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NCAR NATIONAL CENTER FOR ATMOSPHERIC RESEARCH

A standard test case suite for 2D linear transport on the sphere

Peter Hjort Lauritzen (NCAR), W.C. Skamarock (NCAR) ,

M.J. Prather (U.C. Irvine), and M.A. Taylor (SNL)

Solution of Partial Differential Equations on the Sphere
Cambridge (UK), September 24-28, 2012

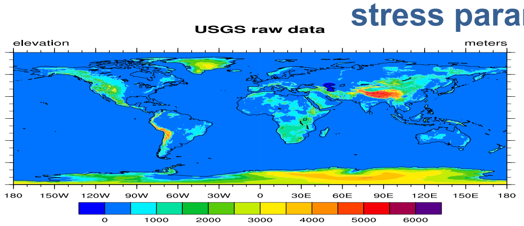
NCAR Earth System Laboratory NESL

New topography generation software

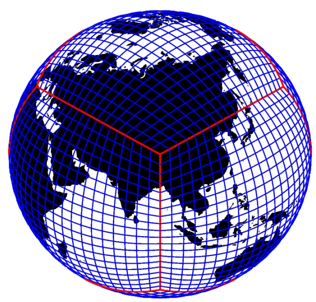
(to be released with NCARs CAM5.2 in November, 2012)

USGS raw data (~1km resolution)

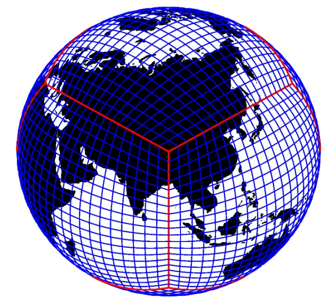
Binning
(compute
sub-grid-scale
variance for turbulent mountain
stress parameterizations)

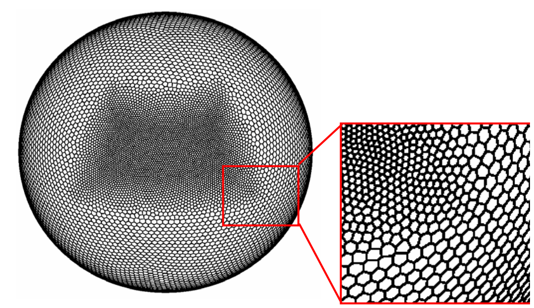


gnomonic cubed-sphere (~3km resolution)



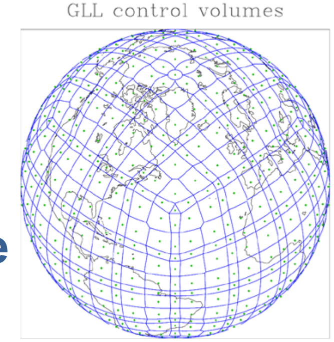
Target grid





(Michael Duda, MMM)

GLL control volumes



Rigorous remapping
(compute sub-grid-scale
variance for gravity wave
parameterizations)

A standard test case suite for 2D linear transport on the sphere

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NCAR Earth System Laboratory

Most widely used test case in the literature (global models) ?

A Standard Test Set for Numerical Approximations to the Shallow Water Equations in Spherical Geometry

DAVID L. WILLIAMSON

The National Center for Atmospheric Research, Boulder, Colorado 80307

JOHN B. DRAKE

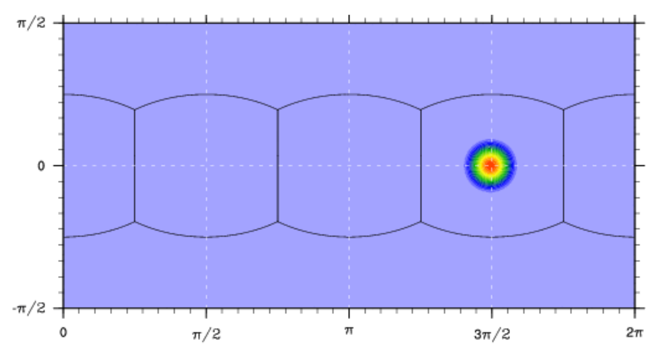
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831

AND

JAMES J. HACK, RÜDIGER JAKOB, AND PAUL N. SWARZTRAUBER

The National Center for Atmospheric Research, Boulder, Colorado 80307

Received June 17, 1991



Test 1: Solid-body advection

Most widely used test case in the literature (global models) ?

Test 1: Solid-body advection

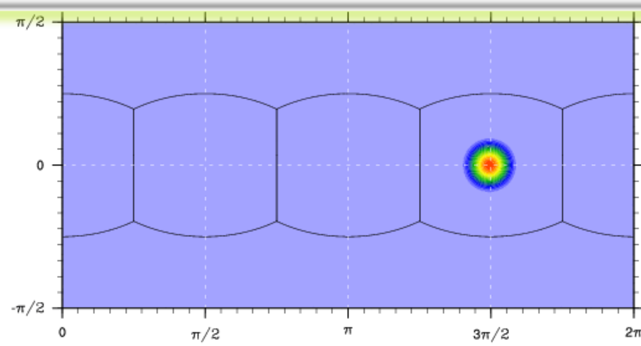
Most widely used test case in the literature (global models) ?

- No deformation only translation:

  -> Flow does not force tracer features to collapse to the grid scale (as it does in nature)

  Note: More recently, Nair and Machenhauer (2002) and Nair and Jablonowski (2008) introduced a highly deformational flow field ("Moving vortices" test case)

- Parcel trajectories are trivial
- No divergence/convergence (see next slide)



Test 1: Solid-body advection

Most widely used test case in the literature (global models) ?

A Standard Test Set for Numerical Approximations to the Shallow Water Equations in Spherical Geometry

No divergence/convergence
$\rightarrow$ modelers basing their schemes on the flux-form of the continuity equations

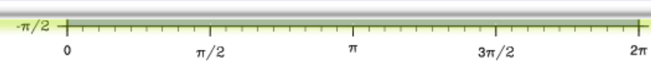
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\vec{v}\rho) = 0, \tag{2}$$

$$\frac{\partial(\rho\,\phi)}{\partial t} + \nabla \cdot (\vec{v}\rho\,\phi) = 0, \tag{3}$$

are not forced to distinguish between $\rho\,\phi$ and ϕ since

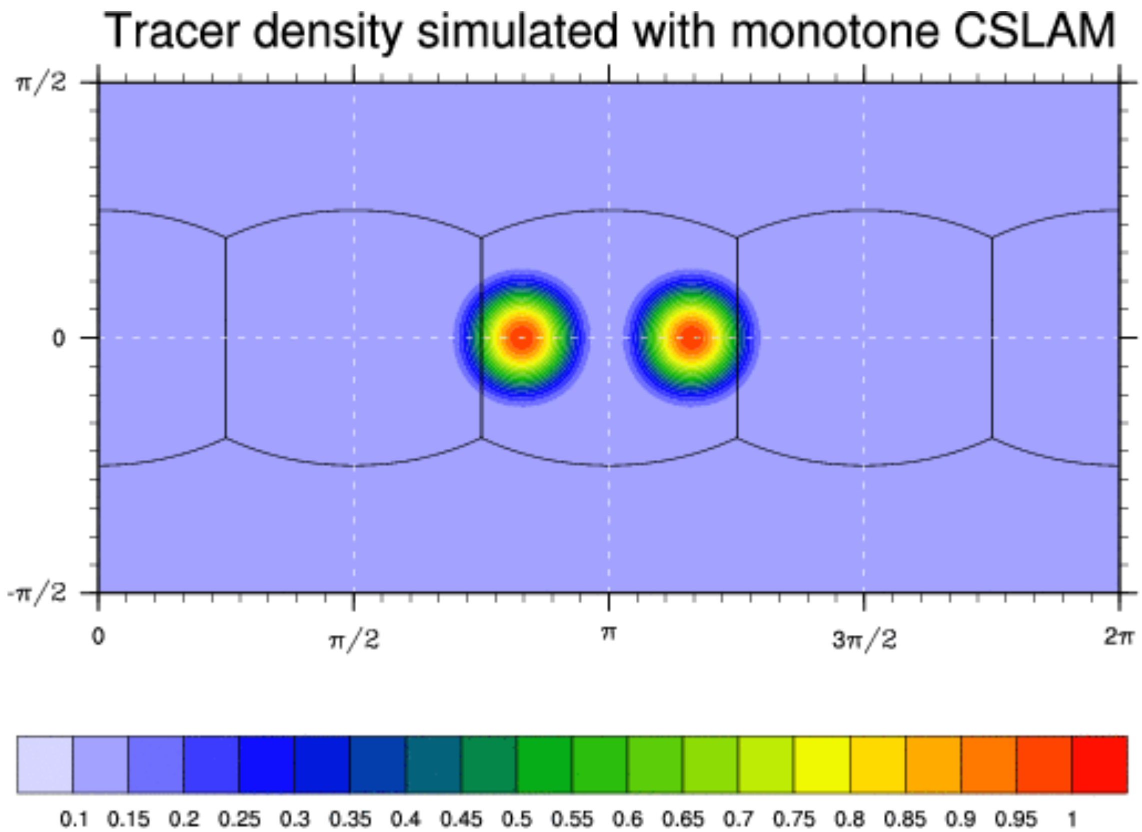
$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\vec{v}\phi) = \phi \nabla \cdot \vec{v} = 0, \tag{4}$$

for non-divergent flow $\nabla \cdot \vec{v} = 0$, that is, (3) and (4) take the same functional form.



Test 1: Solid-body advection

Nair and Lauritzen (2010, J. Comput. Phys.) flow field



CSLAM = Conservative Semi-LAgrangian Multi-tracer scheme

Lauritzen et al. (2010,JCP), Harris et al. (2011), Lauritzen et al. (2011,JCP)

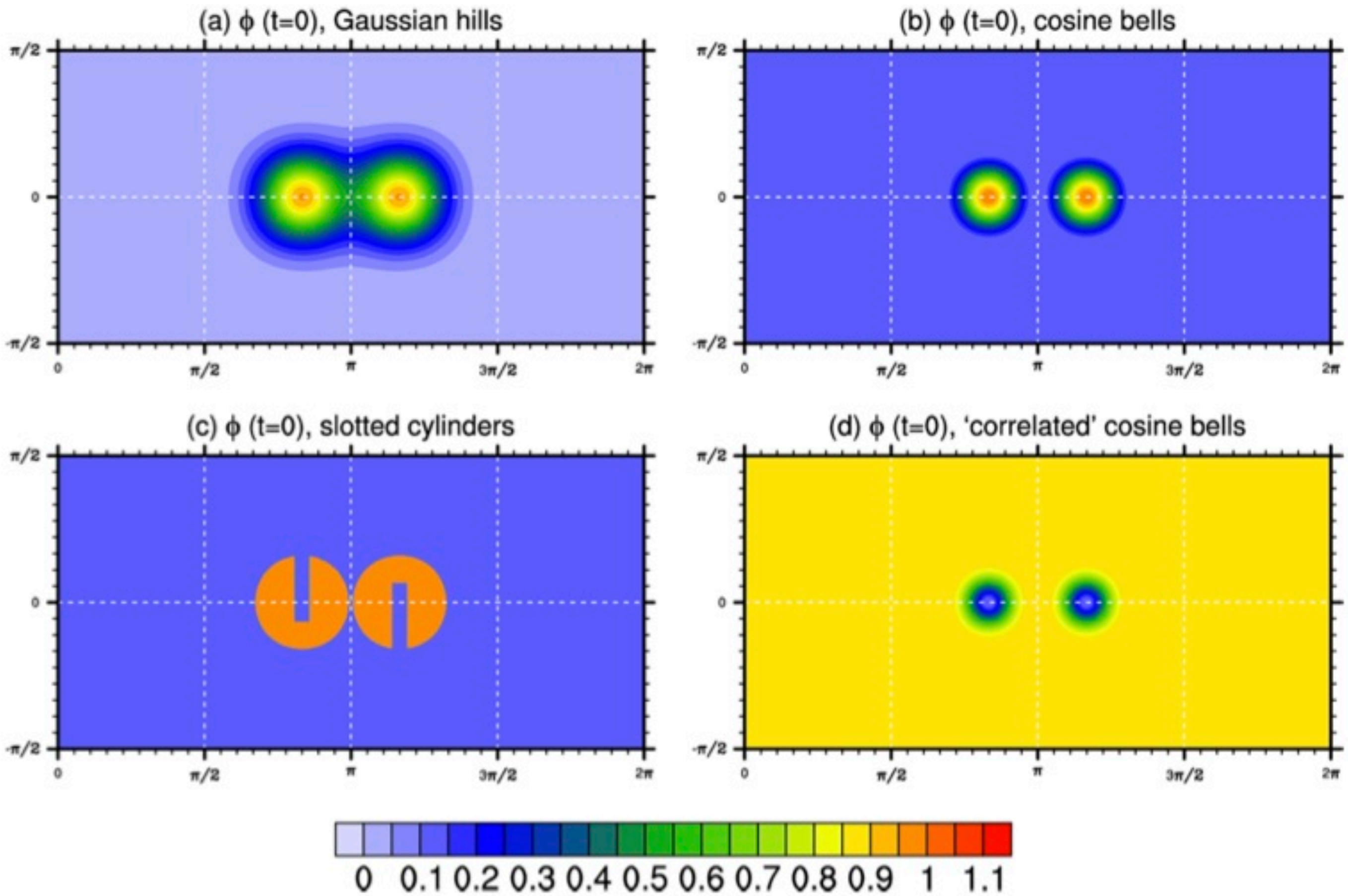
Design objectives

Facilitate scheme intercomparison (model development)
(specific guidelines on resolution, plotting, test case configuration)

Assess important aspects of accuracy in geophysical fluid dynamics using a “minimal” test case suite

Keep things simple !!!!

Only 2 analytical wind fields and 4 initial conditions – the rest is diagnostics!
(almost any test case suite could be extended to include more tests that could provide more insights into specific aspects of accuracy particularly useful for some classes of schemes and applications)


(a) ϕ (t=0), Gaussian hills
(b) ϕ (t=0), cosine bells
(c) ϕ (t=0), slotted cylinders
(d) ϕ (t=0), 'correlated' cosine bells
π/2
0
-π/2
0
π/2
π
3π/2
2π
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1

"Minimal" test case suite

Passive & inert idealized 2D transport test cases designed to assess:

1. Numerical order of convergence (C$^\infty$ initial conditions) – Δx in [0.3^o ,3^o]
2. "Minimal" resolution (C^1 initial conditions)
3. Ability of transport scheme to preserve filaments
4. Ability of transport scheme to transport "rough" distributions
5. Ability of the transport scheme to preserve pre-existing functional relations between species (e.g., N_2O-NO_y, family of species, ...)

under challenging flow conditions

$$u(\lambda, \theta, t) = \kappa \sin^2(\lambda') \sin(2\theta) \cos(\pi t/T) + 2\pi \cos(\theta)/T$$
$$v(\lambda, \theta, t) = \kappa \sin(2\lambda') \cos(\theta) \cos(\pi t/T),$$

(Nair and Lauritzen, 2010, J. Comput. Phys.).

6. Transport under divergent flow conditions (forces modelers to consider coupling between air and tracer mass; at least for finite-volume based schemes)

NCAR Workshop (March, 2011)

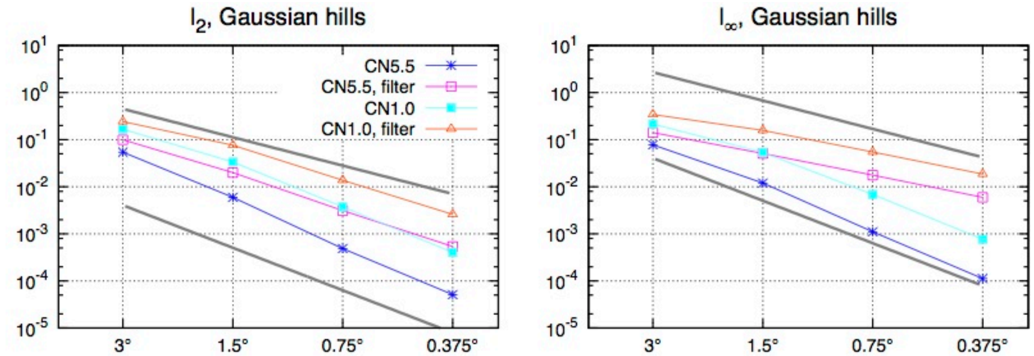
community asked to bring solutions to new test suite

Table 1. A list of acronyms (first column), full names (second column), primary reference (third column), and implementation grid (fourth column) for schemes used in this paper.

scheme acronym	full scheme name	primary reference	implementation grid
CAM-FV	Community Atmosphere Model - Finite Volume scheme	Lin and Rood (1996)	Regular latitude-longitude
CLAW	Wave propagation algorithm on mapped grids	LeVeque (2002)	two-patch sphere grid
CSLAM	Conservative Semi-LAgrangian Multi-tracer scheme	Lauritzen et al. (2010)	Gnomonic cubed-sphere
FARSIGHT	Departure-point interpolation scheme with a global mass fixer	White and Dongarra (2011)	Gnomonic cubed-sphere
HEL	Hybrid Eulerian Lagrangian	Kaas et al. (2012)	Gnomonic cubed-sphere
HEL-ND	HEL - Non-Diffusive	Kaas et al. (2012)	Gnomonic cubed-sphere
HOMME	High-Order Methods Modeling Environment	Dennis et al. (2012)	Gnomonic cubed-sphere (quadrature grid)
ICON-FFSL	ICON - Flux-Form semi-Lagrangian scheme	Miura (2007)	Icosahedral-triangular
LPM	Lagrangian Particle Method	Bosler (2013, in prep)	Icosahedral-triangular
MPAS	Model for Prediction Across Scales	Skamarock and Gassmann (2011)	Icosahedral-hexagonal
SBC	Spectral Bicubic interpolation scheme	Enomoto (2008)	Gaussian latitude-longitude
SFF-CSLAM	Simplified Flux-Form CSLAM scheme	Ullrich et al. (2012)	Gnomonic cubed-sphere
SLFV-SL	Semi-Lagrangian type Slope Limited	Miura (2007)	Icosahedral hexagonal
SLFV-ML	Slope Limited Finite Volume scheme with method of lines	Dubey et al. (2012)	Icosahedral hexagonal grid
TTS	Trajectory–Tracking Scheme	Dong and Wang (2012b)	Spherical Centroidal Voronoi Tessellation
UCISOM	UC Irvine second-order moments scheme	Prather (1986)	Regular latitude-longitude
UCISOM-CS	UC Irvine second-order moments scheme	-	Gnomonic cubed-sphere

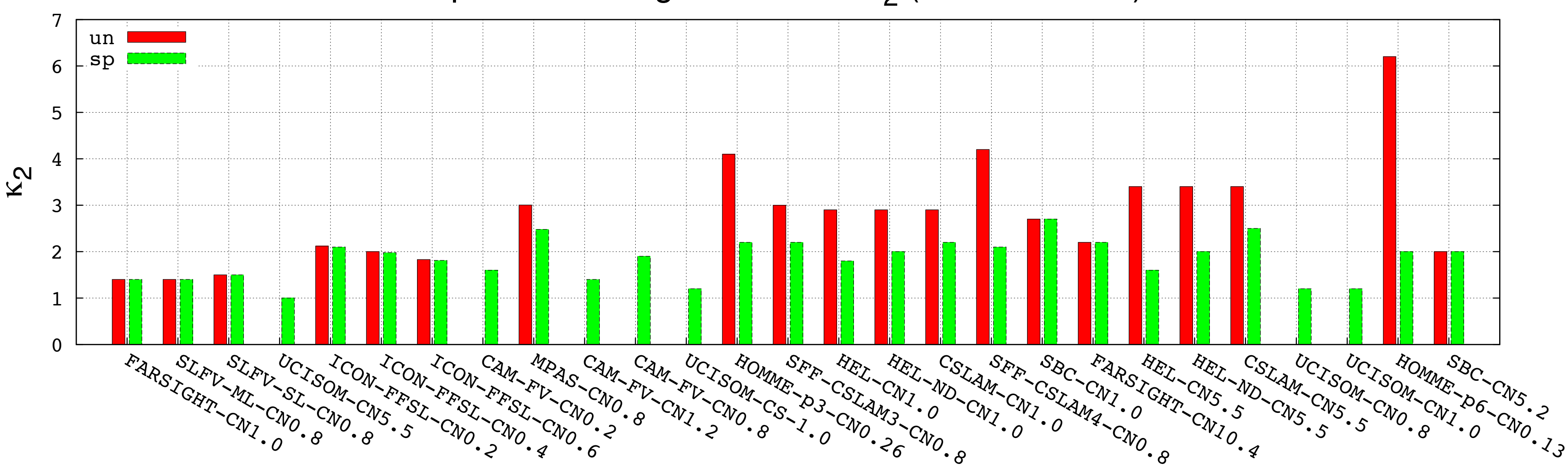
Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

1. Numerical convergence rate



un = unlimited scheme   sp = shape-preserving version of scheme

optimal convergence rate for l_2 (Gaussian hills)

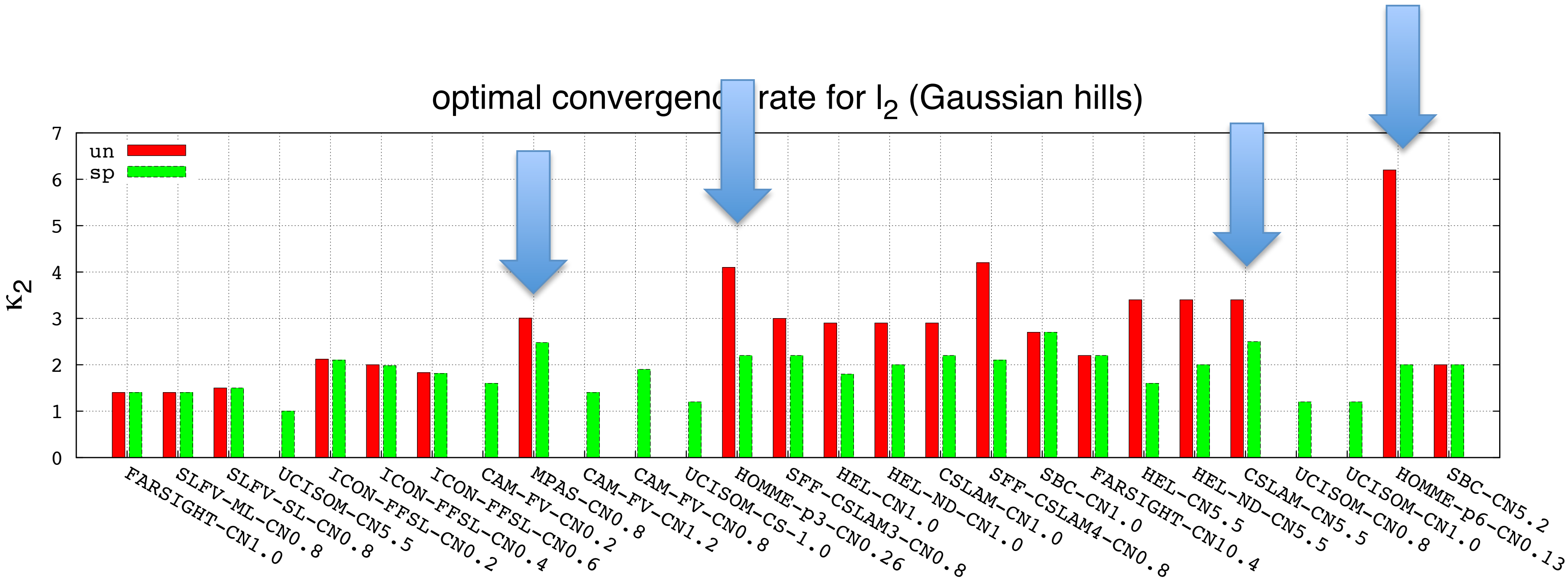


CN = Courant Number

Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

1. Numerical convergence rate

in the resolution range approximately 3° to 0.3° (i.e. from paleo to high resolution climate modeling)



- Not surprisingly shape-preserving filters/limiters reduce order of convergence
- Some shape-preserving filters/limiters are more “invasive” than others

Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

2. "Minimal" resolution

(absolute error)

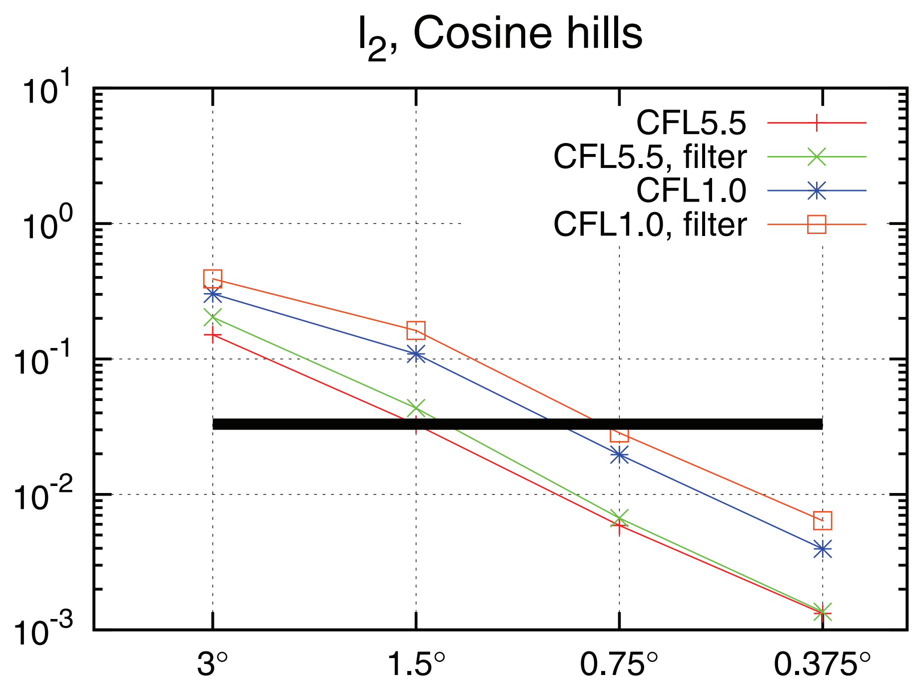
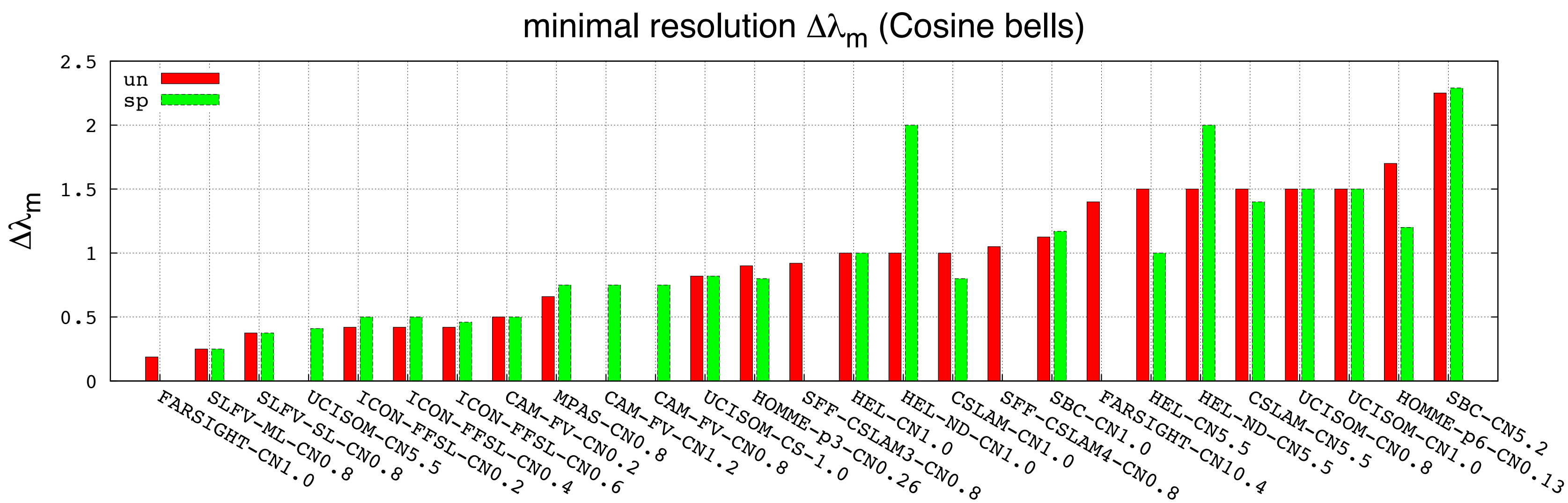


Fig. 5. Convergence plot for ℓ_2 computed with CSLAM with cosine bells initial conditions. The keys are as in Fig. 4. The heavy line is $\ell_2 = 0.033$ and is used to define "minimal" resolution.

Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

2. "Minimal" resolution

(absolute error)



Minimal resolution varies from 0.2° to 2.3°!

Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

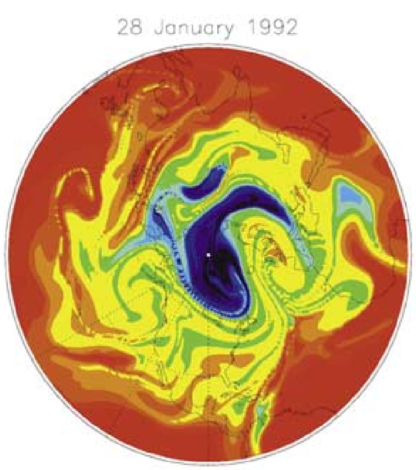
4. Preserving correlations

(the design of schemes that preserve linear correlations was discussed by Lin and Rood (1996) and Thuburn and McIntyre (1997))

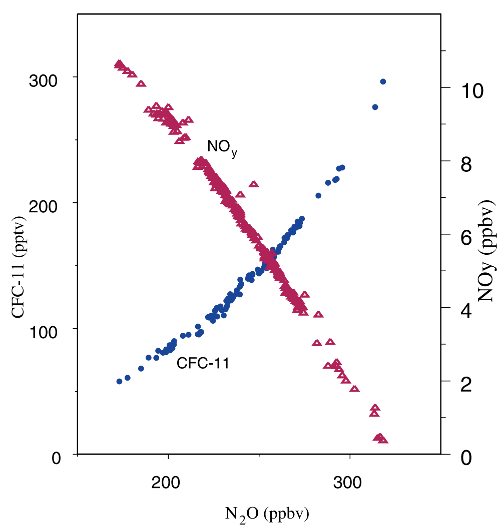
Motivation: Correlations between long-lived species in the stratosphere

Relationships between long-lived stratospheric tracers, manifested in similar spatial structures on scales ranging from a few to several thousand kilometers, are displayed most strikingly if the mixing ratio of one is plotted against another, when the data collapse onto remarkably compact curves. - Plumb (2007)

E.g., when plotting nitrous oxide (N_2O) against 'total odd nitrogen' (NO_y) or chlorofluorocarbon (CFC's)



'inverse filling'



observations

4. Preserving correlations

Motivation: Correlations between long-lived species in the stratosphere

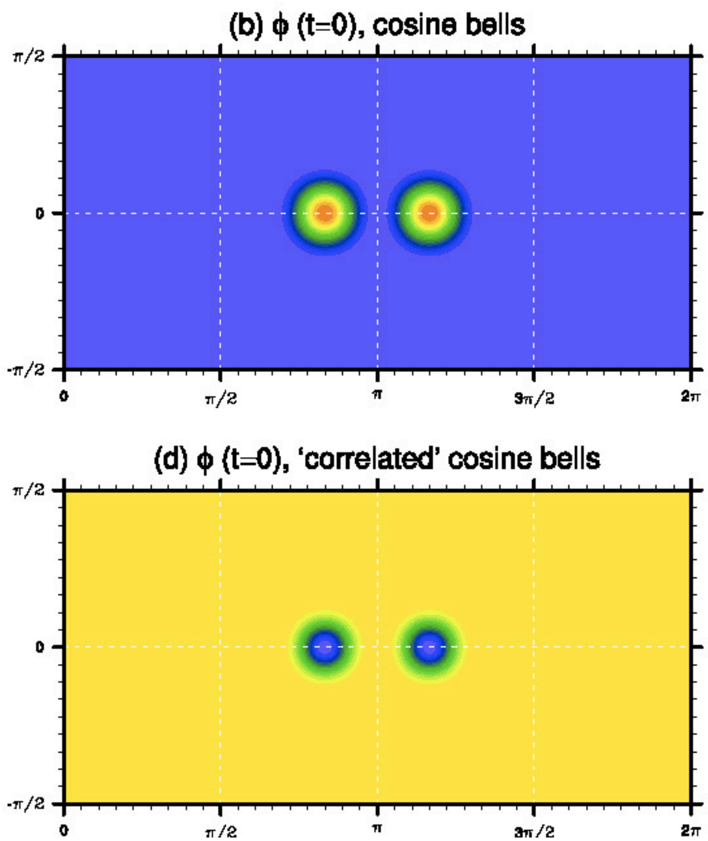
- Such compact scatter plots can be physically or chemically significant; for example, departures from compactness have been used to quantify chemical ozone loss in the ozone hole (Proffitt et al., 1990).
- → It is therefore highly desirable that transport schemes used in modeling the atmosphere should respect such functional relations and not disrupt them in physically unrealistic ways.

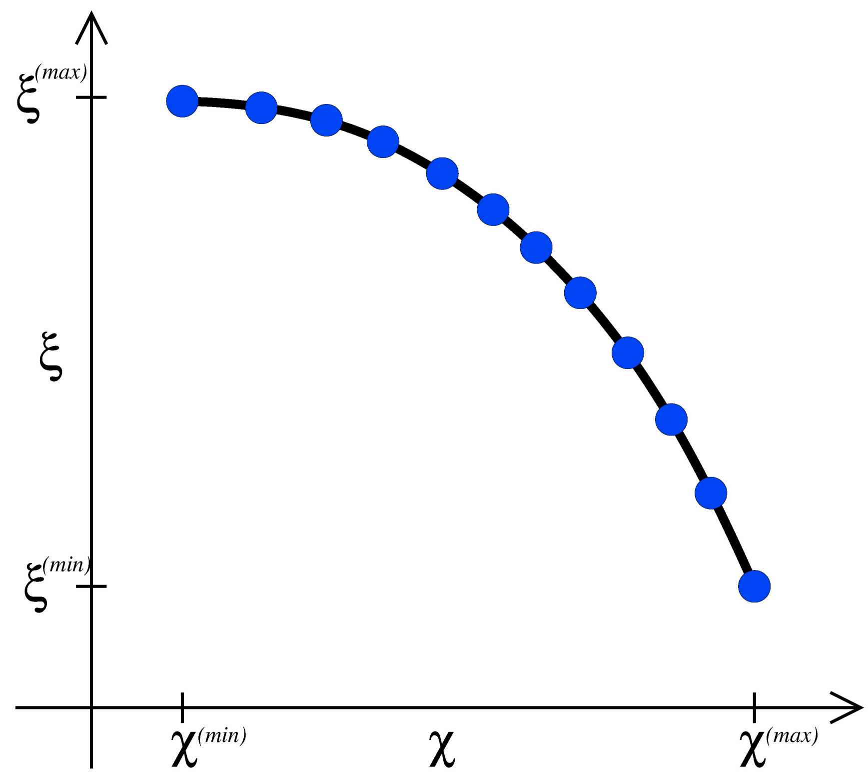
- Similarly, the total of chemical species within some chemical family may be preserved following an air parcel although the individual species have a complicated relation to each other and may be transformed into each other through chemical reactions.
- Similar arguments can be made for aerosol-cloud interactions (Ovtchinnikov and Easter, 2009) where important physical properties are derived from several tracers.

Goal: design idealized test case suite to address some of these aspects of accuracy!

4. Preserving correlations

Initial conditions
tracer 1: cosine bells   tracer 2: correlated cosine bells $\Psi(\chi)=a\chi^2+b$





Lauritzen and Thuburn (2012,QJRMS)

4. Preserving correlations

Initial conditions
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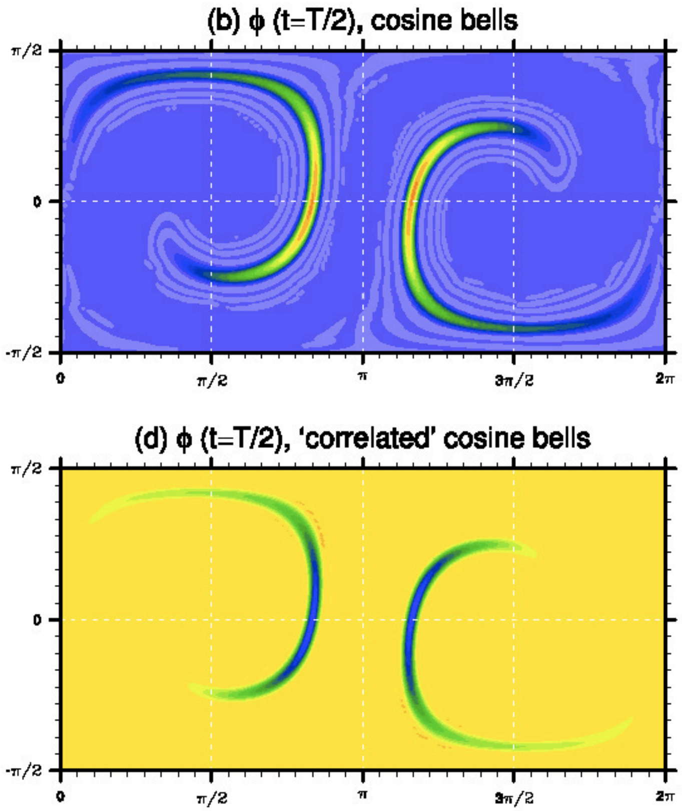
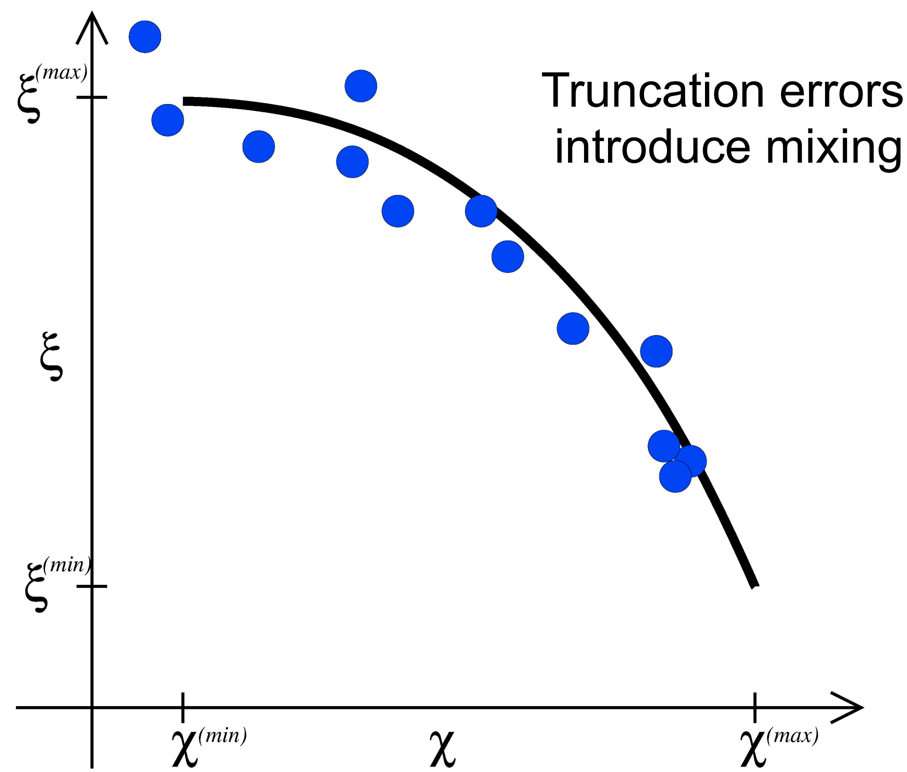


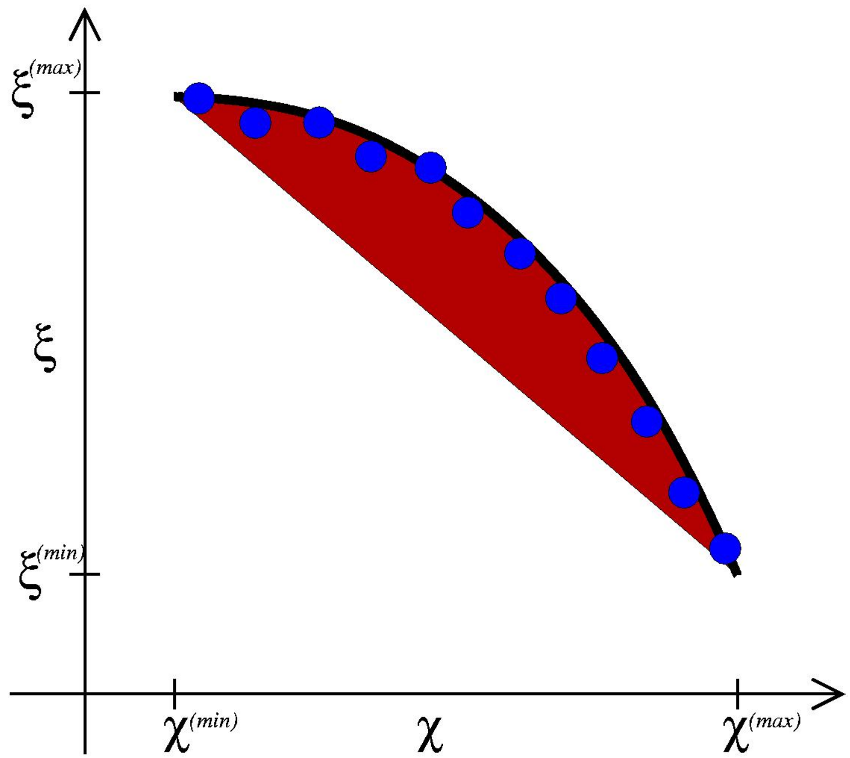


Lauritzen and Thuburn (2012,QJRMS)

4. Preserving correlations

Classification of mixing on scatter plot:

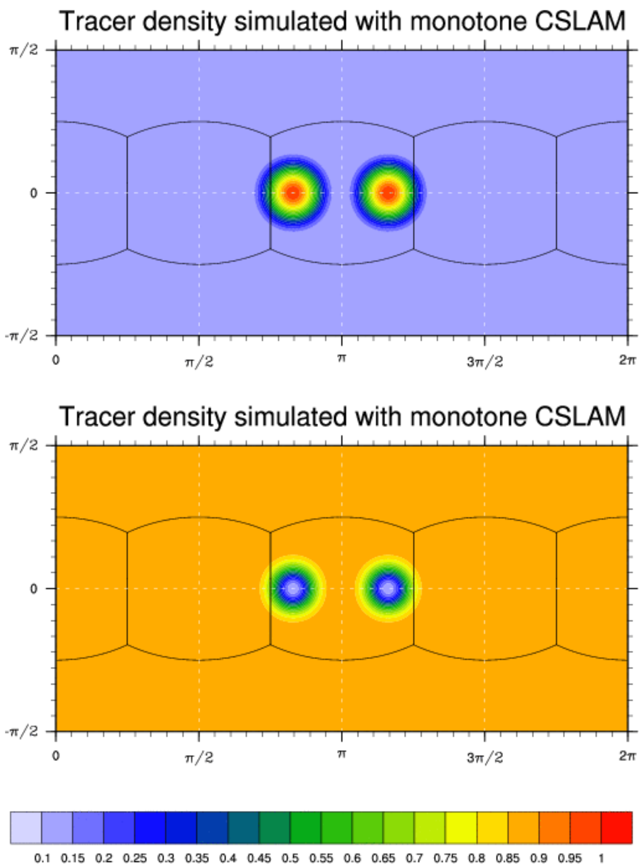
a. Mixing that resembles `real' mixing – convex hull (red area)
b. Everything else is spurious unmixing

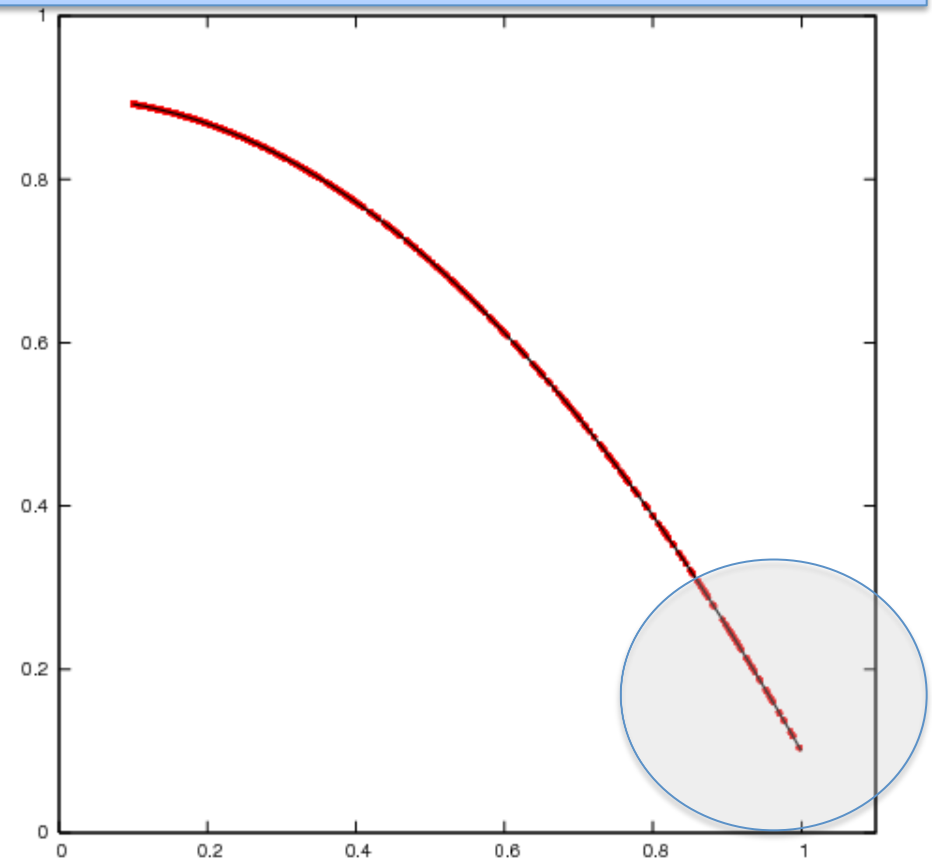


Thuburn and McIntyre (1997,JGR)

4. Preserving pre-existing functional relation between tracers under challenging flow conditions

Note: 1. **Max value decrease**, 2. Unmixing even if scheme is shape-preserving, 3. No expanding range unmixing





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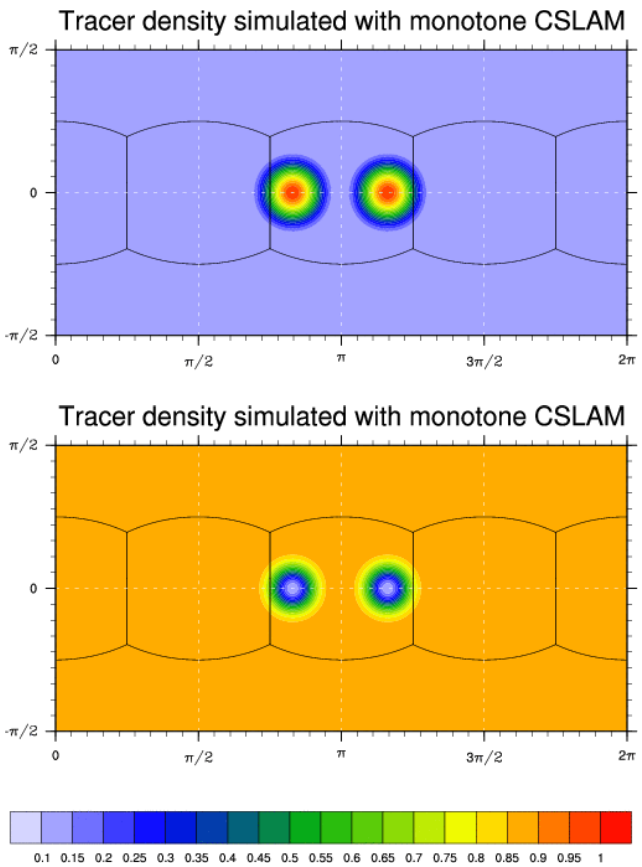

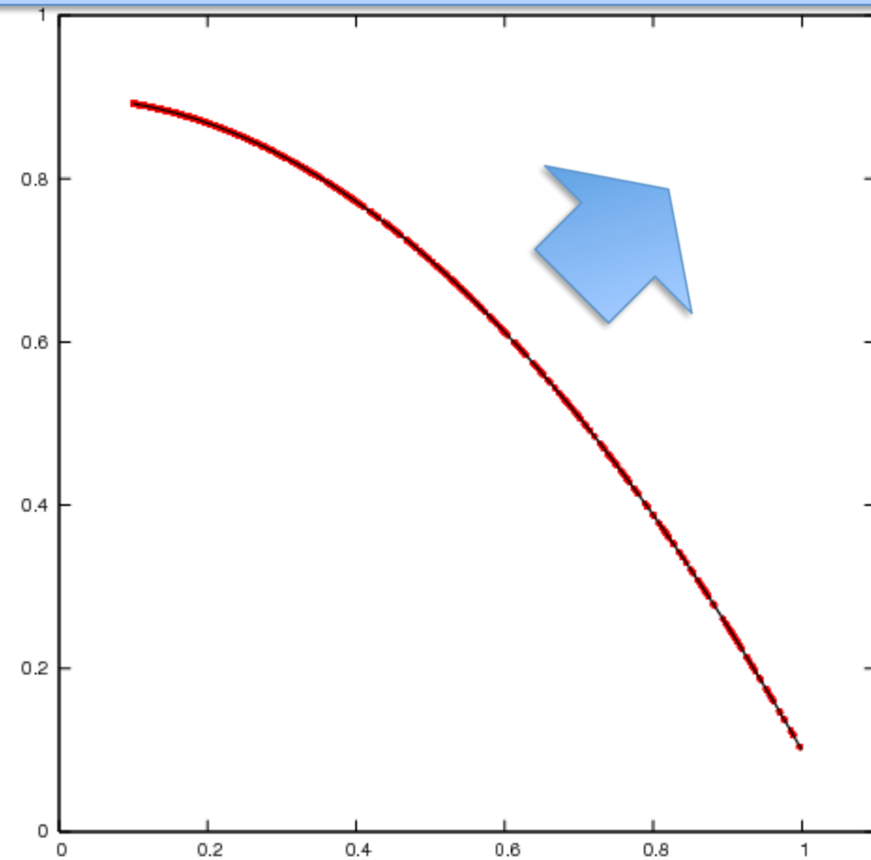

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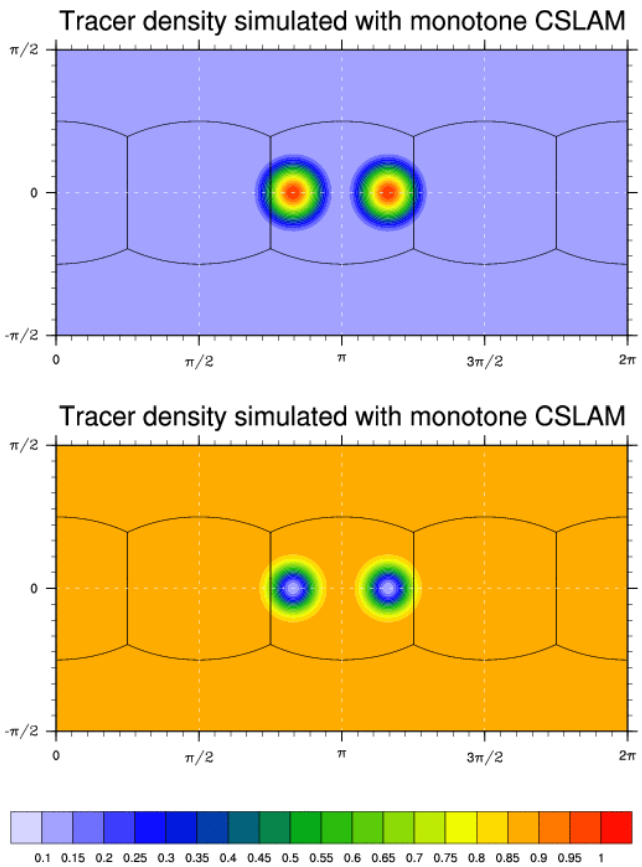
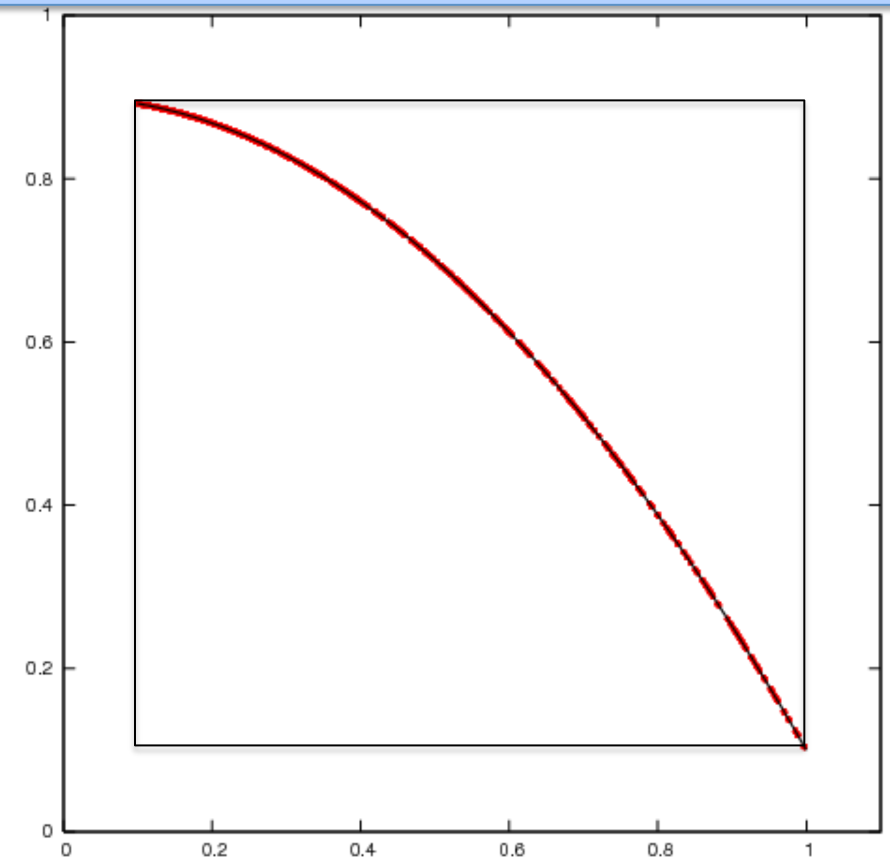
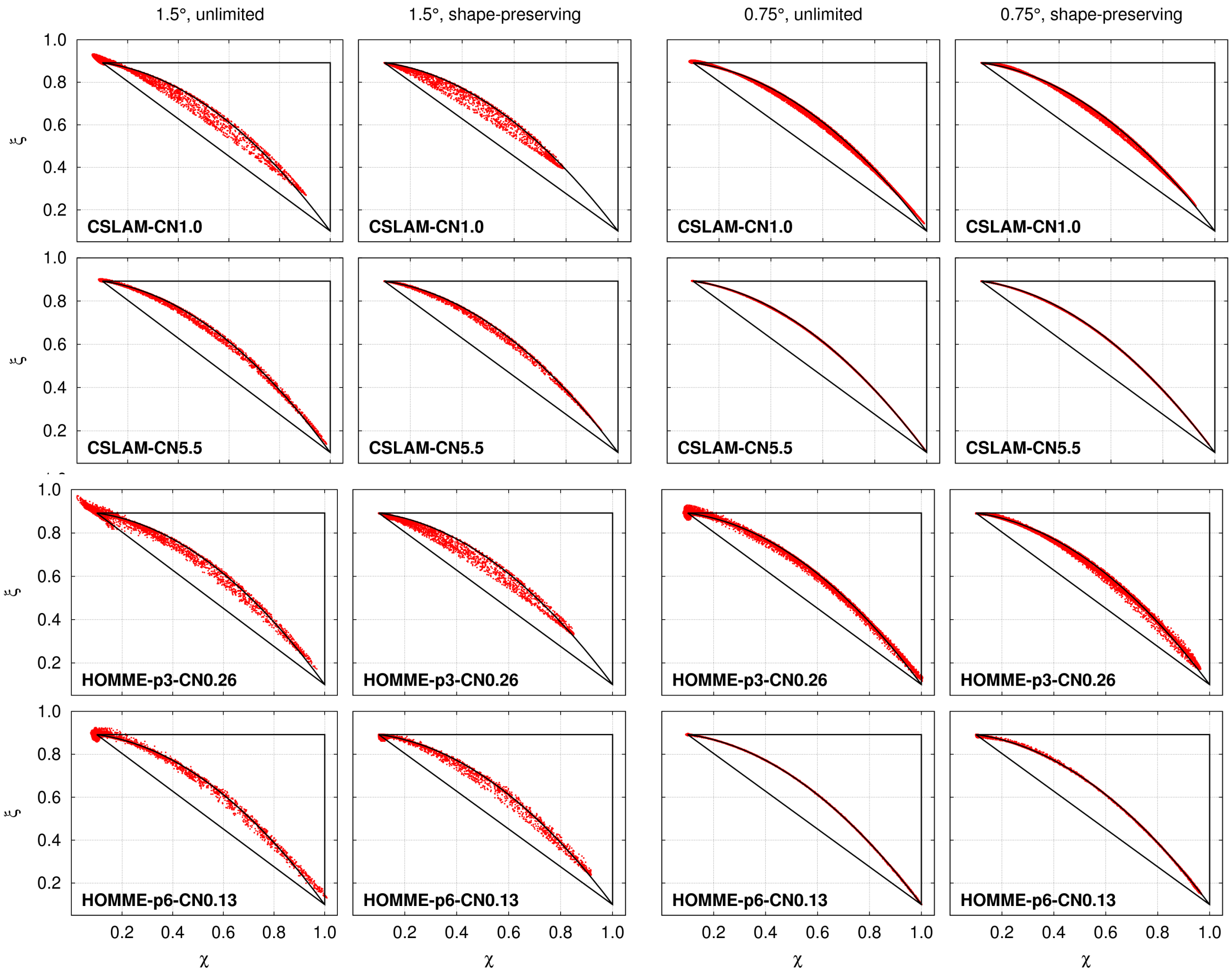


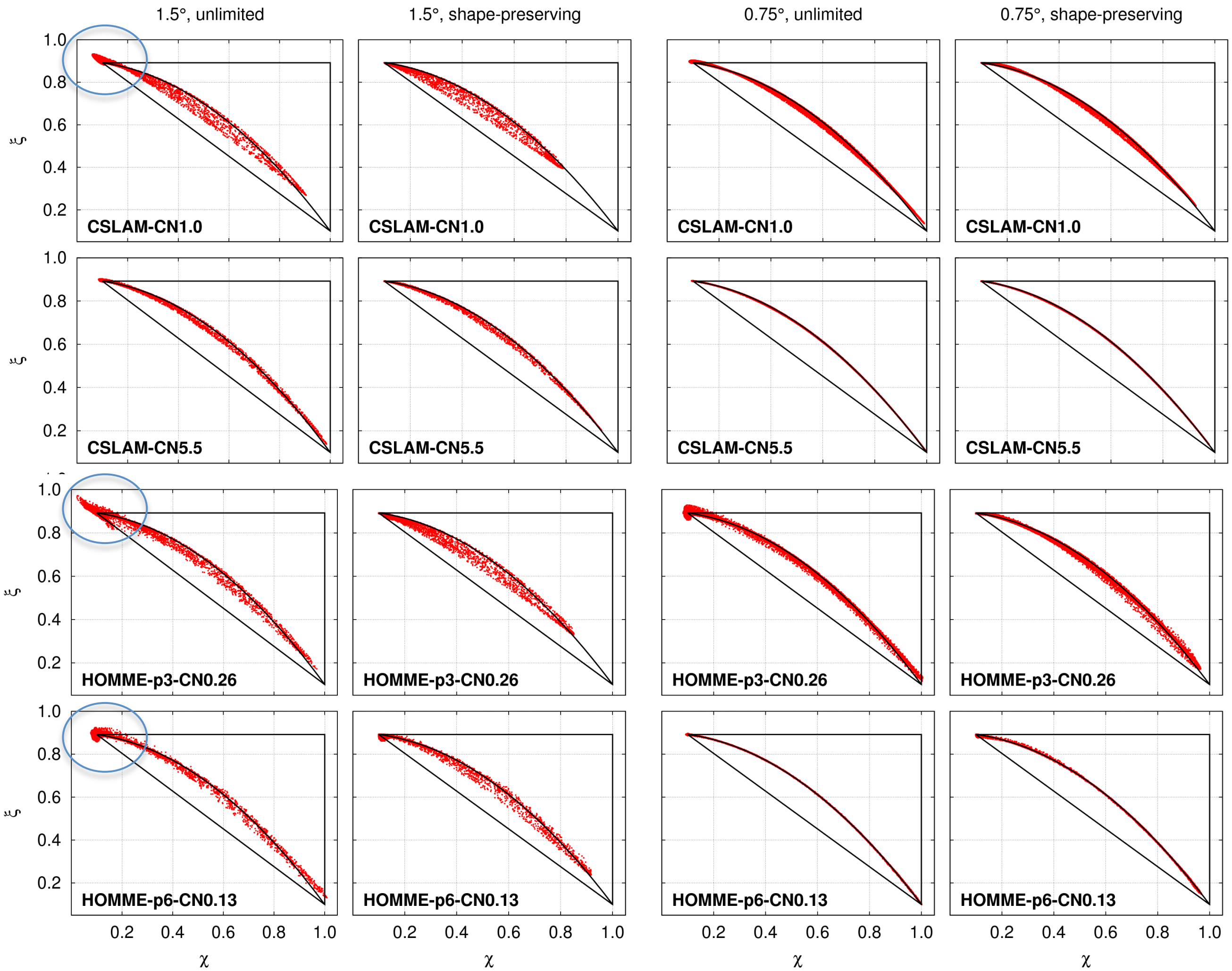


Cubed-sphere models



Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

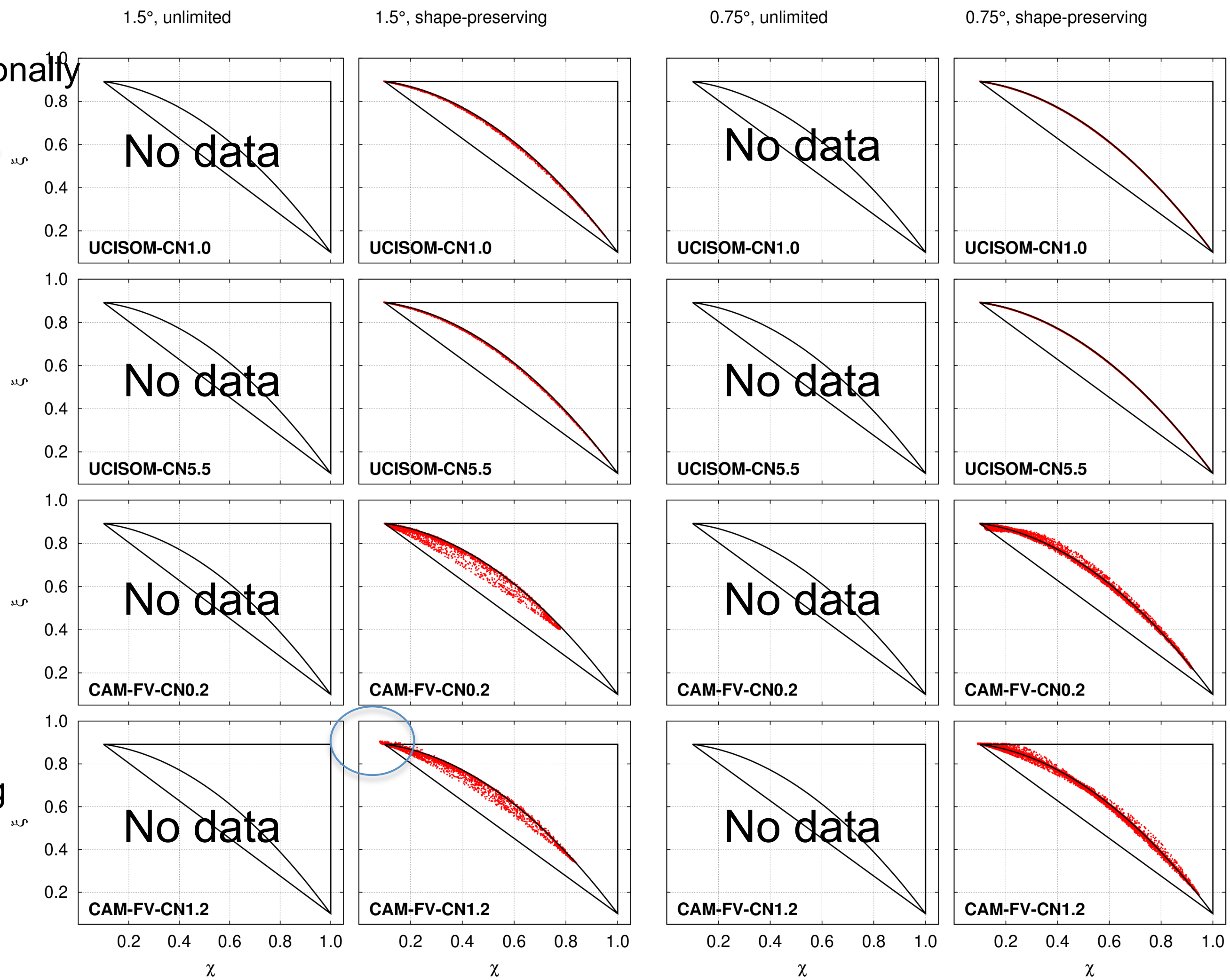
Cubed-sphere models



Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

Reg. lat-lon models

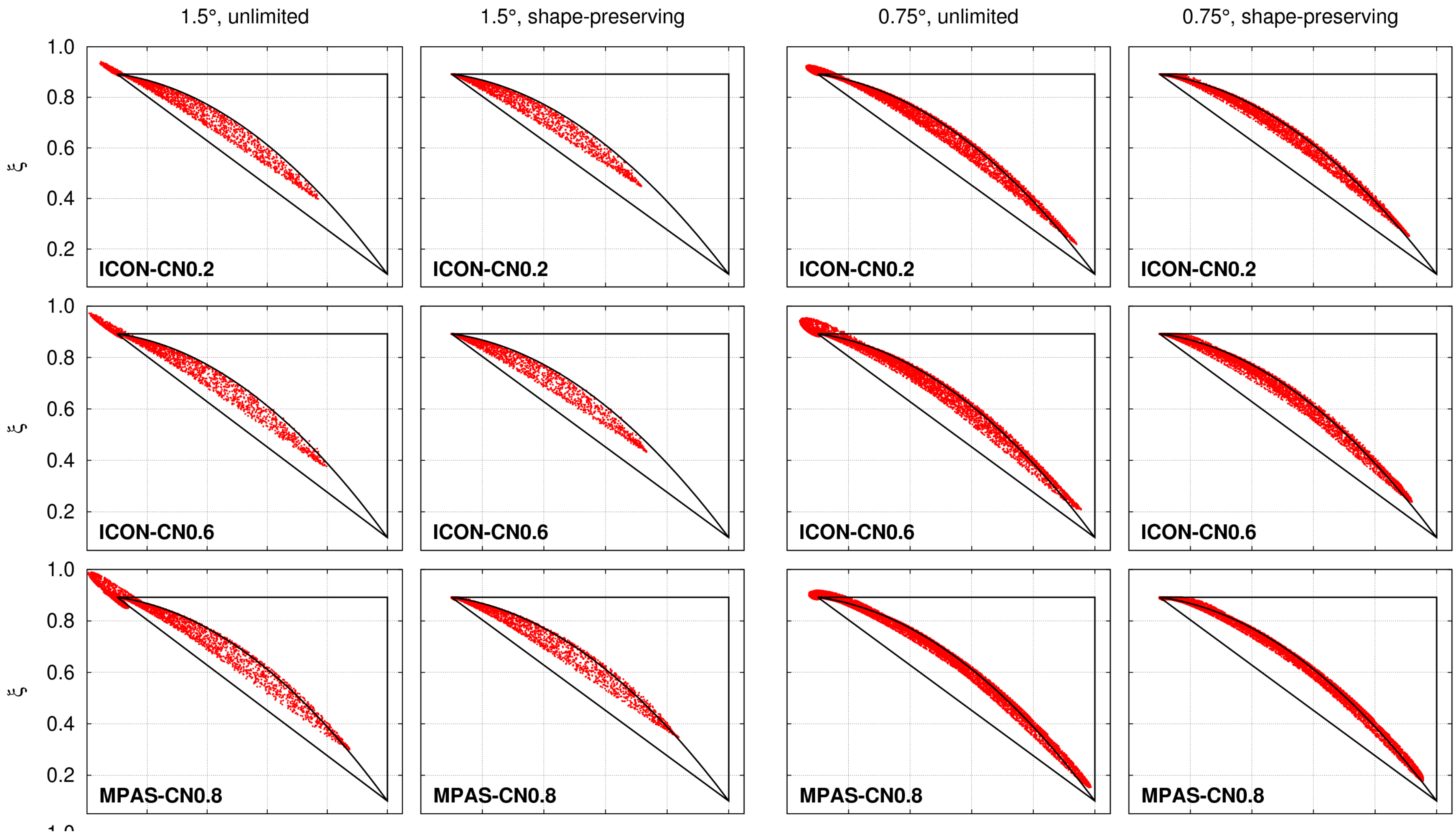
Prather scheme performs exceptionally well



Shape-preserving scheme overshoots!

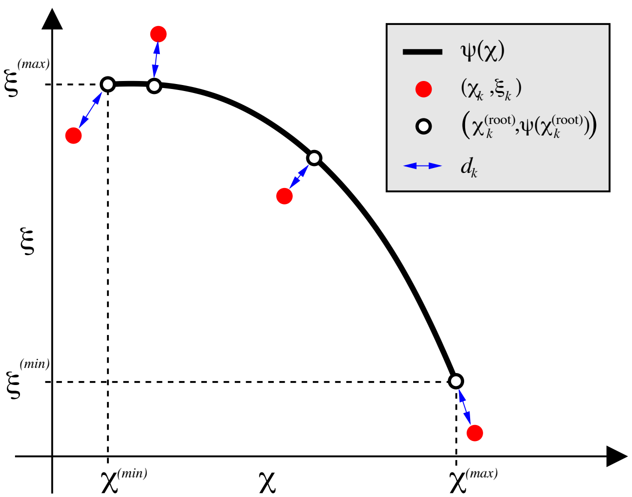
Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

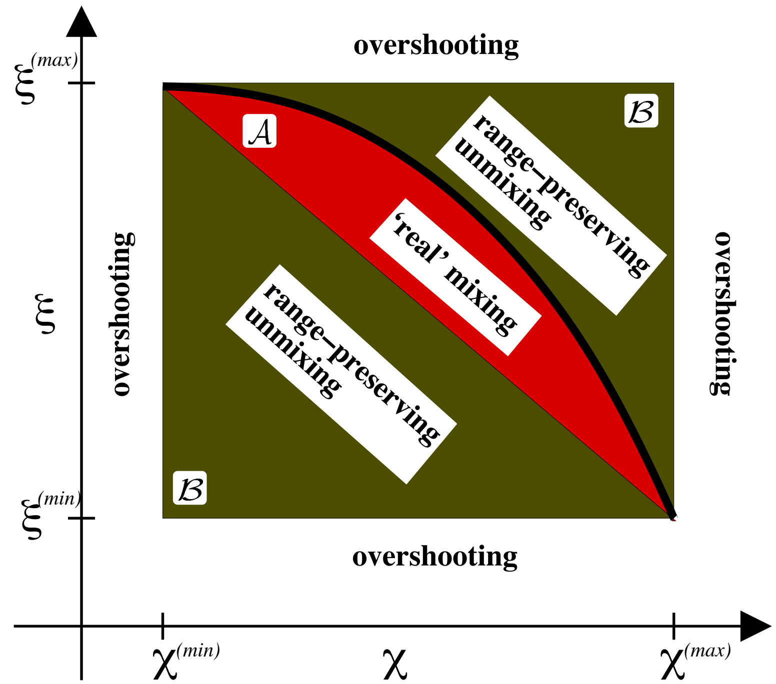
Icosahedral/Voronoi models



Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

Quantifying mixing





$$\ell_r = \frac{1}{A}\sum_{k=1}^{K}\begin{cases} d_k\,\Delta A_k, & \text{if } (\chi_k,\xi_k)\in\mathcal{A}, \\ 0, & \text{else,}\end{cases} \qquad \text{(C1)}$$

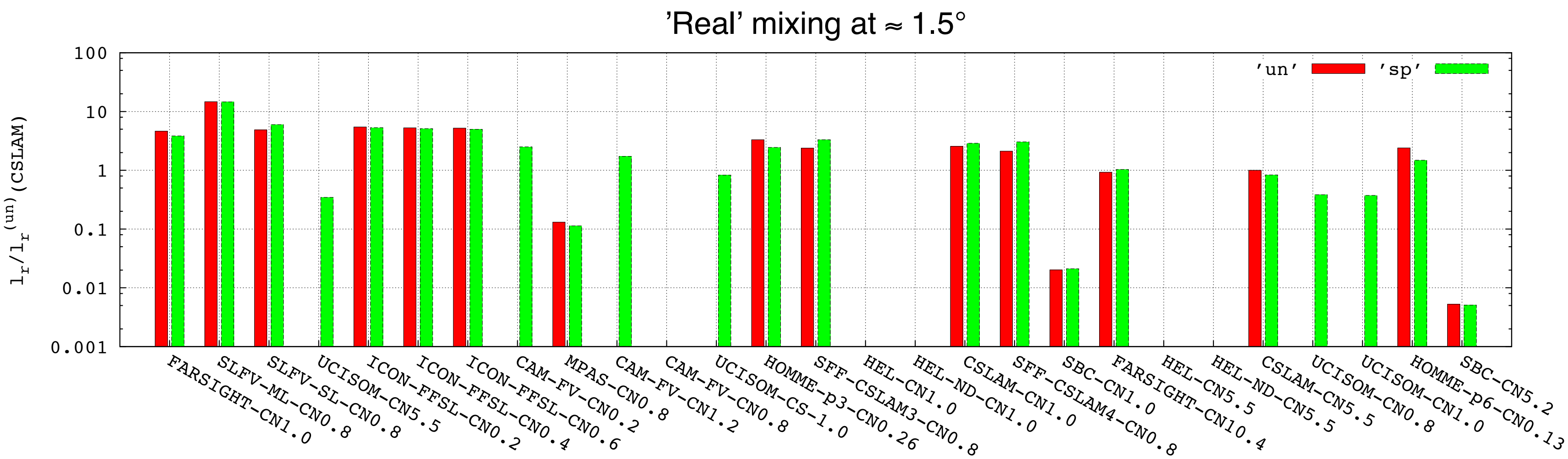
where K is the total numbers of cells/points in the domain, ΔA_k is the spherical area of grid cell k and A is the total area of the domain, $A = \sum_{k=1}^{K} \Delta A_k$. The distance function d_k is the shortest normalized distance between the numerically computed scatter point (χ_k, ξ_k) and the preexisting functional curve within the range of the initial conditions.

This diagnostic does not rely on an analytical solution!

Lauritzen and Thuburn (2012, QJRMS)

Quantifying “real” mixing

un = unlimited scheme   sp = shape-preserving version of scheme



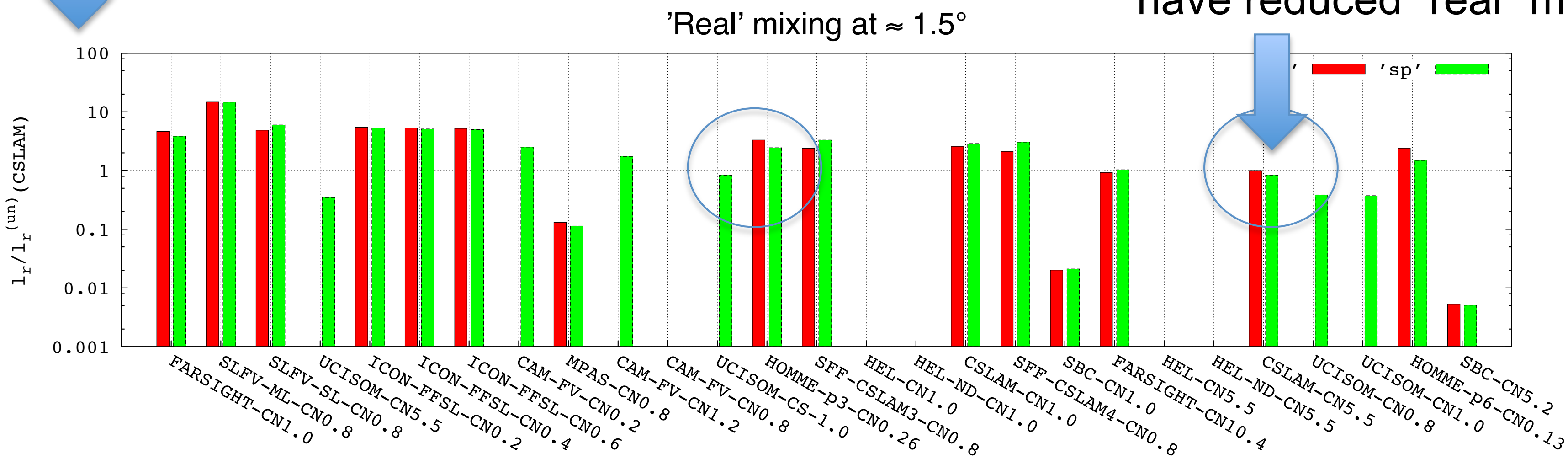
Y-axis: Normalized by CSLAM unlimited l_r at 1.5°

Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

Quantifying "real" mixing

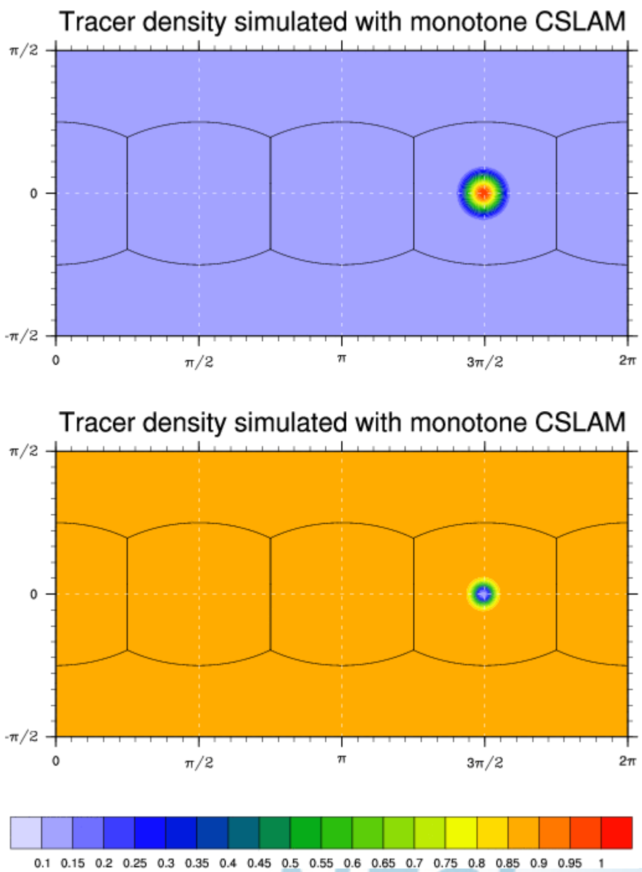
Log-scale!

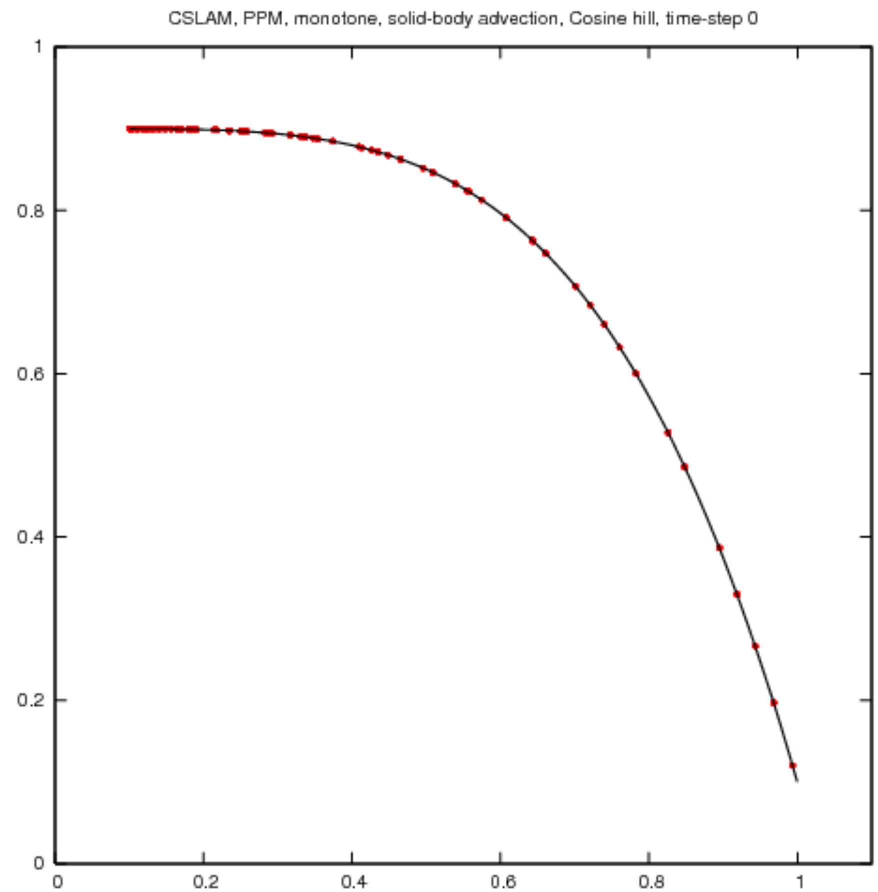
Shape-preserving schemes have reduced "real" mixing



Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

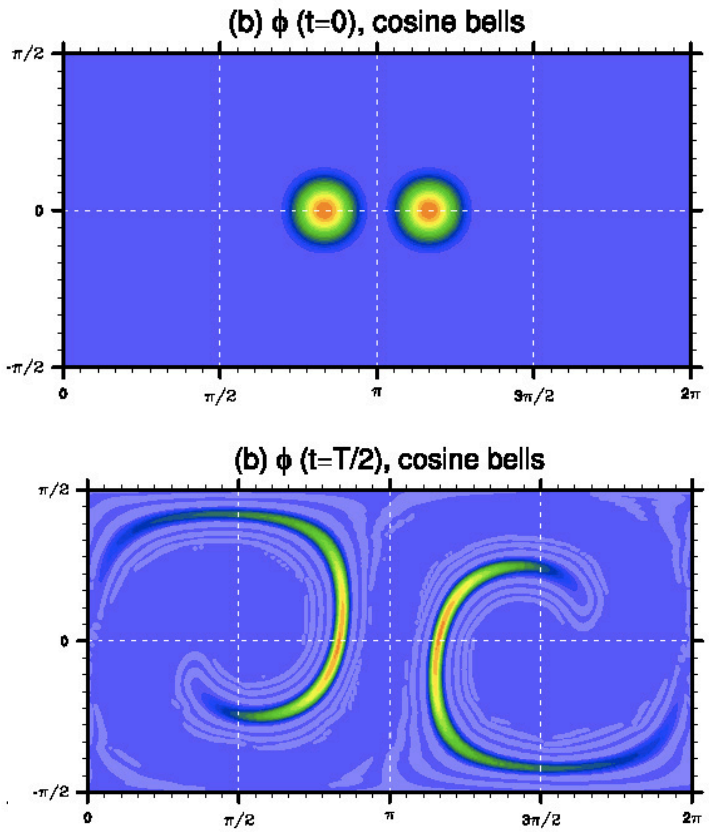
It is key that tracer features collapse to smaller scales (as in nature)





This setup uses a 4th-order non-linear relation $\Psi(\chi)=a\chi^4+b$

3. Filament diagnostic (Prather)



The "filament" preservation diagnostic is formulated as follows. Define $A(\tau,t)$ as the spherical area for which the spatial distribution of the tracer $\phi(\lambda,\theta)$ satisfies

$$\phi(\lambda,\theta) \geq \tau, \tag{27}$$

at time t, where τ is the threshold value. For a non-divergent flow field and a passive and inert tracer ϕ, the area $A(\tau,t)$ is invariant in time.

The discrete definition of $A(\tau,t)$ is

$$A(\tau,t) = \sum_{k\in\mathcal{G}} \Delta A_k, \tag{28}$$

where ΔA_k is the spherical area for which ϕ_k is representative, K is the number of grid cells, and $\mathcal{G}$ is the set of indices

$$\mathcal{G} = \{k \in (1,\ldots,K) | \phi_k \geq \tau\}. \tag{29}$$

For Eulerian finite-volume schemes ΔA_k is the area of the k-th control volume. For Eulerian grid-point schemes a control volume for which the grid-point value is representative must be defined. Similarly for fully Lagrangian schemes based on point values (parcels) control volumes for which the point values are representative must be defined. Note that the "control volumes" should span the entire domain without overlaps or "cracks" between them.

Define the filament preservation diagnostic

$$\ell_f(\tau,t) = \begin{cases} 100.0 \times \frac{A(\tau,t)}{A(\tau,t=0)} & \text{if } A(\tau,t=0) \neq 0, \\ 0.0, & \text{otherwise}. \end{cases} \tag{30}$$

For infinite resolution (continuous case) and a non-divergent flow, $\ell_f(\tau,t)$ is invariant in time: $\ell_f(\tau,t=0) = \ell_f(\tau,t) = 100$ for all τ. At finite resolution, however, the filament

This diagnostic does not rely on an analytical solution!

Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

3. Filament diagnostic

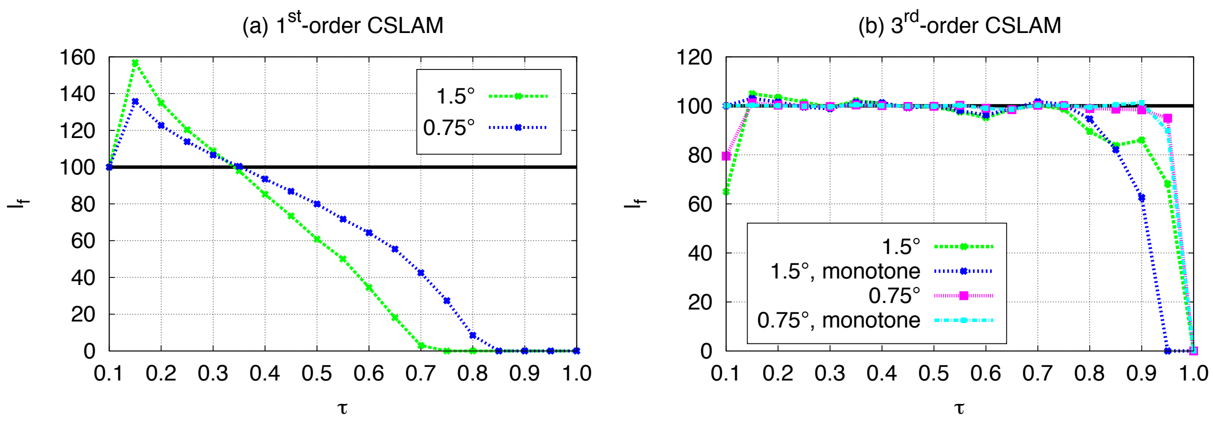
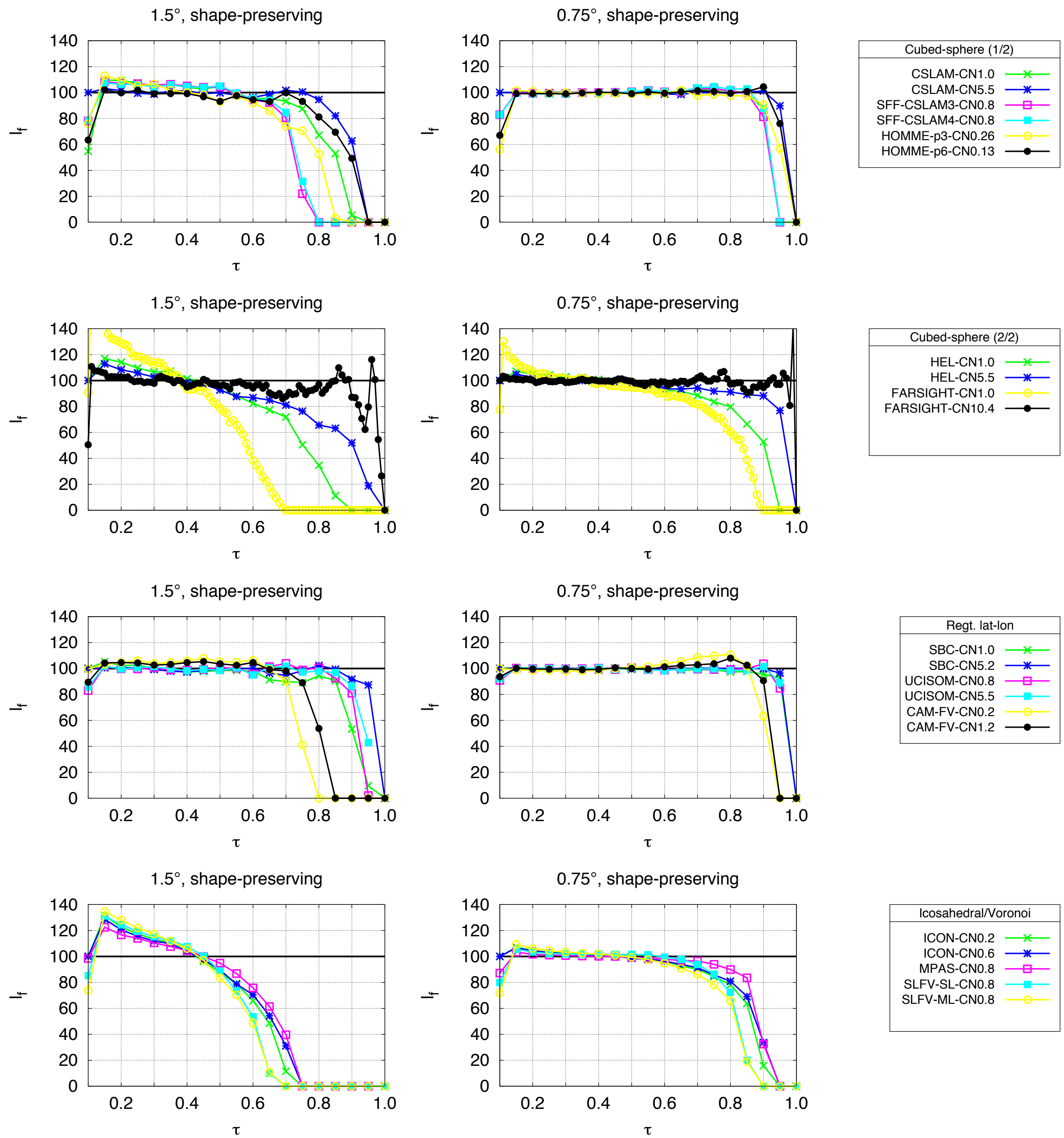


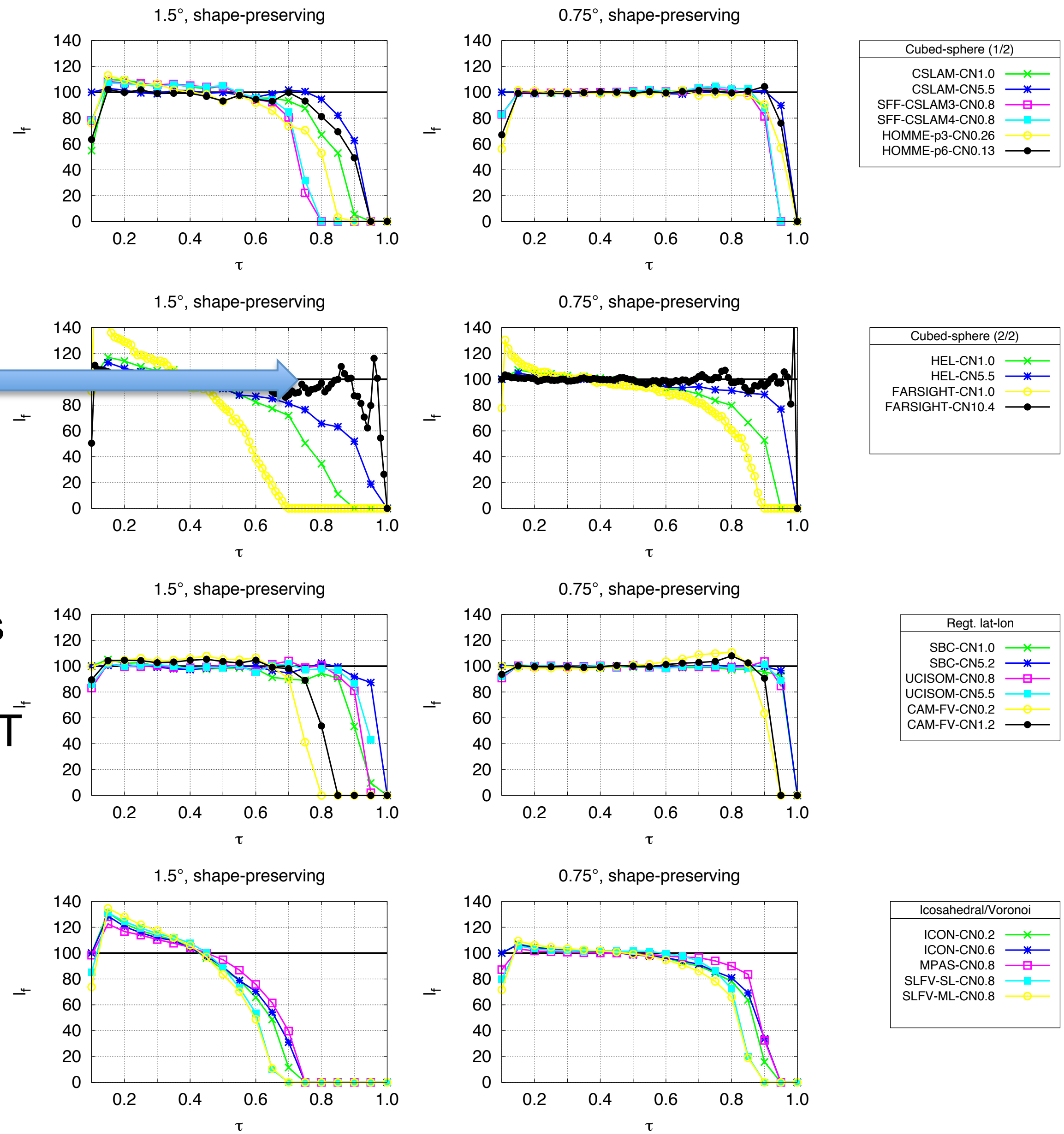
Fig. 6. Filament diagnostics $\ell_f(t=T/2)$ as a function of threshold value τ for different configurations of the CSLAM scheme with Courant number 5.5. (a) 1^{st}-order version of CSLAM at $\Delta\lambda = 1.5^\circ$ and $\Delta\lambda = 0.75^\circ$, and (b) 3^{rd}-order version of CSLAM with and without monotone/shape-preserving filter at resolutions $\Delta\lambda = 1.5^\circ$ and $\Delta\lambda = 0.75^\circ$.

Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)



Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

“ad hoc”
mass-fixer



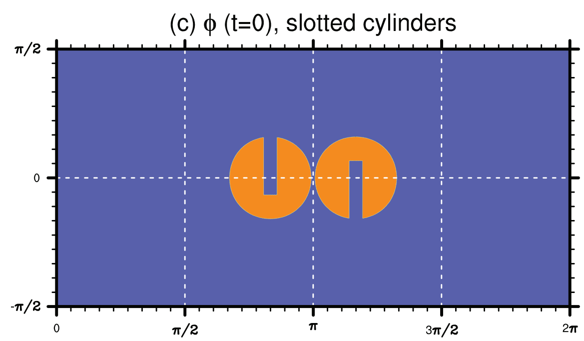
Large differences at 1.5° (some schemes are NOT in an convergent regime at this resolution)

Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

3. "Rough" distribution

(to challenge limiters/filters)

Initial condition



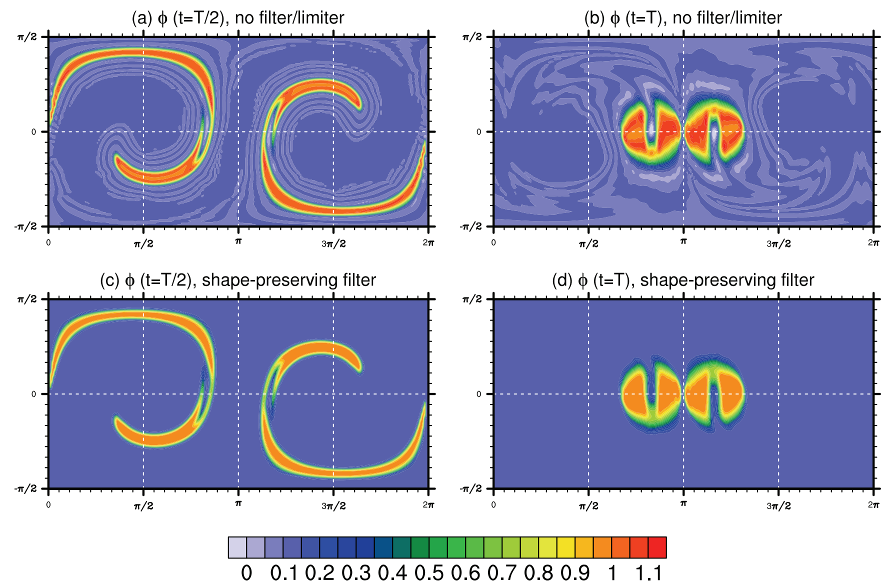
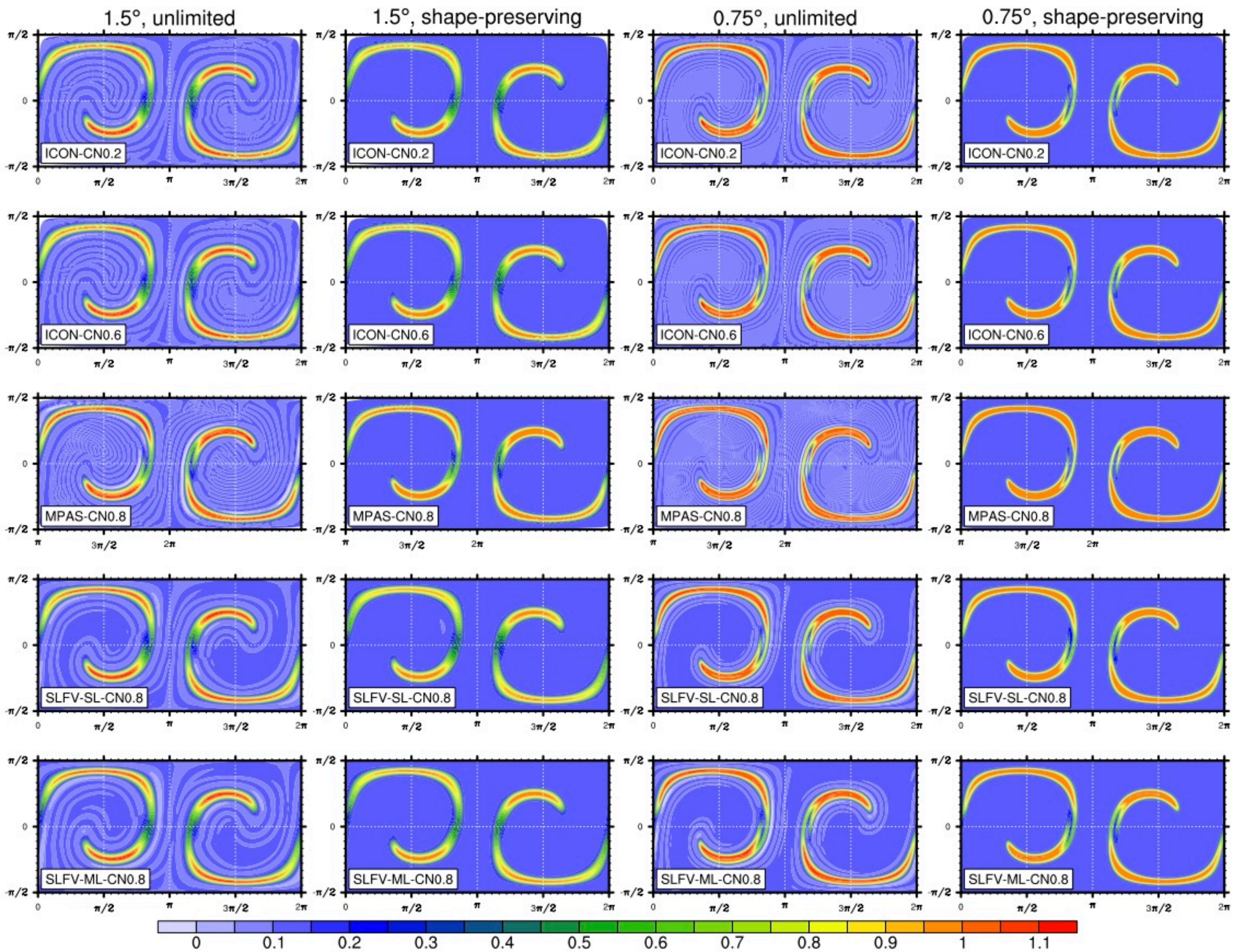


Fig. 7. Contour plot of the CSLAM numerical solution ϕ at resolution $\Delta\lambda = 1.5^\circ$ and time-step $T/120$ using the slotted-cylinders initial condition at time $t = T/2$ (**a** and **c**) and $t = T$ (**b** and **d**) using no filter/limiter (**a** and **b**) and a shape-preserving filter (**c** and **d**). The standard error norms for the unfiltered/unlimited solution are $\ell_2 = 0.24$, $\ell_\infty = 0.79$, $\phi_{min} = -0.19$, and $\phi_{max} = 0.15$, and for the shape-preserving solution they are $\ell_2 = 0.26$, $\ell_\infty = 0.80$, $\phi_{min} = 0.0$, and $\phi_{max} = -4.34 \cdot 10^{-3}$.

Background value is non-zero so positivity preserving filters do not alleviate undershoots!

Lauritzen et al. (2012, Geo. Geosci. Model Dev.,)



Lauritzen et al. (2012, Geo. Geosci. Model Dev., in prep)

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Geoscientific Model Development

A standard test case suite for two-dimensional linear transport on the sphere

P. H. Lauritzen[1], W. C. Skamarock[1], M. J. Prather[2], and M. A. Taylor[3]

[1]National Center for Atmospheric Research, Boulder, Colorado, USA
[2]Earth System Science Department, University of California, Irvine, California, USA
[3]Sandia National Laboratories, Albuquerque, New Mexico, USA

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