Monotone and conservative Cascade Remapping between Spherical grids (CaRS): Regular latitude-longitude and cubed-sphere grids

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Introduction

The cubed-sphere grid system (see Fig.1) has been adopted for many new generation dynamical cores. However, coupling these dynamical cores with model components developed in conventional latitude-longitude based spherical geometry requires reliable conservative re-gridding schemes between the two grid systems. It is highly desirable that the remapping process is conservative and optionally monotone. A monotone and conservative remapping algorithm specifically designed to remap between the cubed-sphere and regular latitude-longitude grids is presented herein (Lauritzen and Nair 2007).

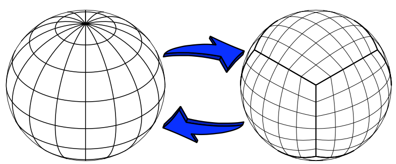

Fig.1: *The regular latitude-longitude (left) and the cubed-sphere grid (right)*

Cascade interpolation

This algorithm is based on the *cascade interpolation* method developed for semi-Lagrangian advection schemes (Purser and Leslie 1991) in which a two-dimensional interpolation problem is split into two one-dimensional problems. This allows for high-order sub-grid-cell reconstructions as well as the application of advanced monotone filters (Zerroukat et al. 2005).

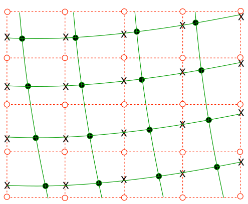

Fig. 2: *A schematic illustration of a two-dimensional cascade interpolation from a source grid to a target grid. The intermediate grid intersections are defined as the intersection of source grid longitudes with target grid latitudes and are marked by cross marks (×).*

The cascade interpolation procedure is split into two 'sweeps':

1. Interpolation from '○' to '×' along the source grid longitudes.
2. The resulting field is interpolated from '×' to '○' along the target grid latitudes.

Note that the cascade algorithm will be most accurate when the two sweeps are approximately orthogonal to each other. The cascade method can be applied when the source and target grids are structured, that is, they can be defined in terms of grid lines similarly to the latitudes and longitudes of the regular latitude-longitude grid.

Cubed-sphere latitudes and longitudes

Traditionally the grid-lines of the cubed-sphere are defined panelwise by employing a local Cartesian grid system. For easy implementation of the cascade method we reconstruct the entire cubed-sphere grid system with a family of horizontal and vertical grid lines similar to the longitudes and latitudes of the regular latitude-longitude grid (see Fig.3).

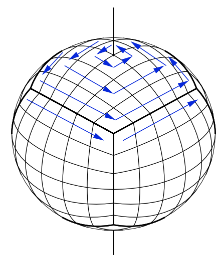

Fig. 3: *We define the cubed-sphere latitudes as vertically stacked closed curves (squared patterns) centered about the vertical axis of the sphere. Note that the cubed-sphere latitude lines are only C^0 continuous at the four corners. The cubed-sphere latitude belts are intersected by a set of cubed-sphere longitudes. On the equatorial panels each cubed-sphere latitude belt has the same number of cubed-sphere longitudes. But for the polar panels the number of cubed-sphere longitude intersections decreases from the side to the center.*

Having defined the source and target grids in terms of 'longitudes' and 'latitudes' the cascade procedure conceptually explained in Fig.2 can be applied.

Higher-order sub-grid-cell reconstruction

For the one-dimensional remapping sweeps we use polynomial reconstruction methods commonly used in finite-volume advection schemes: We apply the piecewise cubic method (PCM) introduced by Zerroukat et al. (2005) as well as the piecewise spline method (PSM; Zerroukat et al. 2006).

The monotone filter reduces the order of the sub-grid-cell reconstruction until monotonicity violating sub-grid-cell variation is eliminated (Fig.4), for example, if the PCM has a spurious under- or overshoot in a grid cell the cubic polynomial in that cell is reduced to a parabola (corresponding to a piecewise parabolic method (PPM) reconstruction); if needed the parabola is further reduced to a linear profile and similarly to a piecewise constant function until monotonicity is obtained.

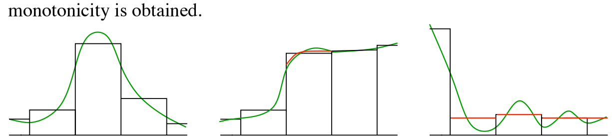

Fig.4: *Unmodified polynomial sub-grid-cell reconstruction (green) computed from cell averages (black). Red lines show the reconstruction in cells where the monotone filter alters the sub-grid-cell polynomial order.*

Results

CaRS is tested by remapping analytic functions (Fig.5) from a coarse regular latitude-longitude (128×64) grid to a high-resolution cubed-sphere grid (130×130×6). Results are compared to the more general remapping method SCRIP (Jones 1999) in which the two-dimensional mass-integrals are converted to line integrals using Gauss's divergence theorem. The method is second-order and based on cell-averages and user-specified estimates of the gradient. Here we approximate the gradient using the polynomial reconstruction methods used for CaRS which allow for the application of monotone filters as described in previous section.

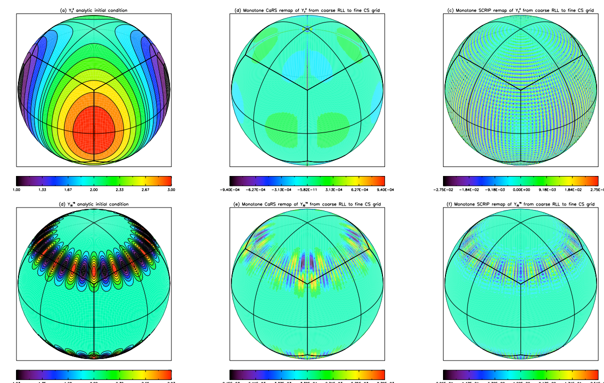

Fig.5: *Exact field (left column) and difference plots for remapping with CaRS (middle) and SCRIP (right) employing PCM with monotone filter. In Table below are standard error measures.*

Method	Field	l_1	l_2	l_∞	Field	l_1	l_2	l_∞
SCRIP-PCM-M	Y_2^2	4.4915×10^{-3}	3.0460×10^{-5}	9.1806×10^{-3}	Y_{32}^{16}	1.1458×10^{-2}	5.4858×10^{-4}	8.3619×10^{-2}
CaRS-PCM-M	Y_2^2	2.7011×10^{-5}	1.4830×10^{-9}	3.1279×10^{-4}	Y_{32}^{16}	4.8992×10^{-4}	9.2326×10^{-7}	3.1809×10^{-3}
CaRS-PSM-M	Y_2^2	2.7026×10^{-5}	1.3803×10^{-9}	2.8937×10^{-4}	Y_{32}^{16}	3.7711×10^{-4}	5.9458×10^{-7}	3.1124×10^{-3}

References

Jones, P. W., 1999: First- and second-order conservative remapping schemes for grids in spherical coordinates. *Mon. Wea. Rev.*, **127**, 2204–2210.

Lauritzen, P. and R. Nair, 2007: Conservative and monotone cascade remapping between spherical grids: Regular latitude-longitude and cubed-sphere grids. *Mon. Wea. Rev*, in revision.

Purser, R. and L. Leslie, 1991: An efficient interpolation procedure for high-order three-dimensional semi-Lagrangian models. *Mon. Wea. Rev.*, **119**, 2492–2498.

Zerroukat, M., N. Wood, and A. Staniforth, 2005: A monotonic and positive-definite filter for a semi-Lagrangian inherently conserving and efficient (SLICE) scheme. *Q. J. R. Meteorol. Soc.*, **131**, 2923–2936.

— 2006: The parabolic spline method (PSM) for conservative transport problems. *Int. J. Numer. Meth. Fluids*, **51**, 1297–1318.