Assessing accuracy of transport schemes in global climate-weather models: new idealized test case suite

Peter Hjort Lauritzen[1] , W.C. Skamarock[1], M.J. Prather[2], M.A. Taylor[3] and C. Jablonowski[4]

[1] National Center for Atmospheric Research, Boulder [2]University of California, Irvine [3]Sandia National Laboratories, Albuquerque [4]University of Michigan, Ann Arbor

Introduction

- It is the purpose of this paper to propose a standard test case suite for 2D transport schemes on the sphere intended to be used for model development and facilitating scheme intercomparison.
- Test cases are designed to assess important aspects of accuracy in geophysical fluid dynamics under challenging flow conditions.
- Experiments are designed to be easy to setup, i.e. only 2 analytical wind fields (1 non-divergent, 1 divergent; Nair and Lauritzen, 2010) and four initial conditions are used:

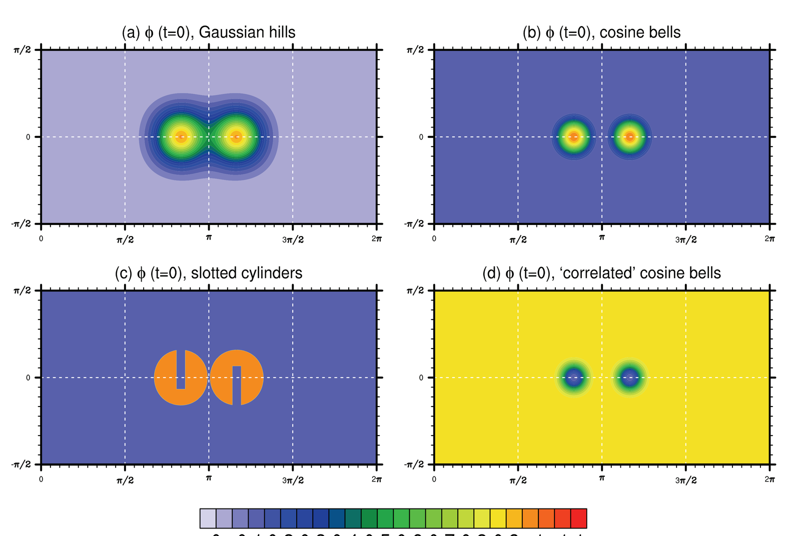


Fig. 1. Contour plots for the four initial conditions for mixing ratio ϕ used in this test suite. **(a)** depicts the infinitely smooth (C^{∞}) initial condition constructed from Gaussian surfaces, **(b)** the cosine bells initial condition which is C^1, **(c)** the non-smooth slotted cylinders initial condition, and **(d)** is the initial condition which is nonlinearly correlated with **(b)**.

Sample results are shown for these schemes:

Table 1. A list of acronyms (first column), full names (second column), primary reference (third column), and implementation grid (fourth column) for schemes used in this paper.

scheme acronym	full scheme name	primary reference	implementation grid
CAM-FV	Community Atmosphere Model - Finite Volume scheme	Lin and Rood (1996)	Regular latitude-longitude
CSLAM	Conservative Semi-LAgrangian Multi-tracer scheme	Lauritzen et al. (2010)	Gnomonic cubed-sphere
HOMME	High-Order Methods Modeling Enviroment	Dennis et al. (2012)	Gnomonic cubed-sphere (quadrature grid)
UCISOM	UC Irvine second-order moments scheme	Prather (1986)	Regular latitude-longitude

Flow deforms initial conditions into thin filaments and an "overlaid" translational flow transports the filaments as they deform (half way through simulation: t = T/2):

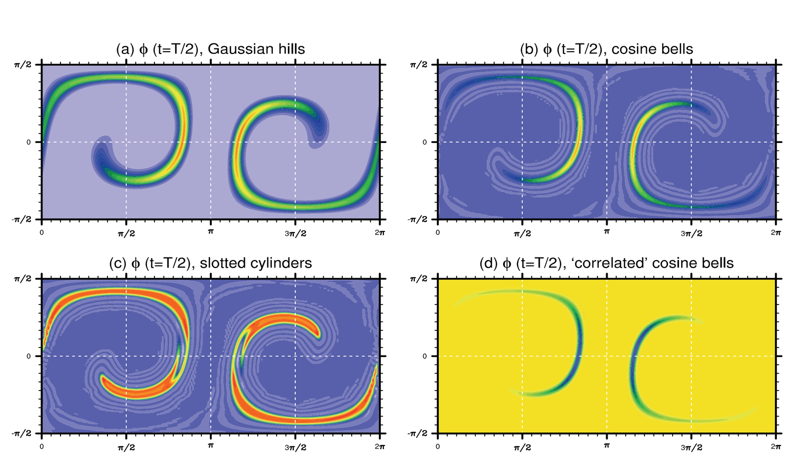


Fig. 2. Same as Fig. 1 but for the numerical solution at $t = T/2$ using CSLAM with a time-step $\Delta t = T/120$ and resolution of $\Delta\lambda = 1.5°$.

Filament diagnostic

Assess how well schemes preserve gradients and thin filaments near the grid scale. Definition from Lauritzen et al. (2012a):

The "filament" preservation diagnostic is formulated as follows. Define $A(\tau,t)$ as the spherical area for which the spatial distribution of the tracer $\phi(\lambda,\theta)$ satisfies

$$\phi(\lambda,\theta) \geq \tau, \tag{27}$$

at time t, where τ is the threshold value. For a non-divergent flow field and a passive and inert tracer ϕ, the area $A(\tau,t)$ is invariant in time.

The discrete definition of $A(\tau,t)$ is

$$A(\tau,t) = \sum_{k\in\mathcal{G}} \Delta A_k, \tag{28}$$

where ΔA_k is the spherical area for which ϕ_k is representative, K is the number of grid cells, and $\mathcal{G}$ is the set of indices

$$\mathcal{G} = \{k \in (1,\ldots,K) | \phi_k \geq \tau\}. \tag{29}$$

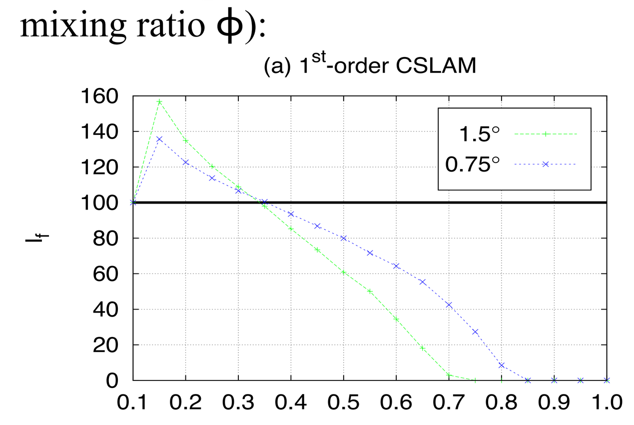
For Eulerian finite-volume schemes ΔA_k is the area of the k-th control volume. For Eulerian grid-point schemes a control volume for which the grid-point value is representative must be defined. Similarly for fully Lagrangian schemes based on point values (parcels) control volumes for which the point values are representative must be defined. Note that the "control volumes" should span the entire domain without overlaps or "cracks" between them.

Define the filament preservation diagnostic

$$\ell_f(\tau,t) = \begin{cases} 100.0 \times \frac{A(\tau,t)}{A(\tau,t=0)} & \text{if } A(\tau,t=0) \neq 0, \\ 0.0, & \text{otherwise.} \end{cases} \tag{30}$$

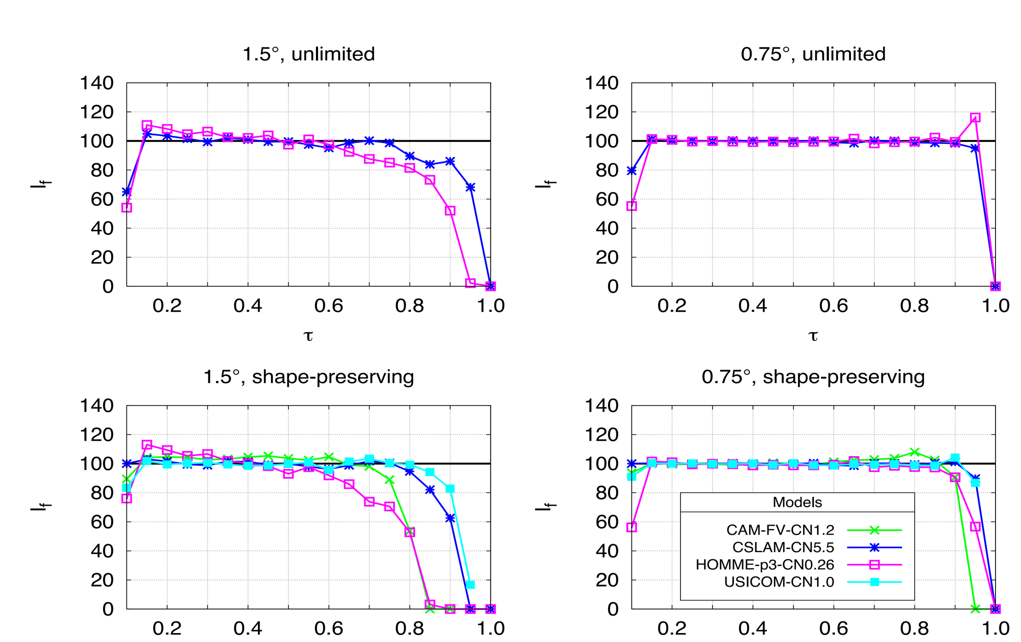
For infinite resolution (continuous case) and a non-divergent flow, $\ell_f(\tau,t)$ is invariant in time: $\ell_f(\tau,t=0) = \ell_f(\tau,t) = 100$ for all τ. At finite resolution, however, the filament

A very diffusive scheme will tend to decrease/increase l_f for high/low values of τ (peak values decrease/more area is covered with lower values of mixing ratio ϕ):



Filament diagnostic at t=T/2 (Fig.2b) for Cosine bells initial conditions (Fig.1b) and non-divergent flow field using 1st-order version of CSLAM

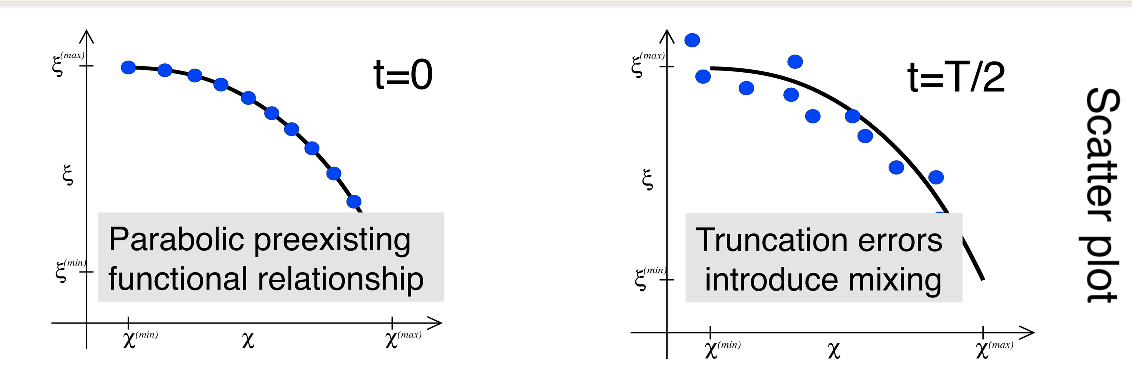
Results for higher-order schemes at t=T/2 without a shape-preserving filter (unlimited) and with one (shape-preserving) at two resolutions (appended "CN" is Courant number):



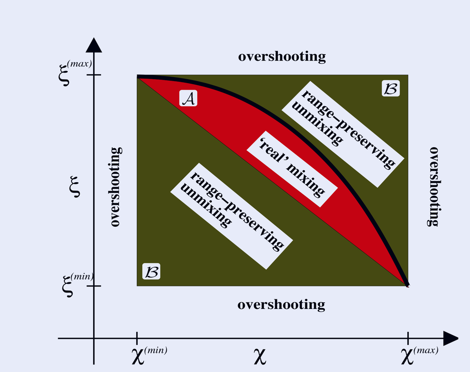
Mixing diagnostics

Atmospheric tracers are often observed to be functionally related, and these relations can be physically or chemically significant. It is therefore highly desirable that transport schemes should not disrupt such functional relations in unphysical ways through numerical mixing or, indeed, unmixing.

Setup: Fig.1b and 1d correlated initial conditions



Classification of mixing:

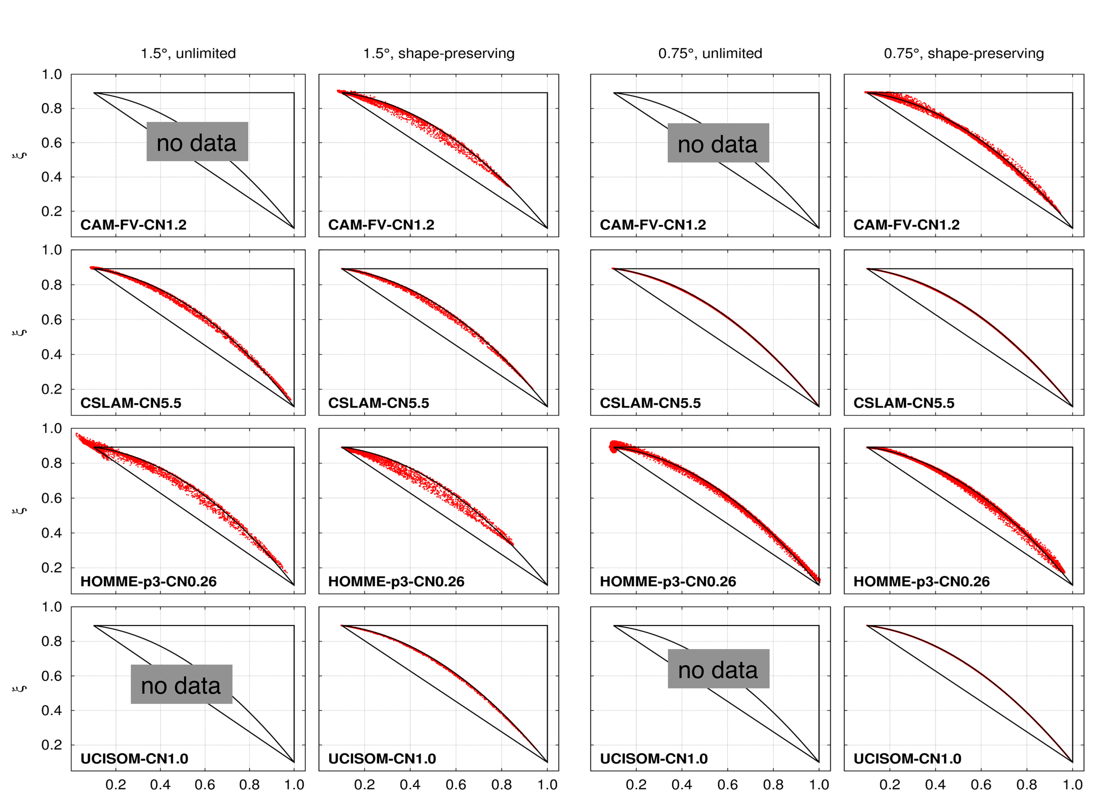


Real mixing: *Resembles mixing as observed in the atmosphere*

Range-preserving umixing: *Spurious unmixing within range of initial conditions*

Overshooting: *Spurious range-expanding mixing*

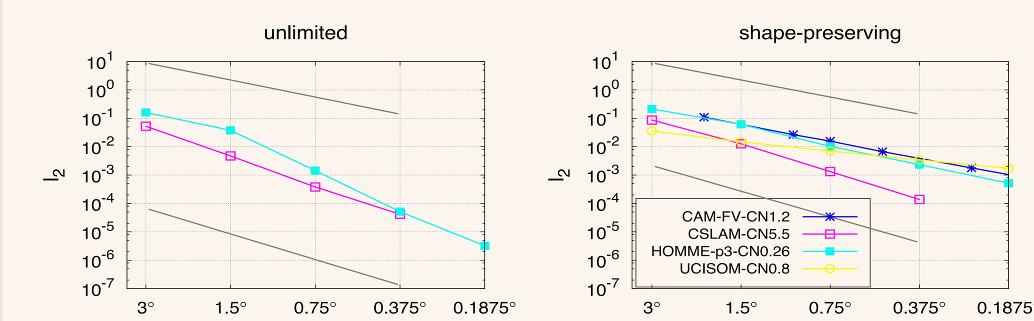
Only 1[st]-order schemes can guarantee that all mixing is "real" mixing. Overshooting can be avoided with properly designed shape-preserving filters. Some range-preserving unmixing must be tolerated with higher-order schemes.



See Lauritzen and Thuburn (2011) for more details on the mixing diagnostics.

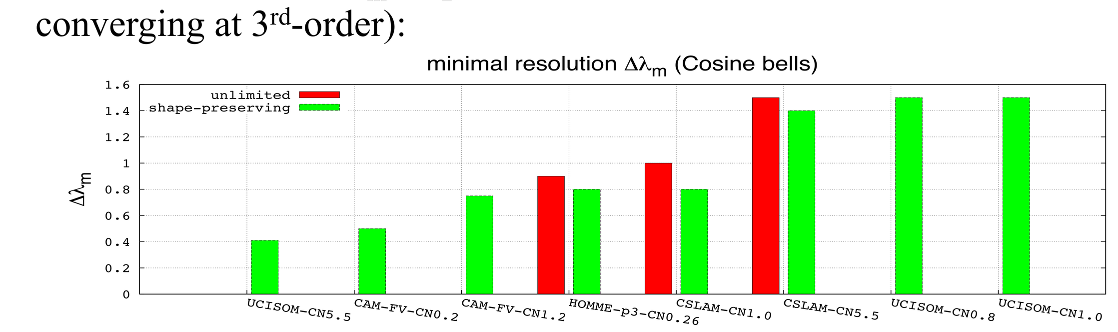
Numerical order of convergence

Compute convergence rates for standard error norms in resolution range Δλ=3°.. 0.3° using C^{∞} initial conditions (Fig.1a):



Minimal resolution

At what resolution $\Delta\lambda_m$ is $l_2 \approx 0.033$ (when CSLAM-CN5.5 start converging at 3[rd]-order):



Transport of rough distribution

To challenge shape-preserving filters transport discontinuous slotted cylinder initial conditions (see Fig.1c and 2c).

Transport in divergent flow

Demonstrate that scheme can transport under divergent flow conditions (usually challenges the coupling between air mass and tracers).

Test case suite was exercised by a dozen state-of-the-art transport schemes at workshop at NCAR in March 2011 (Lauritzen et al., 2012b).

References

Dennis, J. M., and co-authors, 2012: CAM-SE: A scalable spectral element dynamical core for the Community Atmosphere Model, *Int. J. High. Perform. C.*, **26**, 74–89.

Lauritzen P.H., R.D. Nair RD, P.A. Ullrich. 2010. A conservative semi-Lagrangian multi-tracer transport scheme (CSLAM) on the cubed-sphere grid. *J. Comput. Phys*. **229**: 1401–1424.

Lauritzen, P.H. and J. Thuburn, 2011: Evaluating advection/transport schemes using interrelated tracers, scatter plots and numerical mixing diagnostics. *Quart. J. Roy. Meteor. Soc.*: in press

Lauritzen P.H.,W.C.Skamarock, M.J.Prather, M.A.Taylor, 2012a. A standard test case suite for two-dimensional linear transport on the sphere. *Geosci. Model Dev. Discuss*., **5**, 189-228.

Lauritzen P.H. and co-authors, 2012b. A standard test case suite for two-dimensional linear transport on the sphere: results from a collection of state-of-the-art schemes. *Geosci. Model Dev.*, in prep.

Lin, S. J. and R.B.Rood, 1996: Multidimensional Flux-Form Semi- Lagrangian Transport Schemes, *Mon. Wea. Rev.*, **124**, 2046–2070.

Nair, R.N., P.H. Lauritzen 2010. A Class of Deformational Flow Test Cases for Linear Transport Problems on the Sphere. *J. Comput. Phys.* **229**: 8868–8887.

Prather, M.J., 1986. Numerical advection by conservation of second-order moments, *J. Geophys. Res.*, **91**, 6671–6681.

For further information please contact *pel@ucar.edu*
http://www.cgd.ucar.edu/cms/pel