A total energy error analysis of dynamical cores and physics-dynamics coupling in the Community Atmosphere Model (CAM)

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Workshop on the solution to partial differential equations on the sphere, April 29 – May 3, 2019, Montréal, Québec, Canada

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Research question

How large are the spurious total energy sources/sinks in an atmosphere model and where are they coming from?

2018 WCRP workshop: The Earth's Energy Imbalance and its implications (EEI)

The Earth Energy Imbalance (EEI) is one of the most fundamental metrics defining the status of global climate change and expectations for continued global warming. WCRP Core Projects work together for a new WCRP-wide initiative to identify research goals and opportunities for Earth's Energy Imbalance and to strengthen future international scientific collaboration with experts for EEI assessments.

Save the dates for this opportunity for international scientific collaborations: The WCRP workshop will be held on 13-16 November 2018 in Toulouse, France. Further details can be found here and will be updated through the websites of WCRP and its Core Projects.

Total energy (TE) equation
- moist atmosphere

Lauritzen et al. (2018); https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2017MS001257

$$\frac{d\widehat{E}}{dt} = \widehat{F}_{net},$$

Net energy flux calculated by parameterizations

where

Dry mass per unit area

Dry mixing ratio

Kinetic energy

Thermal energy

$$\frac{d\widehat{E}}{dt} = \frac{d}{dt}\left\{\frac{1}{\Delta S}\int_{\eta=0}^{\eta=1}\iint_S\left(\frac{1}{g}\frac{\partial M^{(d)}}{\partial \eta^{(d)}}\right)\sum_{\ell\in\mathcal{L}_{all}}\left[m^{(\ell)}\left(K+c_p^{(\ell)}T+\Phi_s\right)\right]dA\,d\eta^{(d)}\right\},$$

$$\mathcal{L}_{all} = \{\text{`}d\text{'}, \text{`}wv\text{'}, \text{`}cl\text{'}, \text{`}ci\text{'}, \text{`}rn\text{'}, \text{`}sw\text{'}\}$$

and

$$\widehat{F}_{net} = \frac{1}{\Delta S}\int_{\eta=0}^{\eta=1}\iint_S\left(\frac{1}{g}\frac{\partial M^{(d)}}{\partial \eta^{(d)}}\right)\sum_{\ell\in\mathcal{L}_{all}}\left[m^{(\ell)}\right]F_{net}\,dA\,d\eta^{(d)}.$$

where ΔS is the surface area of the sphere, Φ_s is the surface geopotential and $\widehat{(\cdot)}$ refers to the global average.

Total energy (TE) equation
- moist atmosphere

$$\frac{d\widehat{E}}{dt} = \widehat{F}_{net},$$

The continuous equations of motion on which the dynamical core is based conserve TE globally:

$$\frac{d\widehat{E}}{dt} = 0$$

Total energy (TE) equation

- moist atmosphere

$$\frac{d\widehat{E}}{dt} = \widehat{F}_{net},$$

Conserving total energy to within ~0.01 W/m² is considered "good enough" for coupled climate modeling (Boville, 2000; Williamson et al., 2015)

$$\frac{d\widehat{E}}{dt} \leq 0.01 W/m^2$$

Earths energy imbalance is ~1 W/m²

Total energy (TE) equation
- moist atmosphere

$$\frac{d\widehat{E}}{dt} = \widehat{F}_{net},$$

Column physics: TE change in column should be balanced by fluxes in/out of the top and bottom

$$\frac{d\widehat{E}}{dt} = \frac{1}{\Delta S}\iint_S \left(p_{top}F_{net} - p_s F_{net}\right) dA.$$

Potential spurious sources/sinks of total energy in an atmosphere model:

- **Parameterization errors:** Individual parameterizations may not have a closed energy budget. CAM parameterizations are required to have a closed energy budget under the assumption that pressure remains constant during the computation of the subgrid-scale parameterization tendencies. In other words, the TE change in the column is exactly balanced by the net sources/sinks given by the fluxes through the column.

- **Pressure work:** That said, if parameterizations update specific humidity then the surface pressure changes (e.g., moisture entering or leaving the column). In that case the pressure changes which, in turn, changes TE. This is referred to as pressure work [section 3.1.8 in Neale et al., 2012].

- **Continuous TE formula discrepancy:** If the continuous equations of motion for the dynamical core conserve a TE different from the one used in the parameterizations then an energy inconsistency is present in the system as a whole. In CAM this mismatch arose from the evolutionary nature of the model development and not by deliberate design; and should be eliminated in the future.

- **Dynamical core errors:** Energy conservation errors in the dynamical core, not related to physics-dynamics coupling errors, can arise in multiple parts of the algorithms used to solve the equations of motion.

- **Physics-dynamics coupling (PDC):** Assume that physics computes a tendency. Usually the tendency (forcing) is passed to the dynamical core which is responsible for adding the tendencies to the state.

Potential spurious sources/sinks of total energy in an atmo...

- **Paramete...** CAM para... pressure re... In other w... fluxes thro... ...e

- **Pressure** ... changes (e.g., mois... ...nges TE. This is refe...

- **Continuo...** ...ore conserve a TE differe... ...he system as a whole ... not by deliberate design; an...

- **Dynamic...** ...dynamics coupling errors, can...

- **Physics-dyna...** ...e tendency (forcing) is passed to the dynamical core which is responsible for adding the tendencies to the state.

TE errors in the CAM spectral-element dynamical core:

- Horizontal inviscid dynamics: Energy errors resulting from solving the inviscid,adiabatic equations of motion.

- Hyperviscosity: Filtering errors; Note that we use frictional heating:

Let $\delta \mathbf{v}$ be the change in the velocity vector due to diffusion of momentum. Then the change in kinetic energy due to hyperviscosity applied to $\mathbf{v}$ is $\frac{1}{2}\rho\mathbf{v}\cdot\delta\mathbf{v}$. This kinetic energy is converted to a heating rate by adding a heating term $\delta\mathcal{T}$ in the thermodynamic equation corresponding to the kinetic energy change

$$\rho c_p \delta\mathcal{T} = -\frac{1}{2}\rho\mathbf{v}\cdot\delta\mathbf{v} \Rightarrow \delta\mathcal{T} = -\frac{1}{2c_p}(\mathbf{v}\cdot\delta\mathbf{v}), \quad (59)$$

(p.71 in ; Neale et al., 2012). As shown in the results section 4.2 this term is rather large and therefore important for good energy conservation characteristics of the dynamical core.

- Vertical remapping: The vertical remapping algorithm from Lagrangian to Eulerian reference surfaces does not conserve TE.

- Near round-off negative values of water vapor which are filled to a minimal value without compensation.

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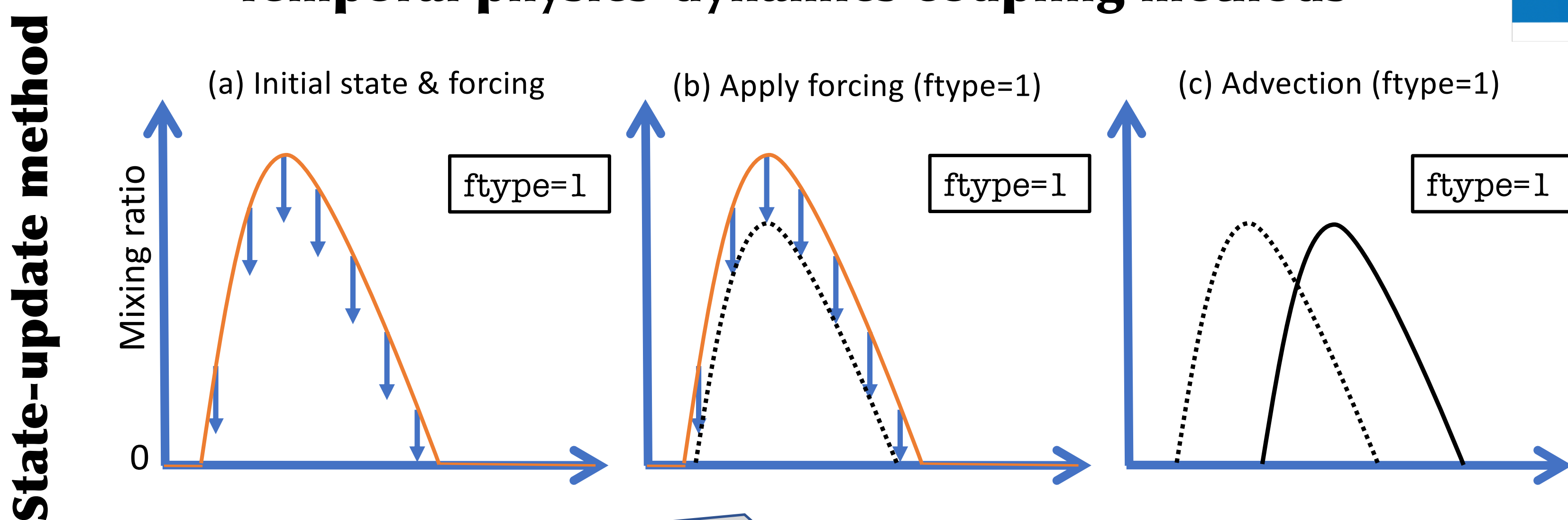
Temporal physics-dynamics coupling methods



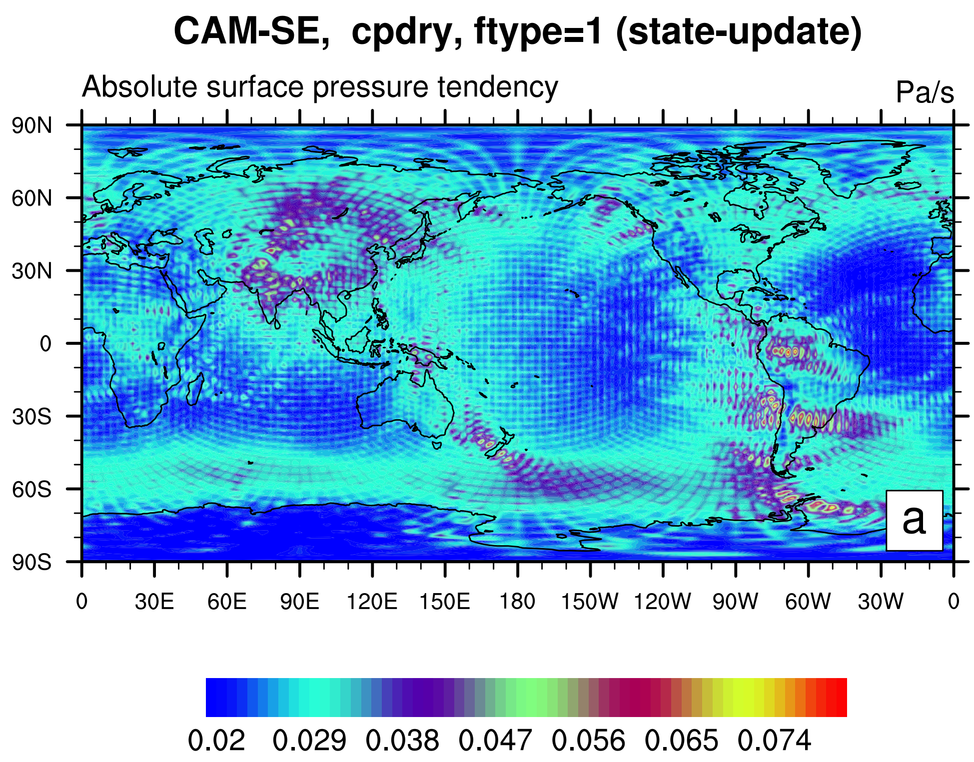
No physics-dynamics coupling error:

(Dry) Energy change due to physics energy increments

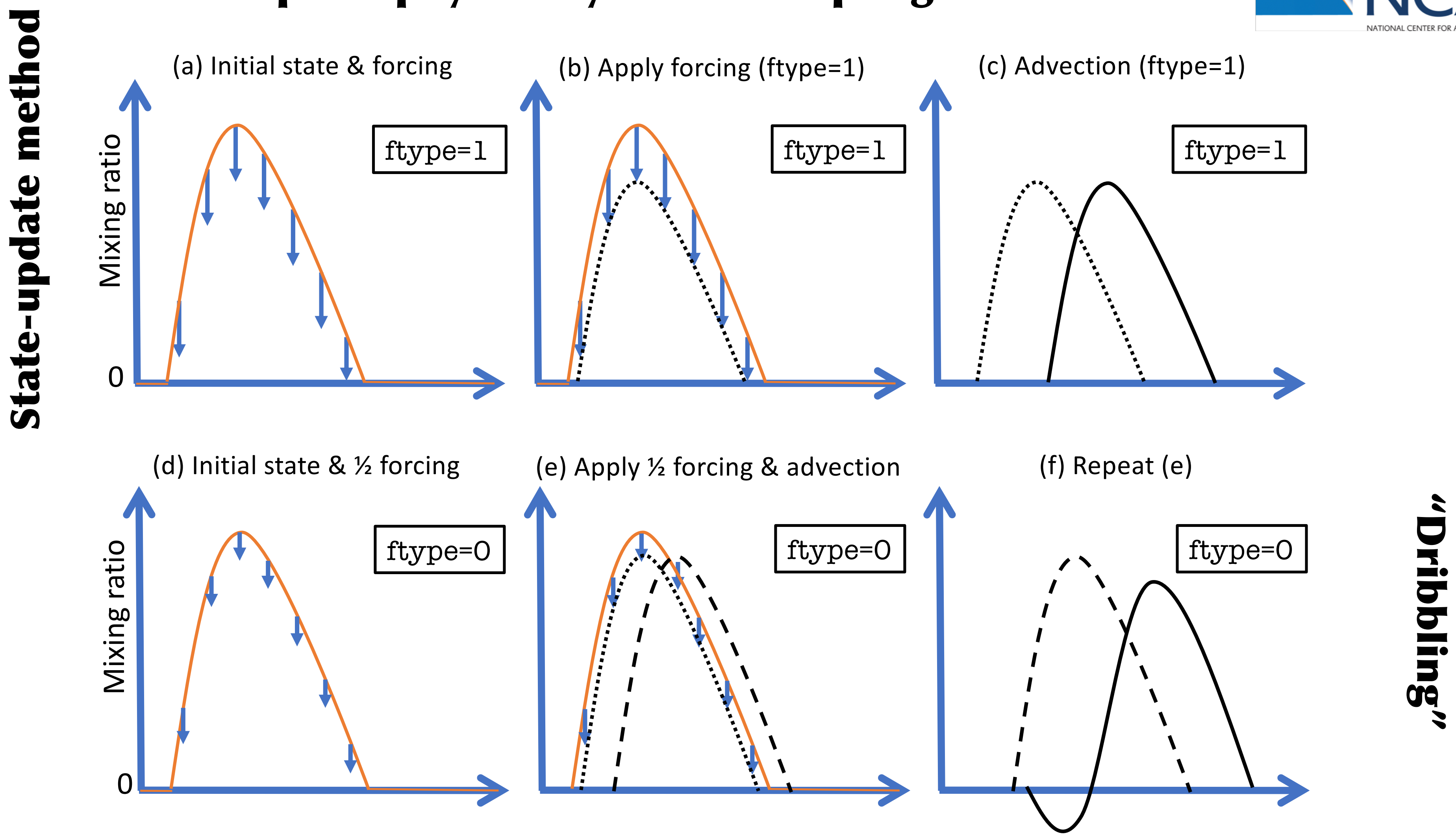
$$\Delta M^{(d)}\Delta T \quad \Delta M^{(d)}\left[(\Delta u)^2+(\Delta v)^2\right] \quad \Delta m^{(\ell)}\Delta M^{(d)}$$

= Dynamics energy change due to physics forcing

1 year average |dps/dt|; AMIP run



Temporal physics-dynamics coupling methods



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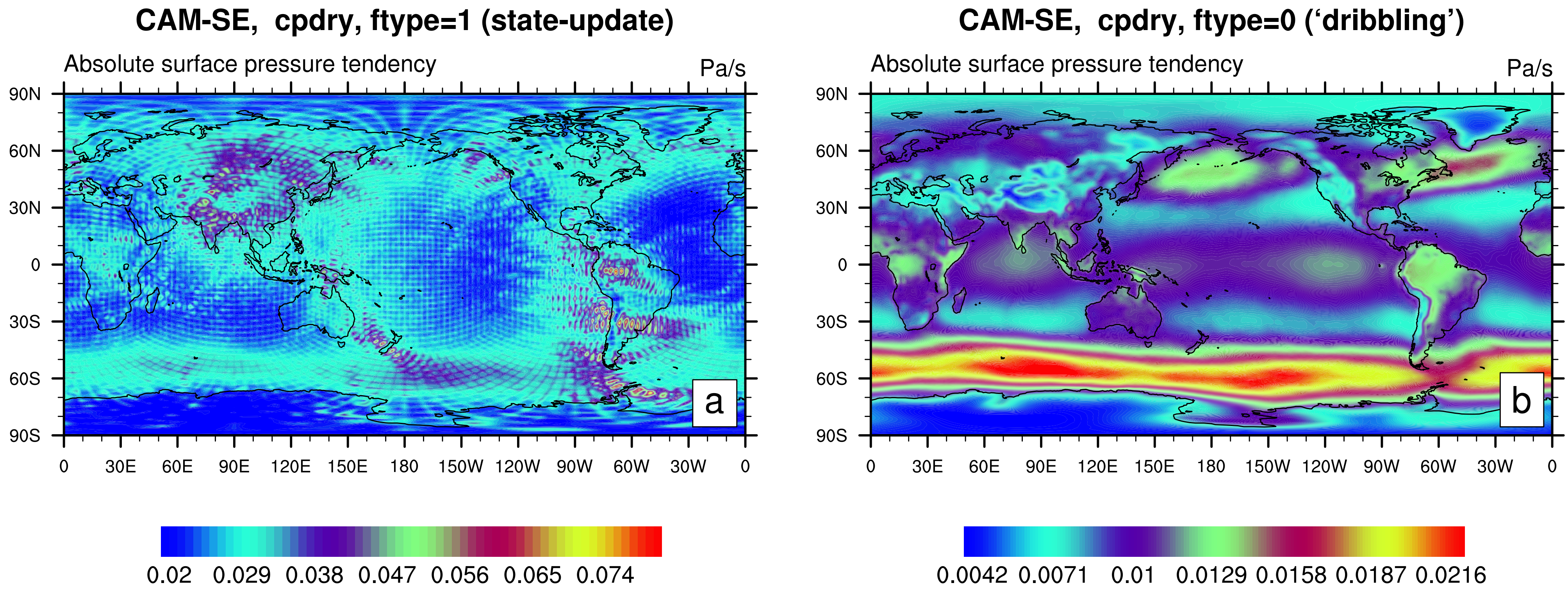


Temporal physics-dynamics coupling methods

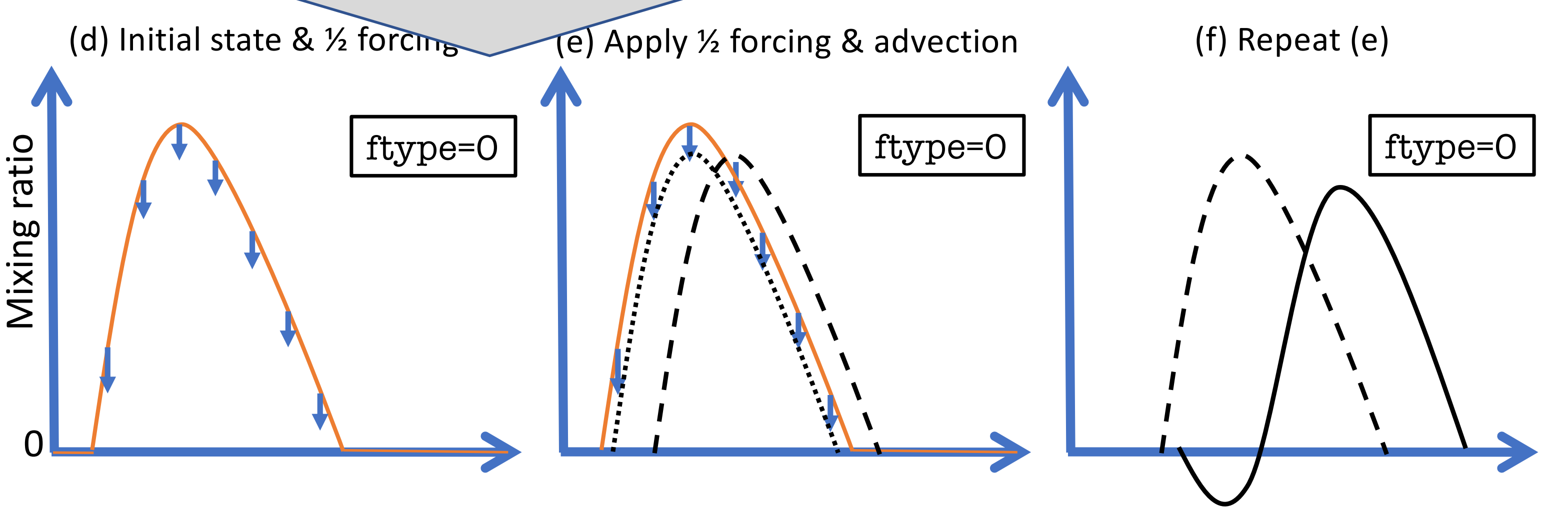
State-update method

- **Thermal energy "dribbling" error: Thermal energy increment from physics**

$$\Delta M^{(d)} \Delta T$$

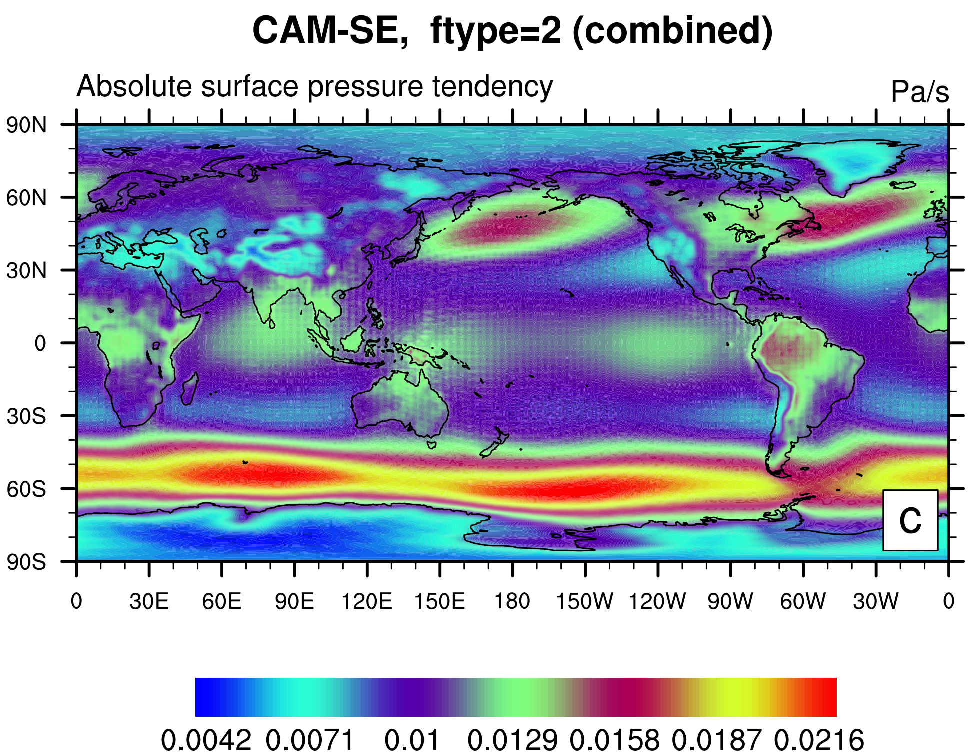
does not match thermal energy change in dycore when tendency is added to dycore state.

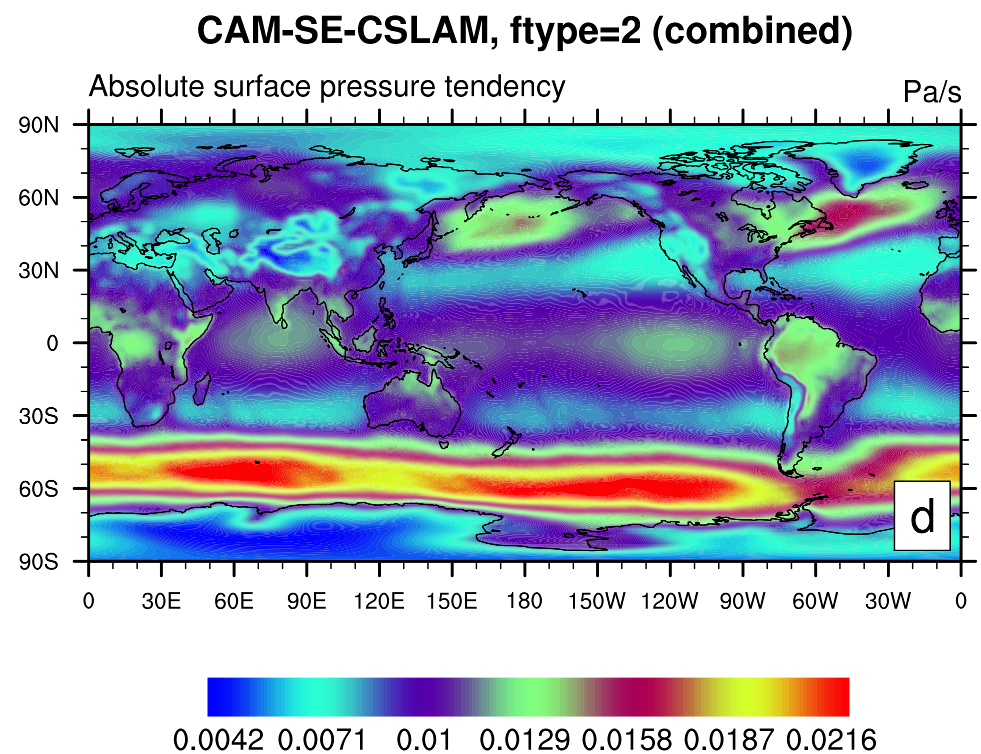
- **Kinetic energy "dribbling" error:** $$\Delta M^{(d)} \left[(\Delta u)^2 + (\Delta v)^2\right]$$
- **Mass "clipping" error: e.g., if logic in dycore to prevent negative mixing ratios**



"Dribbling"

ftype=2: state-updating (type=1) for tracers (i.e. no mass-clipping errors) and “dribbling” (ftype=0) for u,v, and T.





Potential spurious sources/sinks of total energy in an atmosphere model:

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Fixing spurious sources/sinks of total energy in an atmosphere model:

- **Compensating Energy fixers:** To avoid TE conservation errors which could accumulate and ultimately lead to a climate drift, it is customary to apply an arbitrary energy fixer to restore TE conservation. Since the spatial distribution of many energy errors, in general, is not known, global fixers are used. In CAM a uniform increment is added to the temperature field to compensate for TE imbalance from all processes, i.e. dynamical core, physics-dynamics coupling, TE formula discrepancy, energy change due to pressure work, and possibly parameterization errors if present.

$$-\partial \widehat{E}_{phys}^{(efix)} = \partial \widehat{E}_{phys}^{(pwork)} + \partial \widehat{E}_{dyn}^{(adiab)} + \partial \widehat{E}^{(pdc)} + \partial \widehat{E}^{(discr)}$$

- $-\partial \widehat{E}_{phys}^{(efix)}$: Energy fixer
- $\partial \widehat{E}_{phys}^{(pwork)}$: Pressure work
- $\partial \widehat{E}_{dyn}^{(adiab)}$: Dynamical core
- $\partial \widehat{E}^{(pdc)}$: Physics-dynamics coupling
- $\partial \widehat{E}^{(discr)}$: Continuous TE formula discrepancy

Spurious sources/sinks of total energy in atmosphere model:

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- **Parameterization errors:** Indi
  CAM parameterizations are requi
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- **Dynamical core errors:** Energ
  errors, can arise in multiple parts
- **Physics-dynamics coupling (**
  passed to the dynamical core wh

Budget closed in CAM 😀
(except for small "clipping" errors) but …

Pressure work: ~0.3 W/m²

TE formula discr. (CAM-SE only): ~0.6 W/m²

CAM-SE: ~-0.6 W/m² **(decreases to -0.3W/m² with smoother topography)**
CAM-FV and CAM-FV3: ~ -1.1 W/m²

CAM-SE: PDC errors ("dribbling"): ~0.5 W/m²

Spurious sources/sinks of total energy in atmosphere model:

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- **Dynamical**
  errors, can arise
- **Physics-dynamics coupling (**
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TE errors in the CAM spectral-element dynamical core (break-down):

- Horizontal inviscid dynamics: Energy errors resulting from solving the inviscid,adiabatic equations of motion. **~0.010 W/m²**
- Hyperviscosity: Filtering errors (frictional heating is used!). **~-0.587 W/m²**

Frictional heating is ~0.579 W/m²

Note that if no frictional heating is used then TE error would be > 1 W/m²

- Vertical remapping: The vertical remapping algorithm from Lagrangian to Eulerian reference surfaces does not conserve TE. **~-0.012 W/m²**

but …

.6 W/m²

ner topography)

V/m²

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in

Note that there are compensating errors in the system
-> need to do detailed TE budget analysis!

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Spurious sources/sinks of total energy in atmosphere model:

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TE conservation must be assessed with moist physics forcing and 'real-world' topography!

dE_{dycore}/dt for

- **CAM-SE using Held-Suarez forcing (no moisture forcing) : ~-0.02 W/m²**
- **CAM-SE in Aqua-planet setup (no topography but moist physics) : ~-0.14 W/m²**
- **CAM-SE with smoother topography ("real-world" AMIP setup) : ~-0.3 W/m²**
- **CAM-SE default : ~-0.6 W/m²**

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ng" errors) but …

~0.3 W/m²

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CAM-SE: PDC errors ("dribbling"): ~0.5 W/m²

Summary

- Total energy errors in numerical discretizations (dynamical core), physics-dynamics coupling and pressure work errors are ~-0.6 – 0.3 W/m^2
- Local errors can be an order of magnitude larger (at least)!

Outlook

- In next-generation models we should consider formulating physics in dry pressure coordinates (so that coordinate surfaces stay fixed during physics updates)
- Can we close the total energy budget locally in models?
- Integrating weather-climate models: parameterizations for weather models are, in general, not formulated to have a closed TE budget. Major challenge?

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