Conservative Tracer Transport in HOMME

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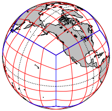
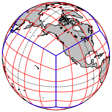




DOE Integrated Climate Change Modeling Science Team Meeting, Washington DC

1. Motivation: Why focus on tracer transport?
2. New challenging test cases for transport schemes on the sphere
3. Conservative Semi-LAgrangian Multi-tracer (CSLAM) scheme
4. Flux-form version of CSLAM
5. Exploratory research (in progress)

Why multi-tracer transport scheme?

- Prognostic mass continuity equations in NCAR's Community Atmosphere Model (CAM) version 5:
  - Air density
  - Water species: Water vapor, cloud liquid water and ice
  - Aerosols: 3 number concentrations, particulate organic matter, dust, sea salt, secondary organic aerosols, ... (**total of 22**)
- Chemistry version of CAM prognoses 126+ chemical species.

- In current and future Earth System Models the computational cost of resolved dynamics is (or is expected to be) dominated by the cost of tracer transport
- Multi-tracer efficiency is becoming increasingly important

Requirements for accuracy

- Consistency:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\, \vec{v}) = 0, \tag{1}$$

$$\frac{\partial (\rho\, q)}{\partial t} + \nabla \cdot (\rho\, q\, \vec{v}) = 0, \tag{2}$$

  where $\vec{v}$ is the velocity vector. If $q = 1$ then (2) reduces to (1).

- Monotonicity: Note that (1) and (2) imply

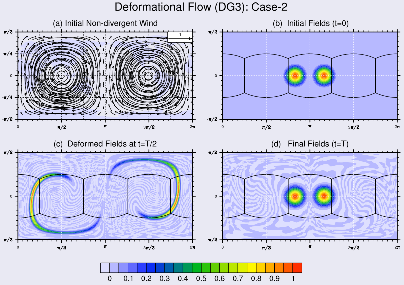
$$\frac{dq}{dt} = 0, \quad \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \vec{v} \cdot \nabla, \tag{3}$$

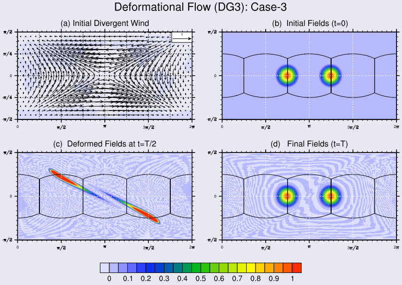
  q is conserved along trajectories/characteristics of the flow.

- Increased chemistry in applications: Relative concentrations controls chemical reactions -
  ideally, transport schemes should preserve relative concentrations (at least in areas with little or no mixing).
- Geometric flexibility
- For full list and discussion see Lauritzen et al. (2010b)

Class of deformational test cases (Nair and Lauritzen, 2010):

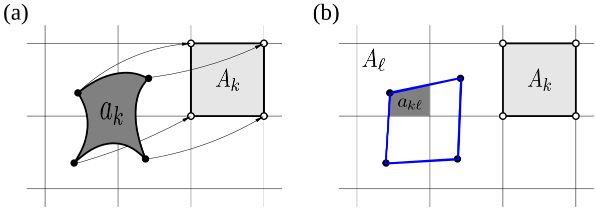
- Idealized transport testing on the sphere has until recently been restricted to solid-body advection (no deformation, no divergence)
- Need more rigorous benchmark test cases in a challenging environment to:
  - test schemes under divergent and deformational flow conditions
  - test schemes on new unstructured spherical grids
  - test static and dynamic mesh refinement algorithms
  - test trajectory algorithms (semi-Lagrangian or Lagrangian methods)





see poster entitled: ‘A Benchmark Test-Case Suite for Transport Problems on the sphere’

Conservative Semi-LAgrangian Multi-tracer (CSLAM)



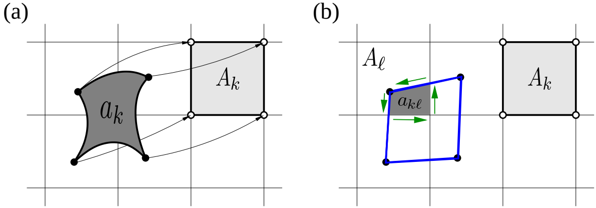
Finite-volume Lagrangian form of continuity equation for $\psi = \rho, \rho\, q$:

$$\int_{A_k} \psi_k^{n+1}\, dx\, dy = \int_{a_k} \psi_k^{n}\, dx\, dy \quad = \quad \sum_{\ell=1}^{L_k} \iint_{a_{k\ell}} f_\ell(x,y)\, dx\, dy,$$

where the $a_{k\ell}$'s are non-empty overlap regions:

$$a_{k\ell} = a_k \cap A_\ell, \quad a_{k\ell} \neq \emptyset; \quad \ell = 1, \ldots, L_k. \tag{4}$$

Conservative Semi-LAgrangian Multi-tracer (CSLAM)



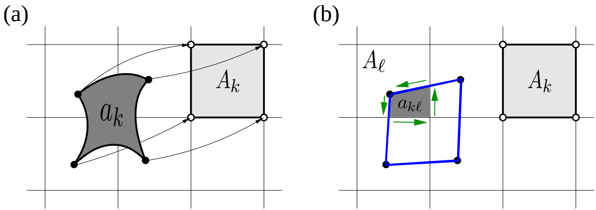
Finite-volume Lagrangian form of continuity equation for $\psi = \rho, \rho\, q$:

$$\int_{A_k} \psi_k^{n+1}\, dx\, dy = \int_{a_k} \psi_k^n\, dx\, dy \quad = \quad \sum_{\ell=1}^{L_k} \oint_{\partial a_{k\ell}} \left[P\, dx + Q\, dy\right],$$

where $\partial a_{k\ell}$ is the boundary of $a_{k\ell}$ and

$$-\frac{\partial P}{\partial y} + \frac{\partial Q}{\partial x} = f_\ell(x,y) = \sum_{i+j\leq 2} c_\ell^{(i,j)} x^i y^j.$$

Conservative Semi-LAgrangian Multi-tracer (CSLAM)

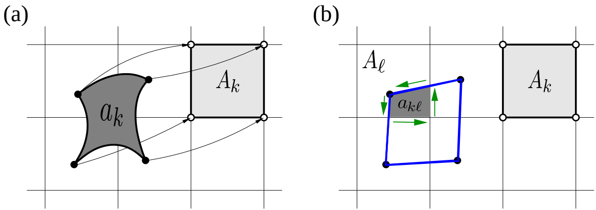


Finite-volume Lagrangian form of continuity equation for $\psi = \rho, \rho\, q$:

$$\int_{A_k} \psi_k^{n+1}\, dx\, dy = \int_{a_k} \psi_k^{n}\, dx\, dy \quad = \quad \sum_{\ell=1}^{L_k} \left[\sum_{i+j\le 2} c_\ell^{(i,j)} w_{k\ell}^{(i,j)} \right],$$

where weights $w_{k\ell}^{(i,j)}$ are functions of the coordinates of the vertices of $a_{k\ell}$.

Conservative Semi-LAgrangian Multi-tracer (CSLAM)

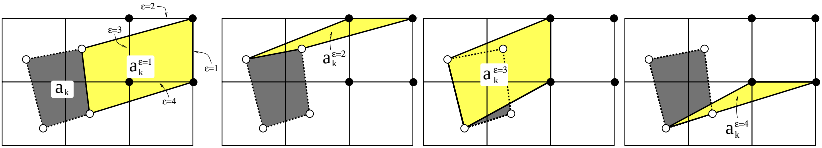


Finite-volume Lagrangian form of continuity equation for $\psi = \rho, \rho\, q$:

$$\int_{A_k} \psi_k^{n+1}\, dx\, dy = \int_{a_k} \psi_k^n\, dx\, dy \quad = \quad \sum_{\ell=1}^{L_k} \left[\sum_{i+j\leq 2} c_\ell^{(i,j)} w_{k\ell}^{(i,j)} \right],$$

- $w_{k\ell}^{(i,j)}$ can be re-used for each additional tracer (Dukowicz and Baumgardner, 2000)
- CSLAM is stable and efficient for long time-steps (CFL$>$1)
- CSLAM is fully two-dimensional and can be extended to any spherical grid constructed from great-circle arcs
- Cubed-sphere extension of CSLAM is documented in Lauritzen et al. (2010a)

Flux-form CSLAM (FF-CSLAM)



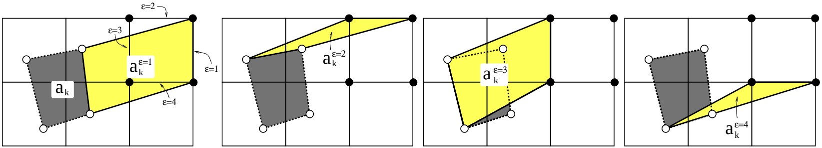
Finite-volume flux-form of continuity equation for $\psi = \rho, \rho\, q$:

$$\int_{A_k} \psi_k^{n+1}\, dx\, dy \quad = \quad \int_{A_k} \psi_k^n\, dx\, dy - \sum_{\epsilon=1}^{4} \left[\sum_{\ell=1}^{L_k^\epsilon} s_{k\ell}^\epsilon \iint_{a_{k\ell}^\epsilon} f_\ell(x,y)\, dx\, dy \right],$$

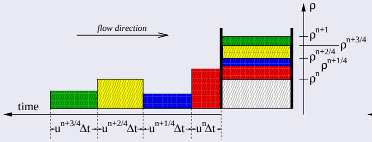
where

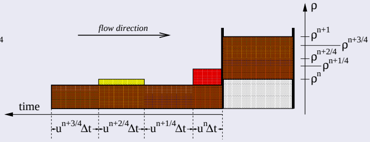
- a_k^ϵ = 'flux-area' (yellow area) = area swept through face ϵ
- L_k^ϵ = number of overlap areas for a_k^ϵ; $a_{k\ell}^\epsilon = a_k^\epsilon \cap A_k$
- $s_{k\ell}^\epsilon$ = 1 for outflow and -1 for inflow.

Flux-form CSLAM (FF-CSLAM)

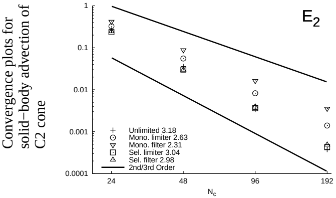


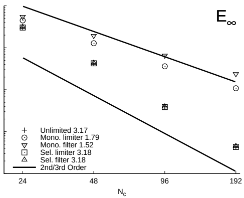
- CSLAM and FF-CSLAM with no filter/limiters are identical. Why FF-CSLAM?
- In CSLAM **shape-preservation** is enforced by filtering the sub-grid-cell reconstructions (also applicable for FF-CSLAM)
- Casting in flux-form one may also apply **flux-limiters** such as FCT (Flux-Correct-Transport, Zalesak 1979).
- Flux-form allows for **sub-cycling (also referred to as super-cycling)**, that is, transport tracers with longer time-steps than what is used for the dynamics.

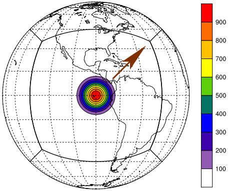


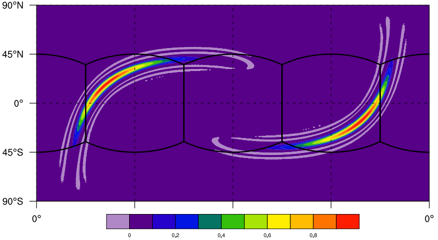


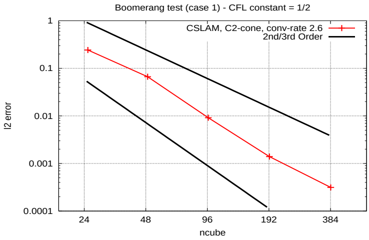
Flux-form CSLAM (FF-CSLAM): Results







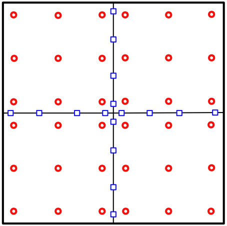


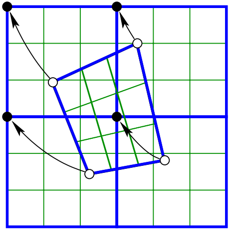


- See poster entitled 'A Flux-form version of the Conservative Semi-Lagrangian Multi-Tracer Transport Scheme (CSLAM)'
- And/or Harris and Lauritzen (2010)

Work in progress (exploratory research)

- Couple CSLAM with ‘spectral-element’ based grids in NCAR’s High-Order Methods Modeling Environment (HOMME).
- HOMME based on 3^{rd}-order Discontinuous Galerkin (DG) method is limited to CFL$<$0.2 for stability (explicit time-stepping).
- Using (FF-)CSLAM with super-cycling of tracers could potentially provide a major speed-up for applications with many tracers





- Prepare HOMME for ultra-high (regional) resolution: Non-hydrostatic extension based on HWENO-based DG methods using compressible Euler equations
- Goal: Perform standard two-dimensional non-hydrostatic tests

References I

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