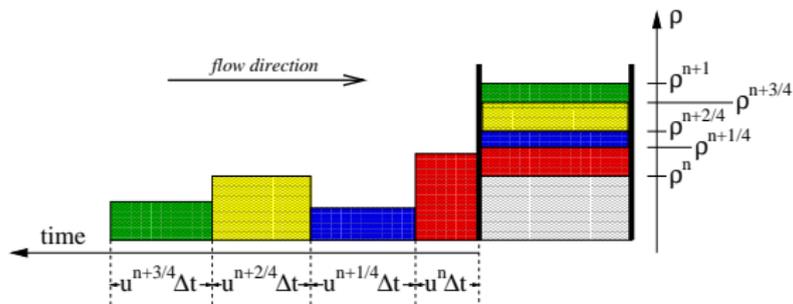
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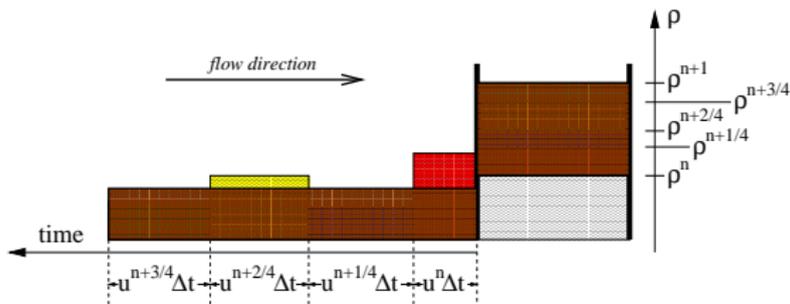
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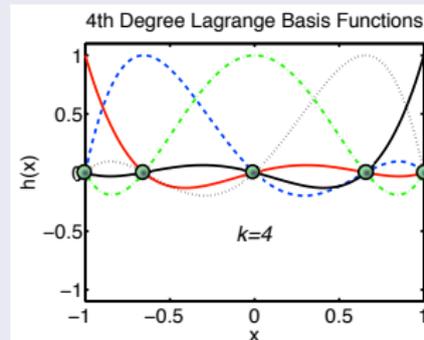
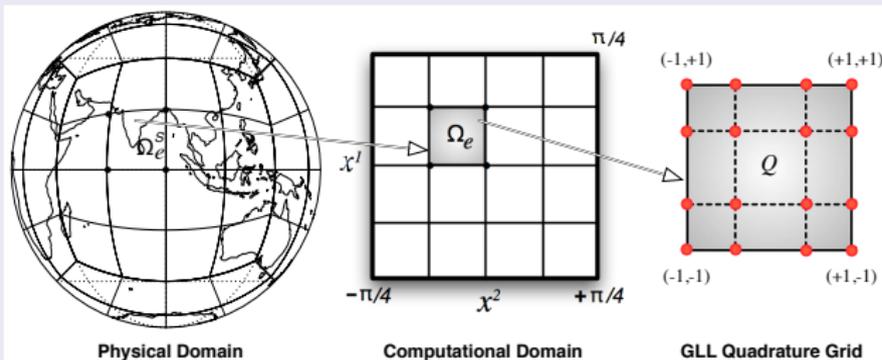


- Solve continuity equation for air  $\rho = \delta p$  together with momentum and thermodynamics equations.
- Repeat *ksplit* times
- Brown area = average flow of mass through cell face.
- Compute time-averaged value of  $q$  across brown area using Lin and Rood (1996) scheme:  $\overline{\langle q \rangle}$ .
- Forecast for tracer is:  $\overline{\langle q \rangle} \times \sum_{i=1}^{ksplit} \delta p^{n+i/ksplit}$
- Yields 'free stream' preserving solution!

# 'Resolved' scale transport: FYI

We are 'switching' dynamical core

November release of CESM → CAM-SE (Spectral Elements)



**Fig. 9.22** A schematic diagram showing the mapping between each spherical tile (element)  $\Omega_e^S$  of the physical domain (cubed-sphere)  $\mathcal{S}$  onto a planar element  $\Omega_e$  on the computational domain  $\mathcal{C}$  (cube). For a DG discretization each element on the cube is further mapped onto a unique reference element  $Q$ , which is defined by the Gauss-Lobatto-Legendre (GLL) quadrature points. The horizontal discretization of the HOMME dynamical cores relies on this grid system.

Figure from Nair et al. (2011)

In default CAM-SE  $n = 3$   
(polynomials of degree 3;  
 $4^{th}$ -order accurate)

Note: On Figure  $n = 4$

Aside: Ongoing DOE project (PI: J.-F. Lamarque)

Among goals: Evaluate CAM-SE as a transport model (Lauritzen and Thuburn, 2012; Lauritzen et al., 2012), add specified dynamics option to CAM-SE and investigate higher-order dynamics-chemistry coupling.

# Assesing 'accuracy' of tracer transport (Rasch et al., 2006)

Test case setup by Rasch et al. (2006) designed to assess transport in region of atmosphere

- (LOW) strongly influenced by sub-grid scale transport processes (convection and boundary layer processes),
- (HIGH) less strongly influenced by sub-grid scale processes; large role being played by resolved scale dynamics
- (MID) 'in the middle'!

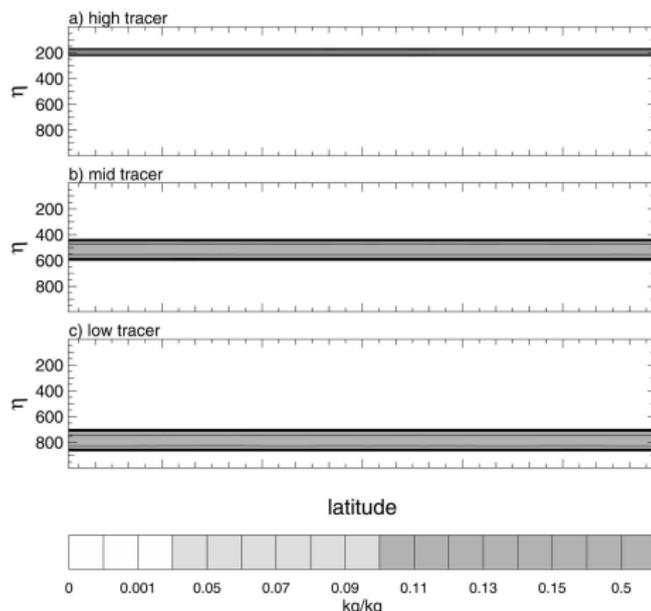


FIG. 1. Initial conditions for (a) HIGH, (b) MID, and (c) LOW tracer.

## LOW tracer (30 day simulation)

- In the tropics: Rapid mixing between surface and tropopause
- Subtropical subsidence region: Low values of tracer mixing ratio
- Midlatitudes-polar regions: Broader mixing
- Quantitative differences between models: FV shows steeper gradient between tropics and subtropics; (day 30) gradient between low and high mixing ratio in subtropics is 2x higher!

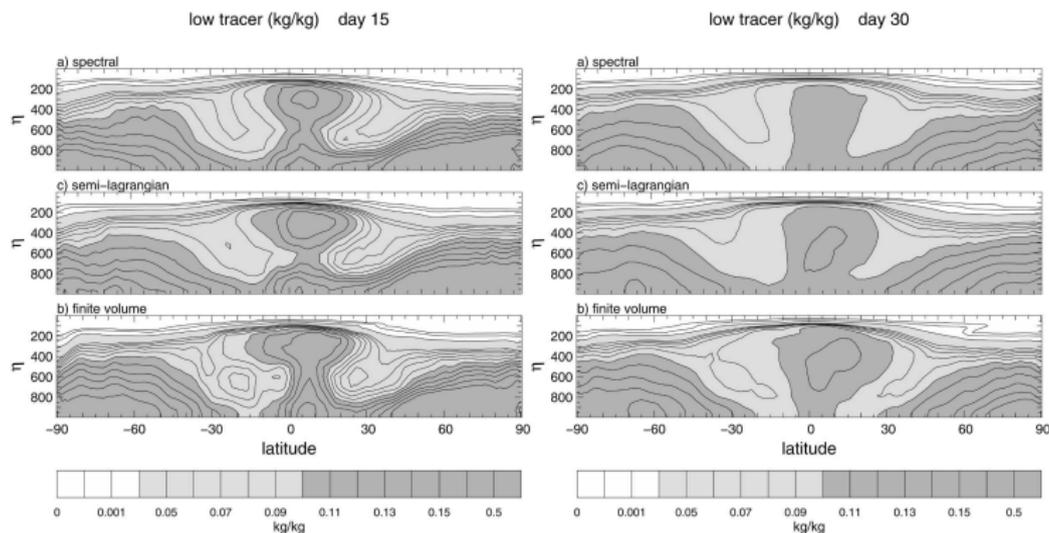


FIG. 2. Zonal averaged mixing ratio for days (left) 15 and (right) 30 for LOW tracer: (a) spectral, (b) semi-Lagrangian, and (c) finite volume.

HIGH tracer (let the model evolve for 30 days):

- Subtropical features seen in LOW tracer also in HIGH tracer
- Substantial differences in mixing in the the upper tropospheric polar region: FV core has preserved initial gradient much more strongly

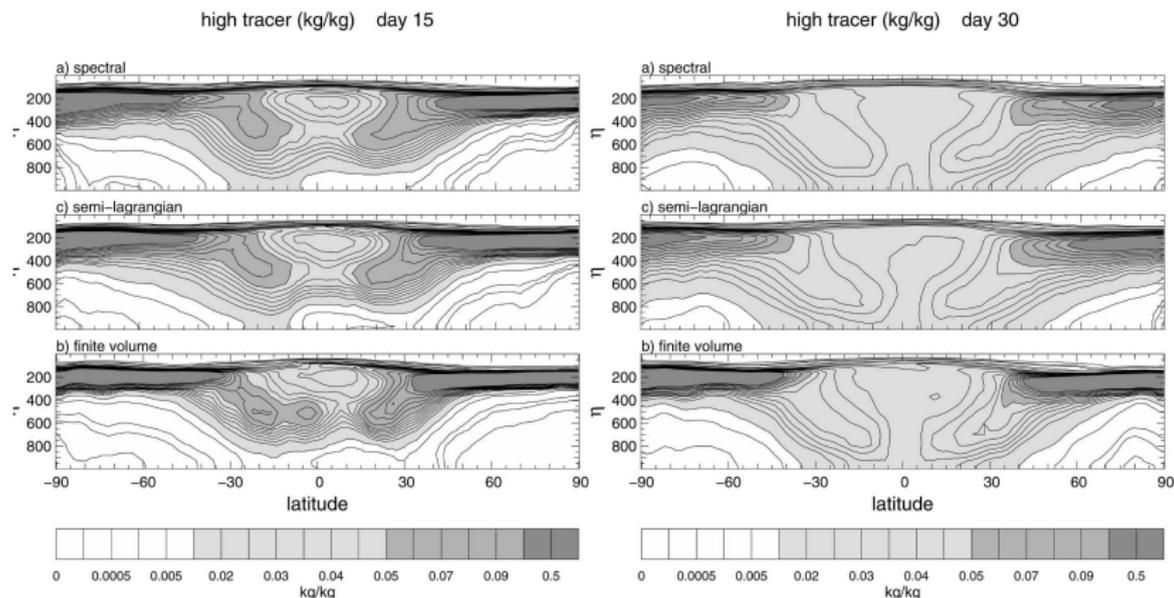
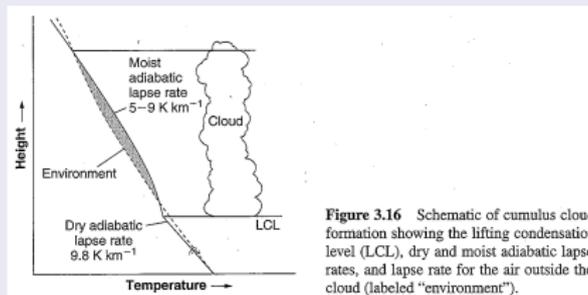


FIG. 3. As in Fig. 2 but for HIGH tracer.





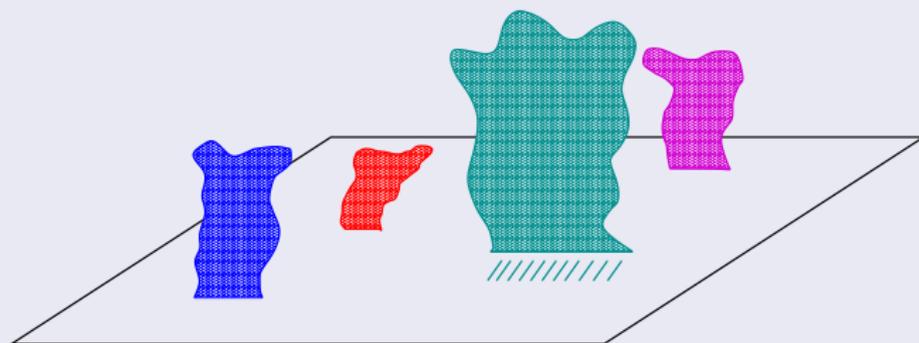
## Simple model of convective cloud:

(ignoring entrainment, increased buoyancy due to  $T$  &  $q$  differences between cloud core updrafts and compensating downdrafts outside cloud, ...)

- parcel, say with 50% relative humidity, near surface starts to rise, say from strong heating → parcel cools at approximately dry adiabatic lapse rate ( $9.8 \text{ K/km}^{-1}$ )
- assume the parcel does not mix (i.e. water vapor content remains the same): **no entrainment**
- as the parcel rises it continues to cool → it can hold less water → relative humidity increases
- when reaching 100% relative humidity the parcel saturates: parcel has reached **Lifting Condensation Level (LCL)**
- from LCL and upward the parcel cools less rapidly (latent heat release of condensation or fusion releases heat):  $5\text{-}6 \text{ K/km}^{-1}$
- at some point well above LCL the parcel is cooler than the environmental air (stops rising - top of cloud); however, buoyancy forces and the parcels upward momentum can make the cloud extend above the cross-over point.

# Deep convection scheme - schematic

Consider a model grid cell with area  $\Delta A$  (typical scale 100km) with several deep convective towers



There is a lot going on sub-grid-scale:

- updrafts, downdraft, entrainment, detrainment, condensation, evaporation, ...

What we know is the cell-averaged model state within that grid cell:  $(\bar{T}, \bar{q}, \bar{u}, \bar{v}, \bar{P}, \dots)$

What the parameterization should give us:

$$\frac{\partial \bar{T}}{\partial t} = \mathcal{F}_T(\bar{T}, \bar{q}, \bar{u}, \bar{v}, \bar{P}, \dots) \quad (8)$$

$$\frac{\partial \bar{q}}{\partial t} = \mathcal{F}_q(\bar{T}, \bar{q}, \bar{u}, \bar{v}, \bar{P}, \dots) \quad (9)$$

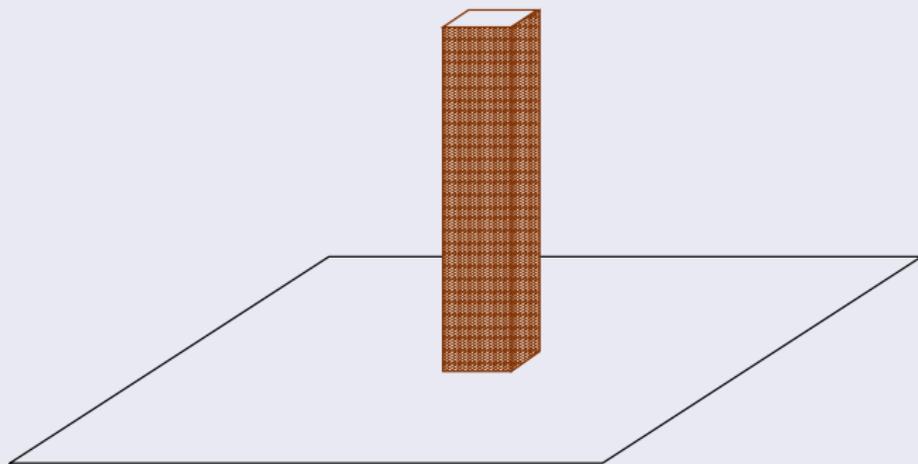
# Deep convection scheme - schematic

Ensemble plume approach (Arakawa and Schubert, 1974)

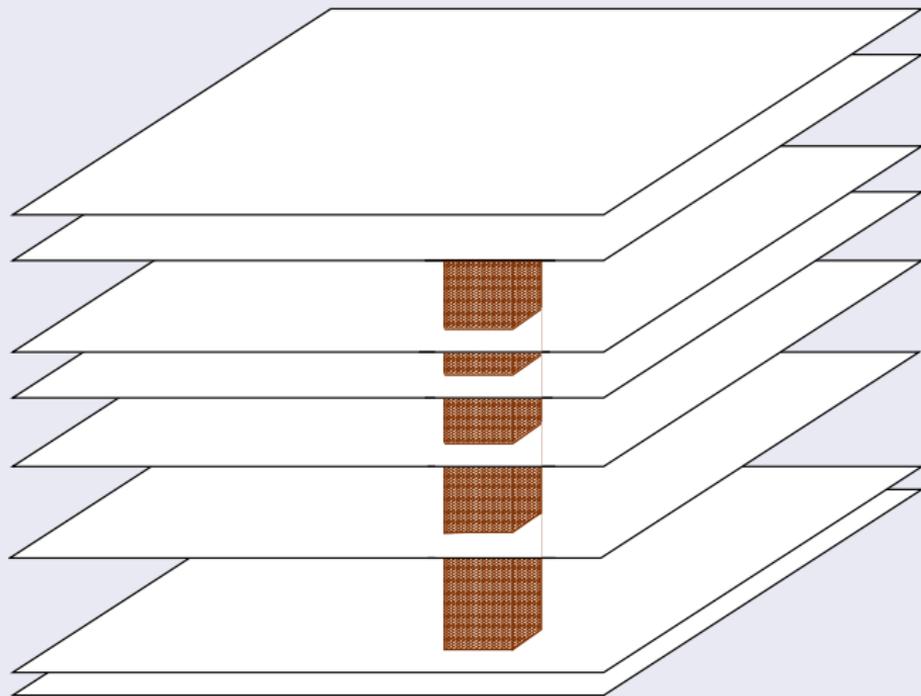
Ensemble of convective updrafts and associated saturated downdrafts exist whenever the atmosphere is conditionally unstable in the lower troposphere → effectively the model 'sees' one 'ensemble column' of convection

Among the assumptions are:

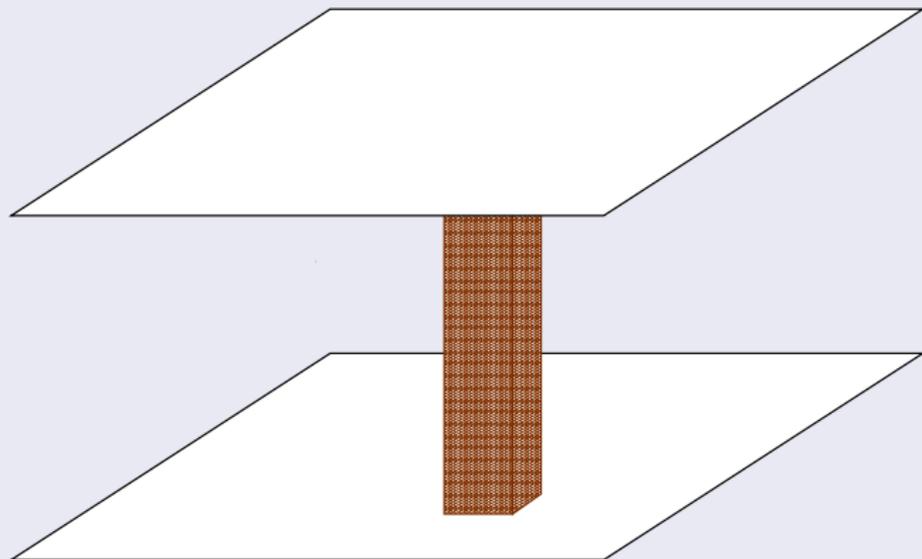
- no tilting of the 'ensemble' convective tower
- area of 'ensemble' convective tower  $\Delta A_c \ll \Delta A$   
→  $q_e = \bar{q}$



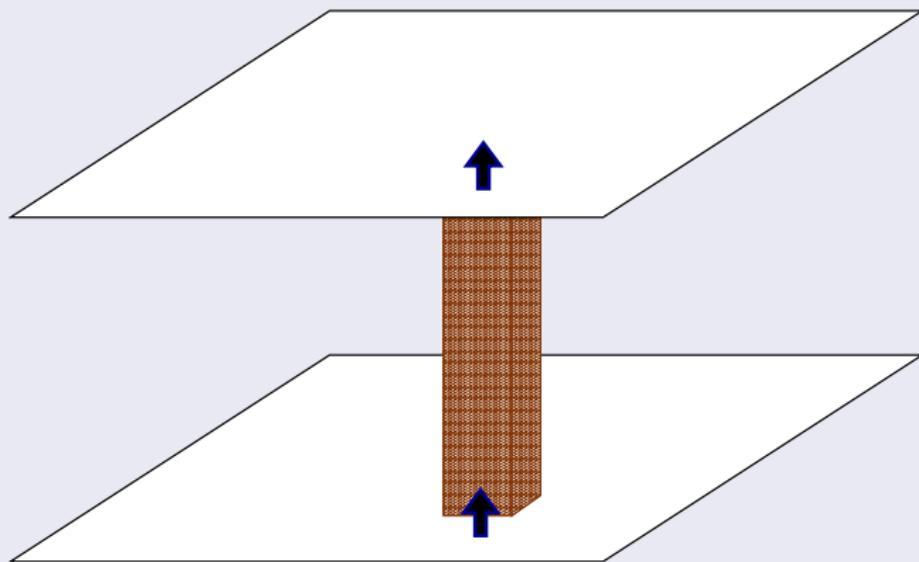
'Ensemble' convective tower may span many vertical levels (even entire column)



Consider 'ensemble plume' in one layer:



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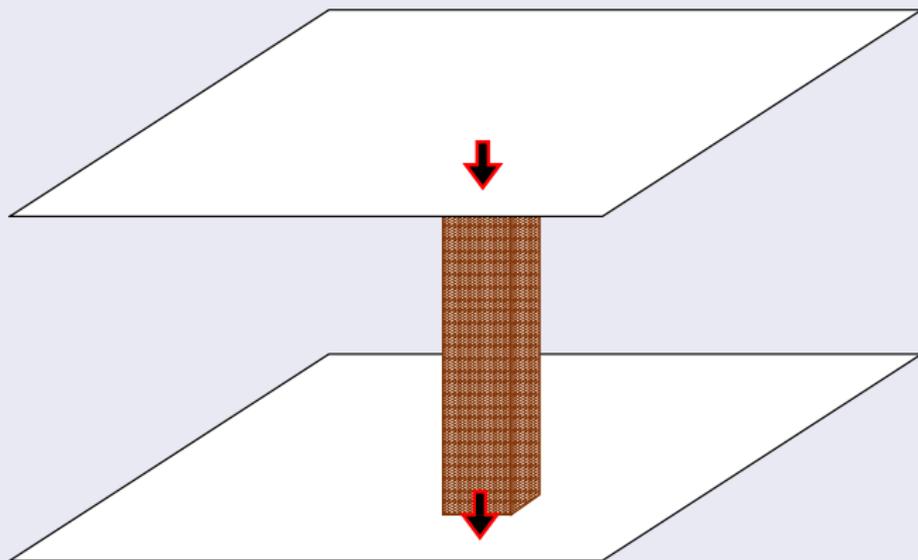


Processes represented:

- $M_U$ : mass flux of 'ensemble' updraft defined at model layer interfaces

# Deep convection scheme - schematic

Consider 'ensemble plume' in one layer:

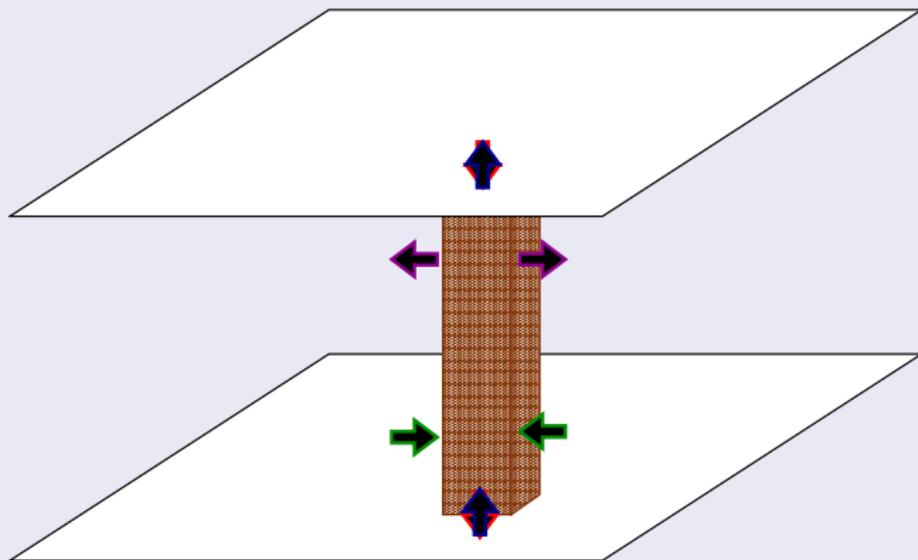


Processes represented:

- $M_u$ : mass flux of 'ensemble' updraft defined at model layer interfaces
- $M_d$ : mass flux of 'ensemble' downdraft defined at model layer interfaces

# Deep convection scheme - schematic

Consider 'ensemble plume' in one layer:

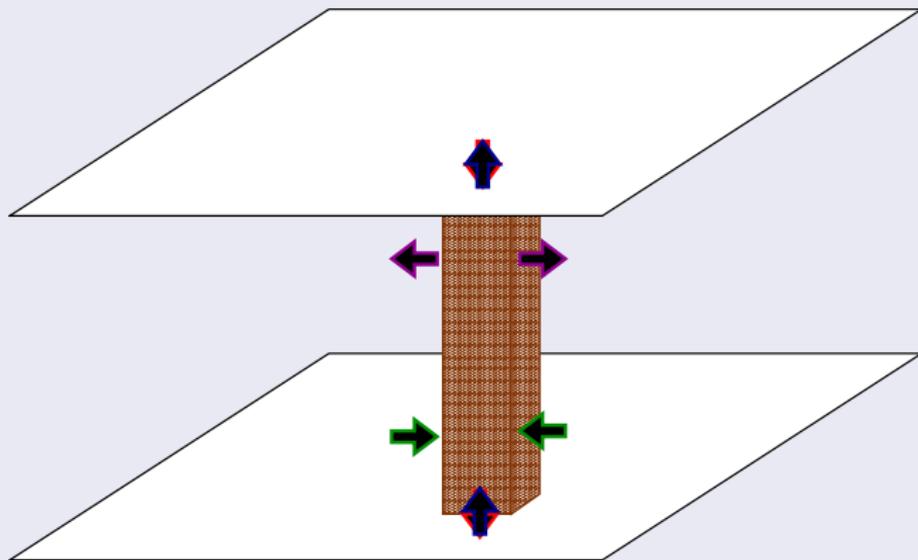


Processes represented:

- $M_u$ : mass flux of 'ensemble' updraft defined at model layer interfaces
- $M_d$ : mass flux of 'ensemble' downdraft defined at model layer interfaces
- $E_x$ ,  $x = u, d$ : entrainment rate of environmental air associated with updrafts and downdrafts, respectively (defined at layer centers)

# Deep convection scheme - schematic

Consider 'ensemble plume' in one layer:



Processes represented:

- $M_u$ : mass flux of 'ensemble' updraft defined at model layer interfaces
- $M_d$ : mass flux of 'ensemble' downdraft defined at model layer interfaces
- $E_x, x = u, d$ : entrainment rate of environmental air associated with updrafts and downdrafts, respectively (defined at layer centers)
- $D_x, x = u, d$ : detrainment rate of 'plume air' associated with updrafts and downdrafts, respectively

# Deep convection scheme - schematic

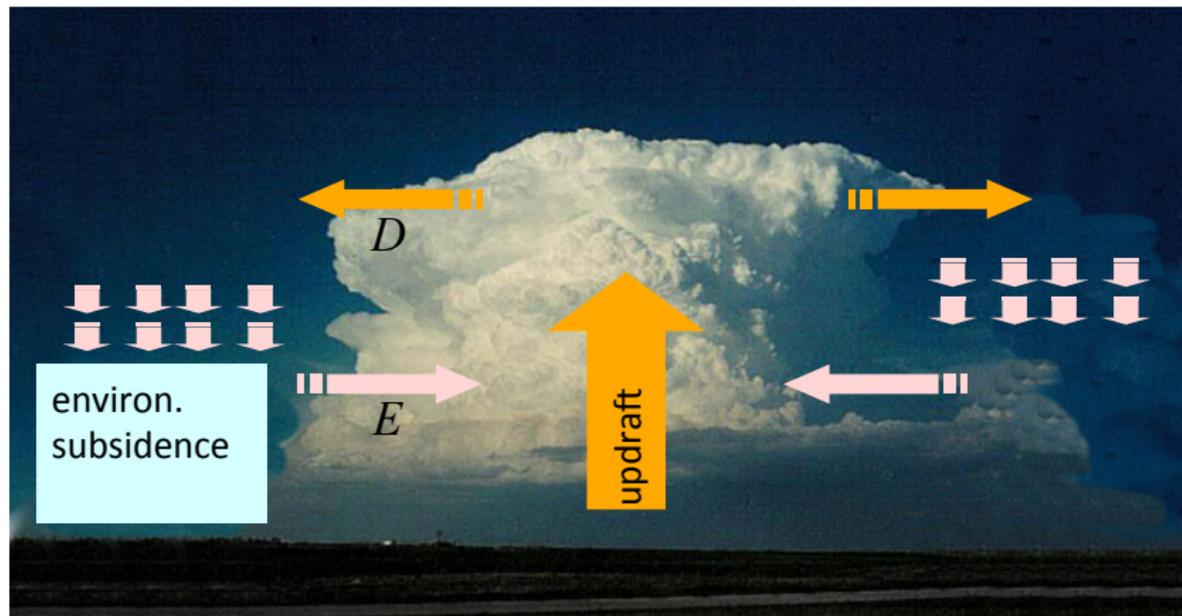
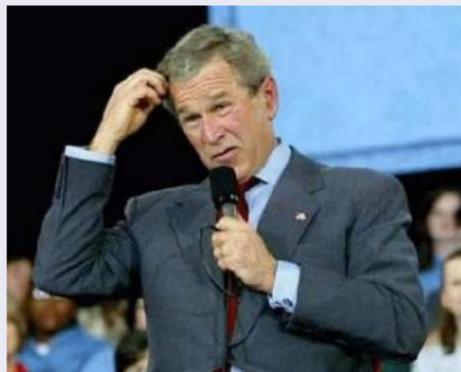


Figure courtesy of J. Bacmeister (NCAR)

# Deep convection scheme - schematic

## CAM5 deep convection scheme

The challenge of cloud researchers is to determine how nature decides which process dominates under different large-scale environmental conditions.



CAM5 deep convection scheme is a simplification of Arakawa and Schubert (1974) for large-scale models (Zhang and McFarlane, 1995) with modified momentum transport by Richter and Rasch (2008) and a modified dilute plume calculation following Raymond and Blyth (1992)

The details of how the deep convection scheme determines  $M_u$ ,  $M_d$ ,  $E_u$ ,  $D_u$ ,  $E_d$ ,  $E_u$  is beyond the scope of this talk (local experts: S. Park, R. Neale, J. Bacmeister), i.e. assume that mass-fluxes, entrainment and detrainment rates are given!

# Deep convective tracer transport

Convection is an effective way of mixing tracers in the vertical (e.g. Mahowald et al., 1995; Collins et al., 1999), e.g., convective updrafts can transport a tracer from the surface to the upper troposphere on time scales of  $\mathcal{O}(1h)$ .

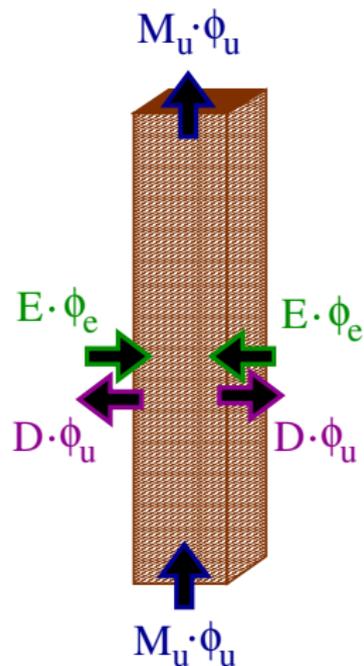
Steady state continuity equation for 'bulk' updraft mixing ratio  $\varphi_u$

$$\frac{\partial (M_u \varphi_u)}{\partial p} = E_u \varphi_e - D_u \varphi_u \quad (8)$$

where

- $M_u$  is mass-flux at layer interfaces
- $\varphi_e$  mixing ratio of environment  
(in CAM:  $\varphi_e = \bar{\varphi}$ ; i.e. we assume that area of updraft  $\ll$  grid cell area)
- $E_u$  and  $D_u$  are entrainment/detrainment rates for the updrafts.

Solve (8) for  $\varphi_u$



# Deep convective tracer transport

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Steady state continuity equation for 'bulk' downdraft mixing ratio

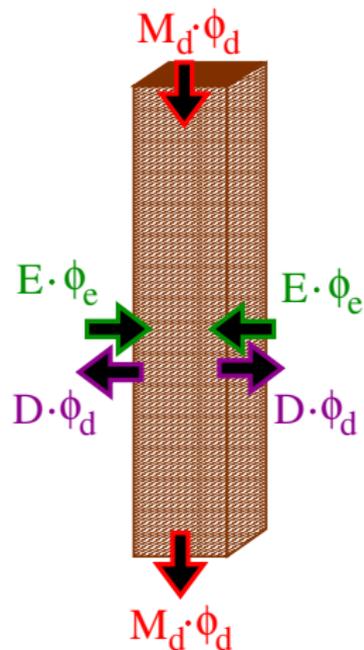
$\varphi_d$

$$\frac{\partial (M_d \varphi_d)}{\partial p} = E_d \varphi_e - D_d \varphi_d \quad (8)$$

where

- $M_d$  is mass-flux at layer interfaces
- $\varphi_e$  mixing ratio of environment  
(in CAM:  $\varphi_e = \bar{\varphi}$ ; i.e. we assume that area of updraft  $\ll$  grid cell area)
- $E_d$  and  $D_d$  are entrainment/detrainment rates for the downdrafts.

Solve (8) for  $\varphi_d$



CAM5 subroutine convtran in physics/cam/zm\_conv.F90 file

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Steady state continuity equation for 'bulk' downdraft mixing ratio

$\varphi_d$

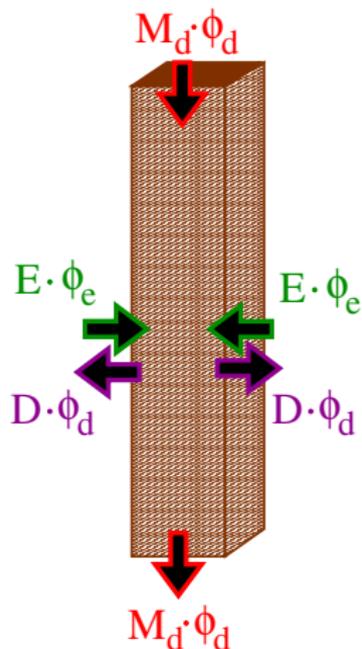
$$\frac{\partial (M_d \varphi_d)}{\partial p} = E_d \varphi_e - D_d \varphi_d \quad (8)$$

where

- $M_d$  is mass-flux at layer interfaces
- $\varphi_e$  mixing ratio of environment  
(in CAM:  $\varphi_e = \bar{\varphi}$ ; i.e. we assume that area of updraft  $\ll$  grid cell area)
- $E_d$  and  $D_d$  are entrainment/detrainment rates for the downdrafts.

Solve (8) for  $\varphi_d$

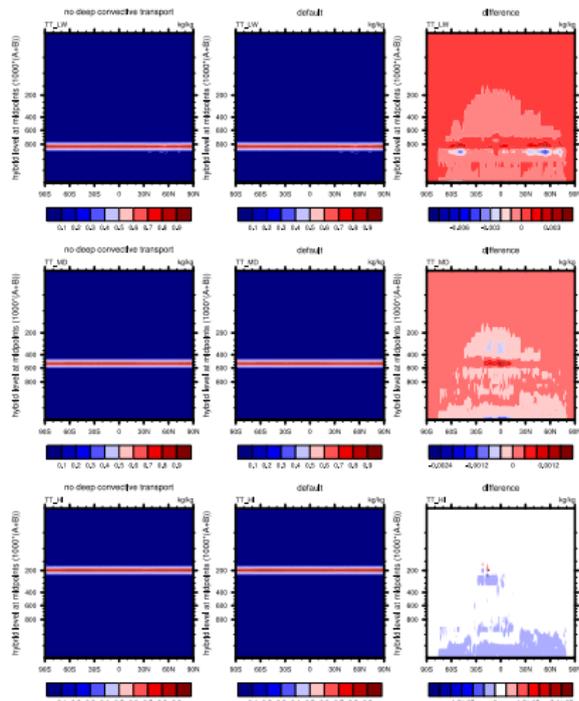
$$\frac{\partial \bar{\varphi}}{\partial t} = \frac{\partial}{\partial p} [M_u (\varphi_u - \bar{\varphi}) + M_d (\varphi_d - \bar{\varphi})] \quad (9)$$



CAM5 subroutine convtran in physics/cam/zm\_conv.F90 file

# How much mixing does deep convection do?

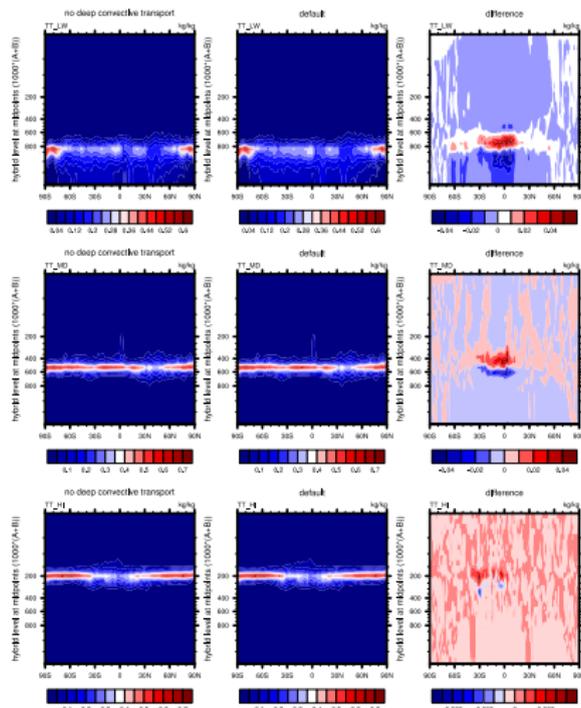
Use Rasch et al. (2006) transport test setup: day 0, zonal average



(left) no deep convective transport of tracers - there is deep convective transport of water variables!, (middle) default, (right) difference

# How much mixing does deep convection do?

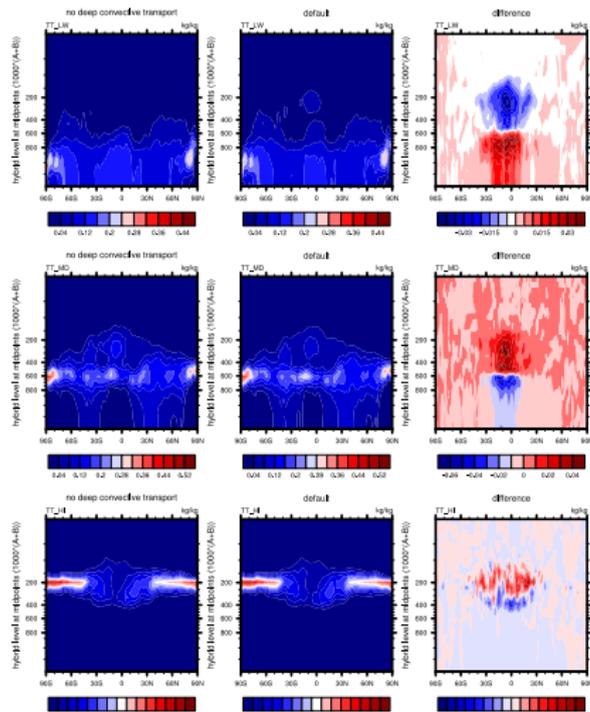
Use Rasch et al. (2006) transport test setup: day 1, zonal average



(left) no deep convective transport of tracers - there is deep convective transport of water variables!, (middle) default, (right) difference

# How much mixing does deep convection do?

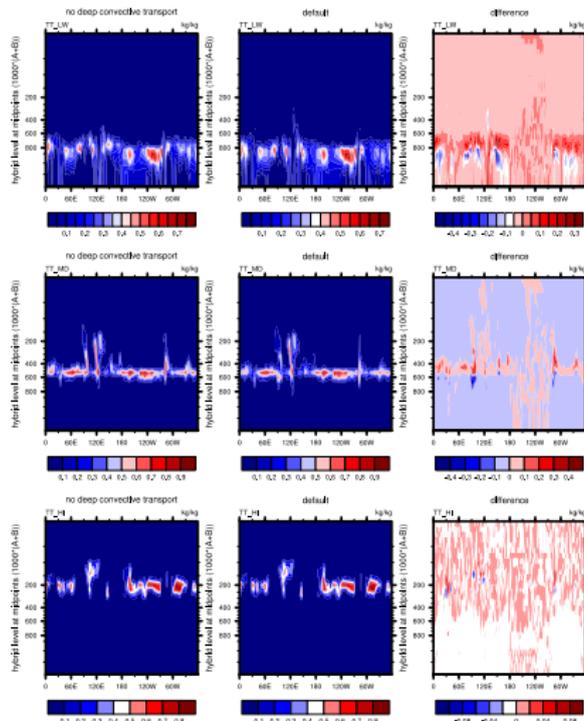
Use Rasch et al. (2006) transport test setup: day 5, zonal average



(left) no deep convective transport of tracers - there is deep convective transport of water variables!, (middle) default, (right) difference

# How much mixing does deep convection do?

Use Rasch et al. (2006) transport test setup: day 1, along Equator



(left) no deep convective transport of tracers - there is deep convective transport of water variables!, (middle) default, (right) difference

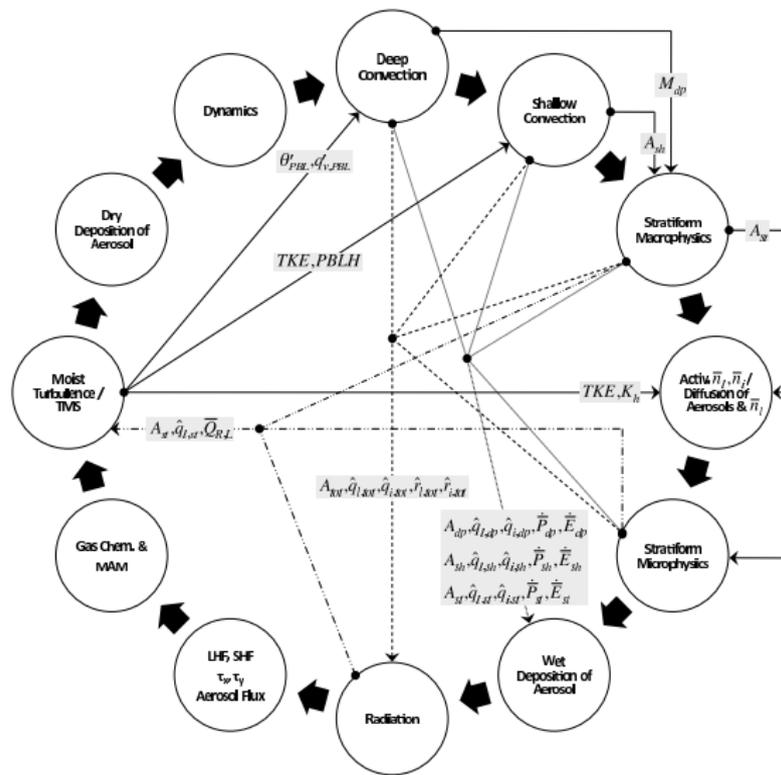


Figure from Park et al. (2012)

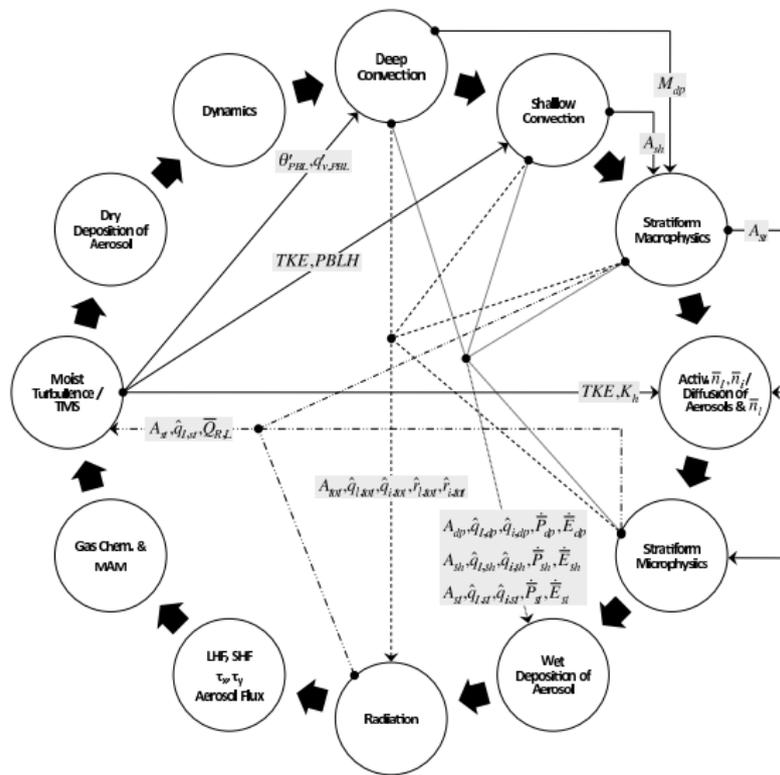


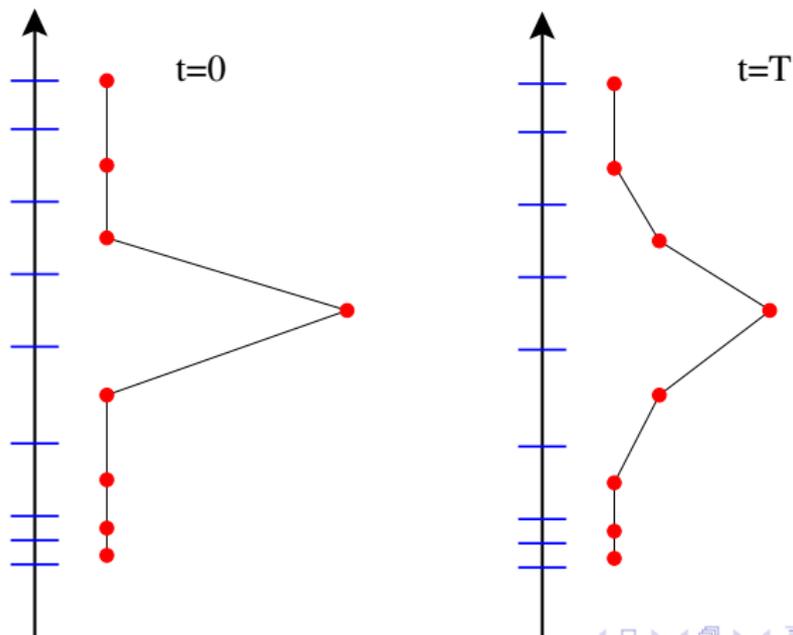
Figure from Park et al. (2012)

# Turbulent diffusion

Given vertical profile of eddy diffusion coefficient  $K(p)$ :

$$\frac{\partial \bar{\varphi}}{\partial t} = \frac{\partial}{\partial p} \left[ K(p) \frac{\partial \bar{\varphi}}{\partial p} \right] \quad (10)$$

Contrary to convective tracer transport turbulent diffusion is a local process!





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