A Locally Mass-Conservative Version of the Semi-Lagrangian HIRLAM

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Model description

- **Continuity equation** Explicit advection with cell-integrated semi-Lagrangian (CISL) advection scheme (Nair and Machenhauer 2002) based on the discretization of

$$\frac{d}{dt}\left(\frac{\partial p}{\partial \eta}\delta V\right) = 0, \qquad (1)$$

where p pressure, η usual hybrid vertical coordinate, and δV is an infinitesimal volume element in the (λ, θ, η) coordinate system; coupled with semi-implicit time-stepping method described in Lauritzen et al. (2006). Remaining equations of motion are discretized in traditional grid-point form.

- **Stability properties** It has recently been shown (Lauritzen 2006) that two-dimensional CISL schemes have superior stability properties compared to Eulerian flux-form-based advection schemes permitting long time steps such as Lin and Rood (1996) scheme. See Fig. 1 below.

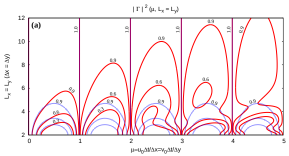
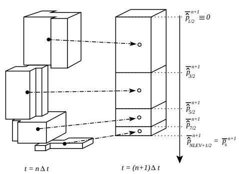


Fig. 1: *Squared modulus of the amplification factor for symmetric modes (u = v=constant) for (blue) cascade CISL scheme of Nair et al. (2002) and (red) flux-form Eulerian scheme of Lin & Rood (1996) as a function of Courant number and wavelength.*

- **Hybrid trajectories** To simplify upstream integrals assume vertically forward and horizontally backward trajectories, that is, cells depart from model layers and move with vertical walls along horizontally backward trajectories (see Fig. on the right). Consequently, the upstream integral is two-dimensional and existing CISL methods are directly applicable. The parcel trajectories do, however, not necessarily arrive at model levels and therefore a vertical remapping/interpolation of the prognostic variables back to model levels is needed after the completion of each time step. The problem is, however, 1D.



- **Consistent vertical displacements** The vertical part of the new model is different from traditional approaches in two ways. Firstly, in the trajectory algorithm the explicit CISL continuity equation is used to track mass departing from a model level and arriving in a regular arrival column. Hereby the vertical displacements are consistent with the hydrostatic assumption and the horizontal flow. Secondly, the discretization of the energy conversion term in the thermodynamic equation is discretized in a Lagrangian fashion and consistent with the discretized semi-implicit CISL continuity equation.

Preliminary tests

Jablonowski-Williamson test case for global dynamical cores:

- The Jablonowski and Williamson (2006) test cases
  1. ***Stationary*** Balanced initial state ($\forall t$: $p_s = 1000$ hPa).
  2. ***Baroclinic wave*** 1. + perturbation in $\vec{v}$. Triggers a well-defined baroclinic wave train in the Northern hemisphere (Fig. 2a).
- **HIRLAM setup** New dynamical core implemented within the framework of HIRLAM, hence numerical domain cannot include the poles, and the elliptic solver cannot deal with periodic boundary conditions. Domain made as global as possible: Active domain (80°W to 280°E)×(80°S,80°N). Dynamic fields relaxed toward initial condition.
- **Effect of boundaries** In test case 1. & 2. the boundary relaxation & elliptic solver in HIRLAM trigger a wave (hereafter referred to as *boundary wave*) that is very similar to the larger scale wave train triggered by the overlaid velocity perturbation (Fig. 2b). To compare limited area model runs with global reference solutions, the spurious *boundary wave* in HIRLAM runs is 'removed' by subtracting the deviation from 1000 hPa in the unperturbed run (1.) from the perturbed run (2.). See Fig. 2c.

CCS	cascade advection scheme of Nair et al. (2002)
CISL	fully 2D advection scheme of Nair and Machenhauer (2002)
COP	consistent discretization of energy conversion term in thermodynamic equation
EUL	Spectral Eulerian dynamical core for CAM.
FV	Finite-volume dynamical core for CAM
R	traditional (inconsistent) discretization of energy conversion term in T-equation
REF-HIRLAM	reference HIRLAM

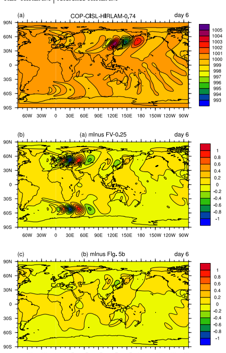


Fig. 2: *p_s (hPa) at day 6 for Jablonowski-Williamson test case 2 for new dynamical core: (a) p_s unmodified, (b) p_s in (a) minus reference solution FV, (c) p_s in (b) minus new dynamical core run for stationary test case 1.*

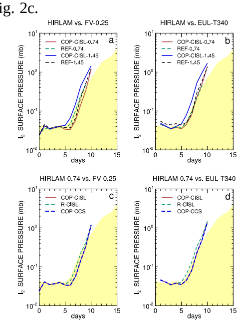


Fig. 3: *l_2 norms of p_s (hPa) between HIRLAM versions and global reference solutions as a function of time. In the left/(right) column the FV/(EUL) is the global reference solution. The yellow region is the uncertainty of the reference solutions defined in Jablonowski and Williamson (2006). (a) and (b) is the l_2 difference for the high(red solid line)/low(blue solid line) resolution run with COP-CISL and the the high(green dashed line)/low(black dashed line) resolution REF-HIRLAM. (c) and (d) is the l_2 difference for COP-CCS (blue dashed line), R-CISL (green dashed line), and COP-CISL (red solid line), respectively.*

high res.	=	$0.74^\circ \times 0.59^\circ, \Delta t = 15$ min., 27 lev.
low res.	=	$1.45^\circ \times 1.15^\circ, \Delta t = 30$ min., 27 lev.

Standalone forecast test:

Dynamical core coupled with DMI-HIRLAM physics package and tested with 24h forecast of severe storm (December 3, 1999).

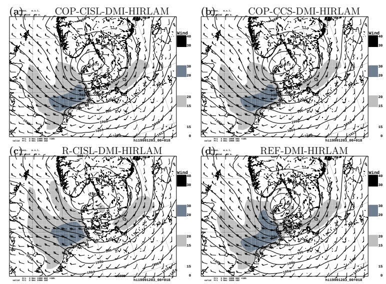


Fig. 3: *Mean sea-level pressure (solid isollines plotted every 4 hPa) and wind field 10 m above ground level (shading in white 0-15 ms^{-1}, light grey 15-20 ms^{-1}, dark grey 20-30 ms^{-1} and black >30 ms^{-1}) for an 18 h forecast with (a) COP-CISL-DMI-HIRLAM, (b) COP-CCS-DMI-HIRLAM, (c) R-CISL-DMI-HIRLAM, and (d) REF-DMI-HIRLAM. Resolution: $\Delta\lambda = \Delta\theta = 0.4^\circ$, 40 levels, $\Delta t = 6$ min.*

Summary

- A semi-Lagrangian semi-implicit limited area model that is locally mass-conservative and that enforces more consistent discretizations has been derived.
- In preliminary tests the new model with and without physics was stable for long time steps and no filters (decentering,etc.) were applied except for ∇^6 horizontal diffusion.
- The importance of consistent discretization of conversion term demonstrated.
- New model can perform *online* transport with a very high level of consistency
- Compared to the unmodified HIRLAM the new model versions are slightly more diffusive.
- Numerical weather prediction and tracer advection tests are currently being performed.

References

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