CAM-SE-CSLAM: Consistent finite-volume transport with spectral-element dynamics

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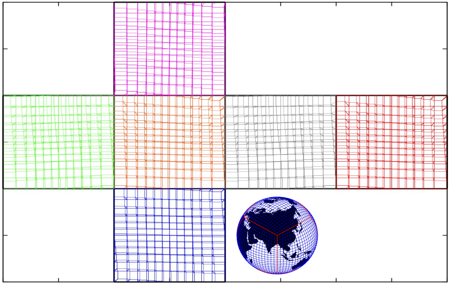


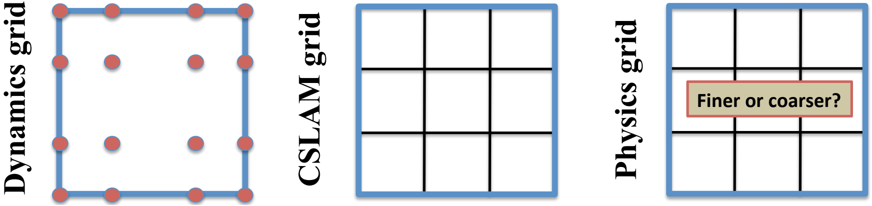
Overview

A new model configuration based on CAM-SE:

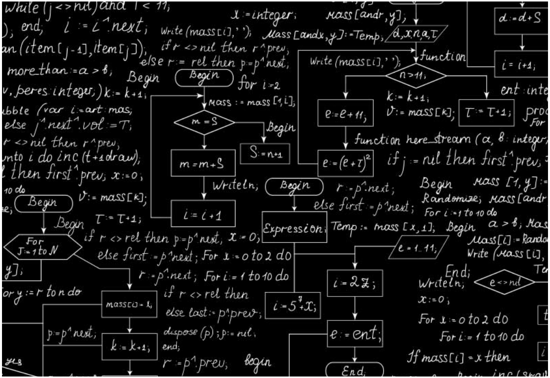
- **SE**: Spectral-element dynamical core solving for $\vec{v}$, T, p_s

  (Dennis et al., 2012; Evans et al., 2012; Taylor and Fournier, 2010; Taylor et al., 1997)
- **CSLAM**: Semi-Lagrangian finite-volume transport scheme for tracers

  (Lauritzen et al., 2010; Erath et al., 2013, 2012; Harris et al., 2010)
- **Phys-grid**: Separating physics and dynamics grids, i.e. ability to compute physics tendencies based on cell-averaged values within each element instead of quadrature points

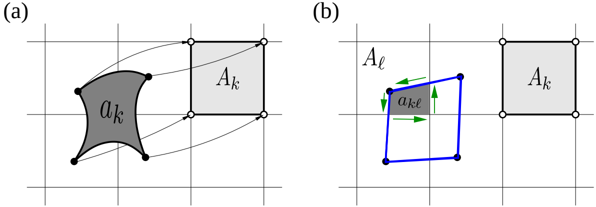


Coupling spectral-element continuity equation for air with CSLAM turned out to be much harder than I had anticipated ...



The ‘spectral-element part’ of this research would not have been possible without the close collaboration with Mark Taylor (DOE), James Overfelt (DOE) and Paul Ullrich (UCDavis).

Conservative Semi-LAgrangian Multi-tracer (CSLAM)

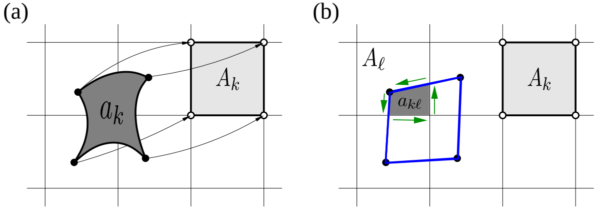


Finite-volume Lagrangian form of continuity equation for air (pressure level thickness, Δp), and tracer (mixing ratio, q):

$$\int_{A_k} \psi_k^{n+1}\, dA = \int_{a_k} \psi_k^n\, dA \quad = \quad \sum_{\ell=1}^{L_k} \left[\sum_{\imath+\jmath\leq 2} c_\ell^{(\imath,\jmath)} w_{k\ell}^{(\imath,\jmath)} \right], \quad \psi = \Delta p,\, \Delta p\, q,$$

where n time-level, $a_{k\ell}$ overlap areas, L_k #overlap areas, $c^{(\imath,\jmath)}$ reconstruction coefficients for ψ_k^n, and $w_{k\ell}^{(\imath,\jmath)}$ weights.

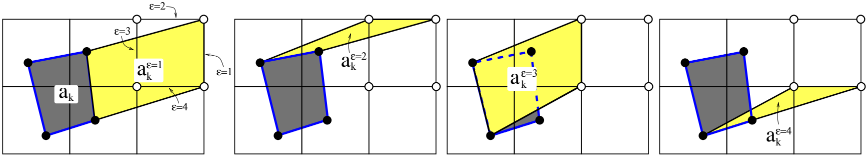
Conservative Semi-LAgrangian Multi-tracer (CSLAM)



$$\int_{A_k} \psi_k^{n+1}\, dA = \int_{a_k} \psi_k^n\, dA \;\; = \;\; \sum_{\ell=1}^{L_k} \left[\sum_{\imath+\jmath\leq 2} c_\ell^{(\imath,\jmath)} w_{k\ell}^{(\imath,\jmath)} \right], \quad \psi = \Delta p,\, \Delta p\, q,$$

- Multi-tracer efficient: $w_{k\ell}^{(i,j)}$ re-used for each additional tracer (Dukowicz and Baumgardner, 2000).
- Scheme allows for large time-steps (flow deformation limited).
- Conserves mass, shape, linear correlations (Lauritzen et al., 2014).

Flux-form CSLAM ≡ Lagrangian CSLAM



$$\int_{A_k} \psi_k^{n+1}\, dA = \int_{A_k} \psi_k^{n}\, dA - \sum_{\epsilon=1}^{4} s_{k\ell}^{\epsilon} \int_{a_k^{\epsilon}} \psi\, dA, \quad \psi = \Delta p, \Delta p\, q.$$

where

- a_k^ϵ = 'flux-area' (yellow area) = area swept through face ϵ
- $s_{k\ell}^\epsilon$ = 1 for outflow and -1 for inflow.

Flux-form and Lagrangian forms of CSLAM are equivalent (Lauritzen et al., 2011).

Requirements for transport schemes

1. Global (and local) Mass-conservation

If Δp is pressure-level thickness and q is mixing ratio, then the total mass

$$M(t) = \int_{\Omega} \Delta p \, q \, dA,$$

is invariant in time: $M(t) = M(t=0)$

2. Shape-preservation

Scheme does not produce new extrema (in particular negatives) in q

3. Consistency

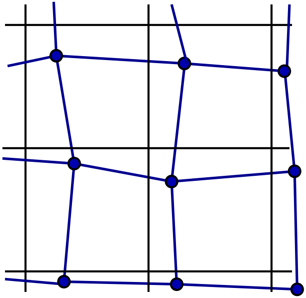
If $q = 1$ then the transport scheme should reduce to the continuity equation for air.

How does CSLAM fulfill requirements?

1. Global (and local) Mass-conservation

- Upstream Lagrangian areas span domain Ω without cracks & overlaps
- $\int_{\Omega_k} \psi_k(x,y)\, dA = \Delta A_k \, \overline{\psi}_k$,

where $\psi_k(x,y)$ is reconstruction function in kth cell Ω_k, ΔA_k is area of Ω_k, $\overline{\psi}_k$ is cell averaged value



How does CSLAM fulfill requirements?

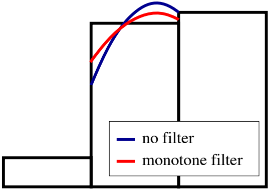
2. Shape-preservation

- Apply limiter to mixing ratio sub-grid cell distribution:

$$q(x,y) = \sum_{\imath+\jmath<3} c^{(\imath,\jmath)} x^{\imath} y^{\jmath},$$

(Barth and Jespersen, 1989) so that extrema of $q(x,y)$ are within range of neighboring $\overline{q}$.

- And upstream areas span domain Ω without cracks & overlaps



How does CSLAM fulfill requirements?

3. Consistency

- Solve continuity equations for air and tracer on the form: (Nair and Lauritzen, 2010):

$$\frac{D}{Dt}\int_{\delta A} \Delta p(x,y)\, dA = 0 \quad (1)$$

$$\frac{D}{Dt}\int_{\delta A} \left\{ \overline{\Delta p}\, q(x,y) + \overline{q}\left[\Delta p(x,y) - \overline{\Delta p}\right]\right\} dA = 0 \quad (2)$$

$\rightarrow$ if $q = 1$ then (2) reduces to (1).

- Note also that limiter acts on $q(x,y)$ and not $q(x,y)\,\Delta p(x,y)$, i.e. no reason to have a limiter on pressure level thickness.

Coupling problem formulation

We need to find a departure grid so that

$$\Delta p^{(CSLAM)} = \Delta p^{(SE)} \tag{3}$$

$\Rightarrow$ requirements 1-3 are fulfilled with existing CSLAM technology.

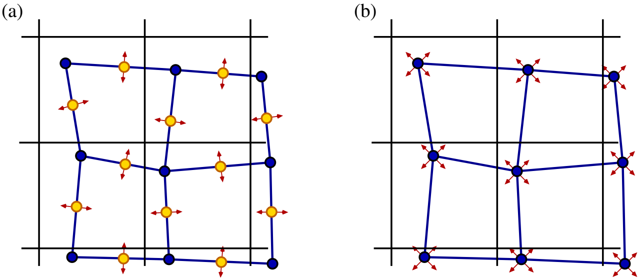


Figure: Global iteration problem ☹ and it is ill-conditioned since any non-divergent perturbation of points yields the same solution ☹☹☹

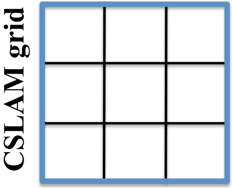
Solution

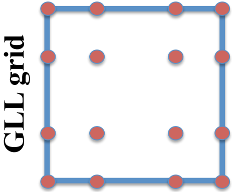
Cast problem in flux-form:

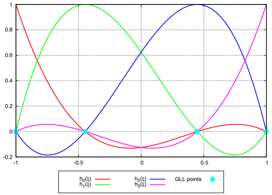
$$\mathcal{F}^{(CSLAM)} = \mathcal{F}^{(SE)} \tag{4}$$

$\Rightarrow$ requirements 1-3 are fulfilled with existing CSLAM technology.

- Spectral-element method does not operate with fluxes: Taylor et al. have derived a method to compute fluxes, $\mathcal{F}^{(SE)}$, through the CSLAM control volume faces! presented at ICMS conference in March, 2015.



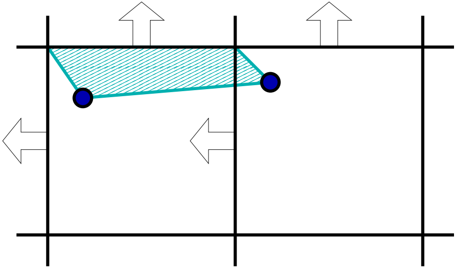




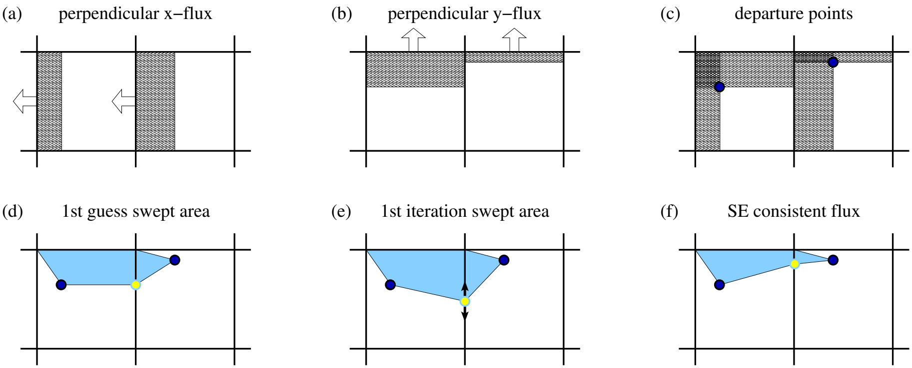
CSLAM fluxes

Given $\mathcal{F}^{(SE)}$ find swept areas, $\delta\Omega$, so that:

1. $$\mathcal{F}^{(CSLAM)} = \int_{\delta\Omega} \Delta p(x,y)\, dA = \mathcal{F}^{(SE)} \qquad \forall\ \delta\Omega.$$
2. The sum of all the swept areas, $\delta\Omega$, span the domain without cracks or overlaps

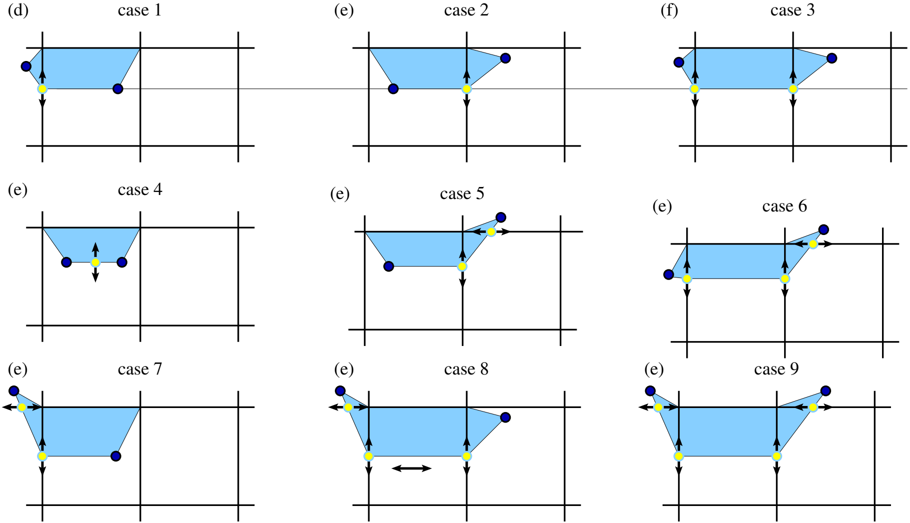


Consistent SE-CSLAM algorithm: step-by-step example

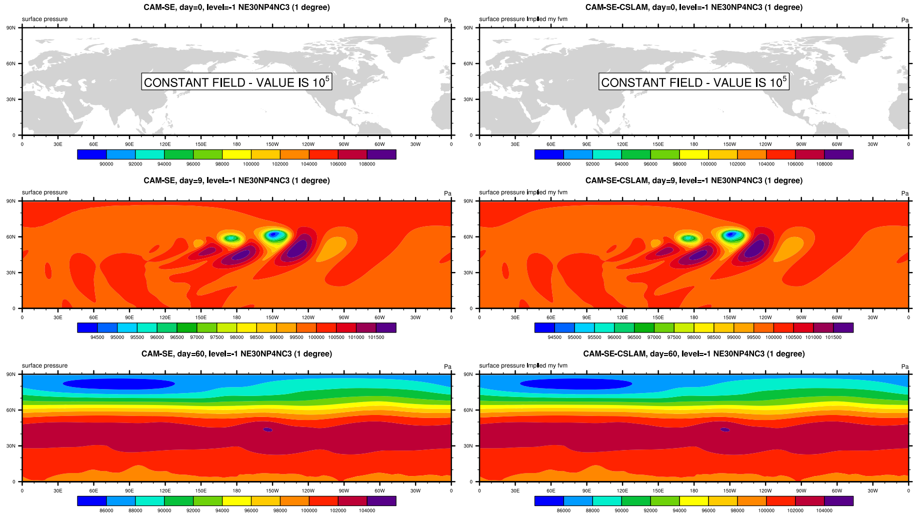


Well-posed? As long as flow deformation $\left|\frac{\partial u}{\partial x}\right| \Delta t \lesssim 1$ (Lipschitz criterion)

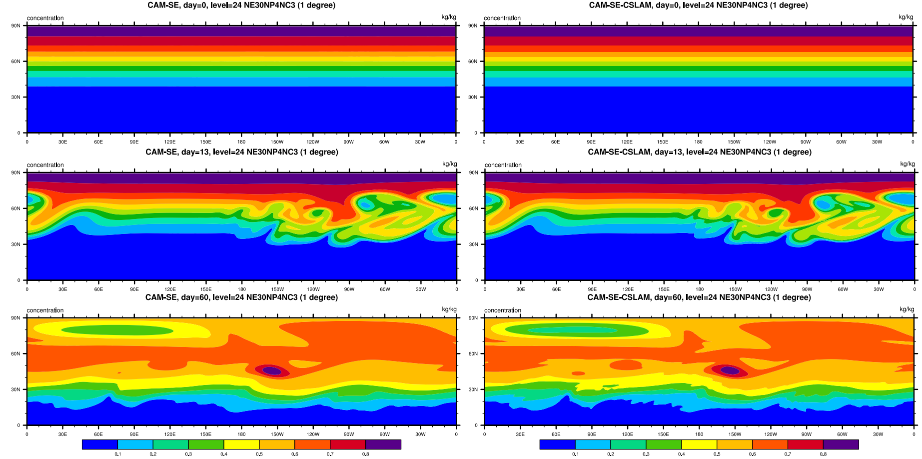
Consistent SE-CSLAM algorithm: flow cases



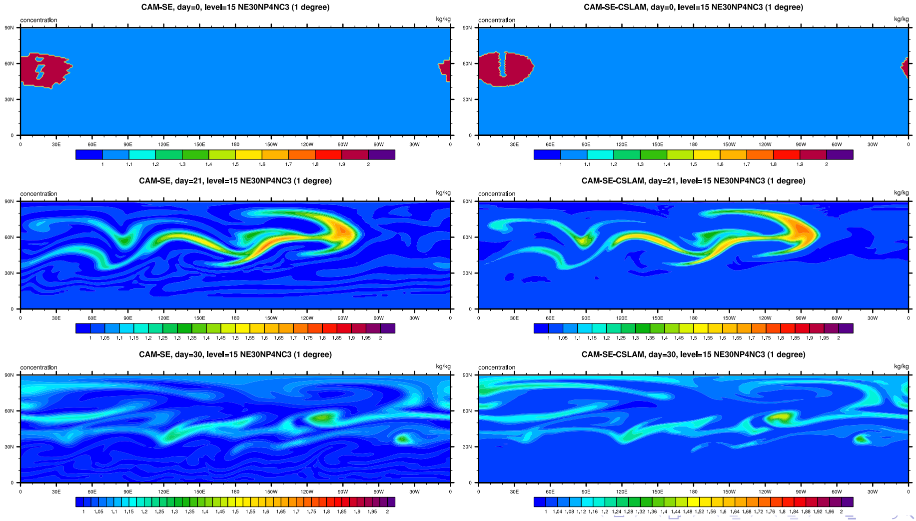
P_s for (left) SE and (right) CSLAM at day 0, 9, 60



Smooth zonally symmetric tracer: (left) SE and (right) CSLAM at day 0, 13 and **60**



Discontinuous tracer: (left) SE and (right) CSLAM at day 0, 21 and **30**



Questions?

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