Isoneutral Mixing Operators In Oceanic Models : Numerical Delicacies

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Grenoble

The MOISE Inria team

Numerical Models

Local mesh refinement, ocean-atmosphere coupling, numerical methods

E. Blayo, L. Debreu, F. Lemarié

Variational Data assimilation

F.-X. LeDimet, A. Vidard, E. Kazantsev, M. Nodet

Uncertainty quantification

C. Prieur, C. Helbert

Outline

1. Spurious diapycnal mixing: The problem and a solution, Marchesiello et al (Ocean Modelling, 2009)
2. On the stability and accuracy of the harmonic and biharmonic isoneutral mixing operators in ocean models, Lemarié et al (Ocean Modelling, 2012)
3. Other problems and perspectives, Demange et al (in preparation)

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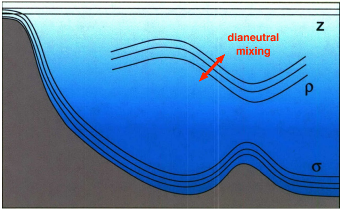
Spurious diapycnal mixing: The problem and a solution

- Tracer mixing in a stratified ocean (away from turbulent boundary layers)
  - $\kappa_{\mathrm{dia}} \approx 10^{-5} \mathrm{m\ s}^{-1}$ (e.g. Ledwell et al., 1993)
  - $\kappa_{\mathrm{iso}} \approx 10^{3} \mathrm{m\ s}^{-1}$ (horizontal scale ≈ 100 km)

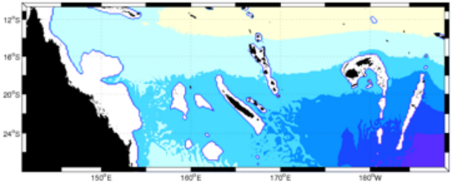
Vertical coordinates systems:



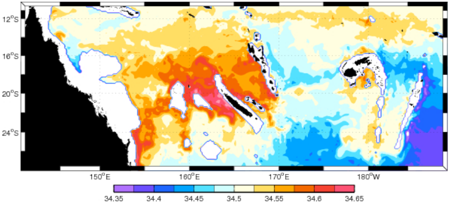
Main problems of terrain following (σ) (and geopotential z) models:
Pressure gradient error and Diapycnal Mixing
Stronger in ocean models than in atmospheric models

Spurious diapycnal mixing: The problem and a solution

ROMS model, terrain following coordinates, Third order upstream biased scheme for tracers. Salinity at 1000m depth



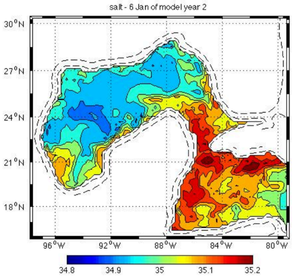
Climatology



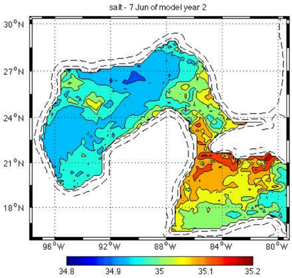
After 2 years of integration

The (implicit) diffusion of the upstream biased scheme acts along horizontal coordinates . . .

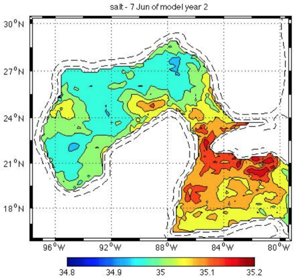
ROMS model, σ coordinates, Third order upwind scheme for tracers
Salitiny at 1000m after 2 years of integration



Upstream order 3



Upstream order 5

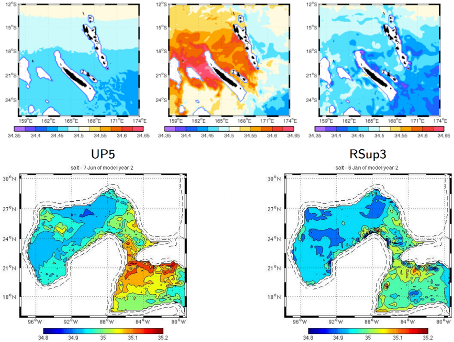


Weno 5

A solution

Rotate the diffuse part of the upstream biased scheme:

$$\left.\frac{\partial q}{\partial x}\right|_{(2n+1)order} = \left.\frac{\partial q}{\partial x}\right|_{(2n+2)order} + (-1)^n K \underbrace{\frac{\partial^{(2n+2)} q}{\partial x^{2n+2}}}_{rotated}$$



Numerical issue

Rotation of the diffusive part introduces mixed (horizontal/vertical) and purely vertical derivatives

Stabilization of a (high order) rotated diffusion operator

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Isoneutral mixing problem (continuous formulation)

▷ Isoneutral Laplacian operator

Tracer field q in an unbounded domain $\Omega = \mathbb{R}^3$

$$\begin{cases} \partial_t q = \mathcal{D}_2(q) = \nabla \cdot (\mathbf{R} \nabla q) = -\nabla \cdot \mathbf{F} & \text{in } \Omega \times [0, T], \\ q|_{t=0} = q_0(x, y, z) & \text{in } \Omega, \end{cases}$$

the matrix form of the tensor $\mathbf{R}$ is [Redi, 1982; Gent & McWilliams, 1990]

$$\mathbf{R} = \begin{pmatrix} \kappa_x & 0 & \kappa_x \alpha_x \\ 0 & \kappa_y & \kappa_y \alpha_y \\ \kappa_x \alpha_x & \kappa_y \alpha_y & \kappa_x \alpha_x^2 + \kappa_y \alpha_y^2 \end{pmatrix}, \quad \text{with} \quad \boldsymbol{\alpha} = (\alpha_x, \alpha_y) = -\frac{(\partial_x \rho, \partial_y \rho)}{\partial_z \rho}, \quad \|\boldsymbol{\alpha}\| \ll 1.$$

Properties

- Orthogonality condition : $\mathbf{F} \cdot \boldsymbol{\rho}_\perp = 0$
- Satisfy monotonicity principle [Mathieu and Deleersnijder, 1998]
- Satisfy global tracer variance dissipation [Griffies et al., 1998]

Isoneutral mixing problem (continuous formulation)

▷ Isoneutral biharmonic operator

The rotated biharmonic operator reads

$$\begin{cases} \partial_t q &= \mathcal{D}_4(q) = -\mathcal{D}_2(\Psi) = -\nabla \cdot \mathbf{F_4}, \qquad \text{with } \Psi = \mathcal{D}_2(q), \\ q\big|_{t=0} &= q_0(x,y,z), \end{cases}$$

composition of two isoneutral Laplacian operators

Properties

- Orthogonality condition : $\mathbf{F_4} \cdot \boldsymbol{\rho}_\perp = 0$
- ~~Satisfy monotonicity principle [Mathieu and Deleersnijder, 1998]~~
- Satisfy global tracer variance dissipation [Griffies, 2004]

Time discretization

[Lemarié et al., 2012]

Objectives :

- same stability limit of the non-rotated operators
- only the vertical direction can be implicit

Proposed scheme :

→ **Laplacian**

$$\mathcal{D}_2(q) = \underbrace{\partial_x(\kappa_x [\partial_x q + \alpha_x \partial_z q]) + \partial_z (\alpha_x \kappa_x \partial_x q)}_{explicit} + \underbrace{\partial_z \left(\kappa_x \alpha_x^2 \partial_z q\right)}_{G_3(q)}$$

$$\begin{cases} q^\star &= q^n + \Delta t \mathcal{D}_2(q^n) \\ q^{n+1} &= q^\star + \theta \Delta t \left[G_3(q^{n+1}) - G_3(q^n)\right] \end{cases}$$

Method of Stabilizing Corrections [van der Houwen & Verwer, 1979; Hundsdorfer, 2002]

Time discretization

[Lemarié et al., 2012]

Objectives :

- same stability limit of the non-rotated operators
- only the vertical direction can be implicit

Proposed scheme :

→ **Biharmonic**

$$\begin{cases} q^\star &= q^n + \Delta t \mathcal{D}_4(q^n) \\ q^{n+1} &= q^\star + \Delta t \partial_z \left[\widetilde{\kappa} \partial_z q^{n+1} - \widetilde{\kappa} \partial_z q^n \right] \end{cases}$$

$\widetilde{\kappa}$ chosen through linear stability analysis

$$\widetilde{\kappa} = 8 \Delta z^2 \sigma_x s_x^2 (1 + s_x^2)/\Delta t \qquad \Rightarrow \qquad \sigma_x \leq \frac{1}{8}$$

with $s_x = \alpha_x \dfrac{\Delta x}{\Delta z}$, $\qquad \sigma_x = \kappa_x \dfrac{\Delta t}{\Delta x^4}$

Conclusions of Lemarié et al (2012)

- the rotated operators can be advanced with the same time step as the non-rotated ones !!
- the rotated biharmonic can be a viable operator for use in high-resolution global models
- clipping/tapering should act on **s** rather than α
- a slope-dependent discretization provides more accuracy notably for large grid slope ratios
- has been implemented in ROMS and NEMO (also for rotation of hyperviscosity)

Alternatives to the use of rotated operators:

- No major improvements so far with higher-order advection schemes ...
- Arbitrary-Lagrangian-Eulerian (ALE) vertical coordinate $\rightarrow$ strong ongoing effort

Remaining open issue:

- Monotone advection schemes and diapycnal mixing

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Other sources of artificial diffusion (and dispersion)

- Amount of numerical viscosity and vertical advection schemes
- Barotropic/Baroclinic time splitting
- Internal Gravity Wave propagation

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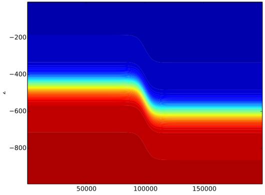
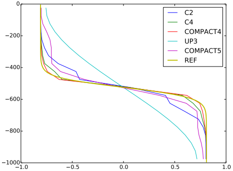
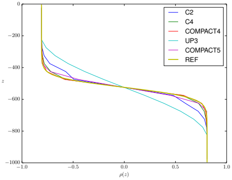
- Amount of numerical viscosity and vertical advection schemes



Initial condition



Momentum : UP3



Momentum : UP1

- Barotropic/Baroclinic time splitting
- Internal Gravity Wave propagation

Other sources of artificial diffusion (and dispersion)

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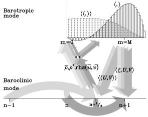
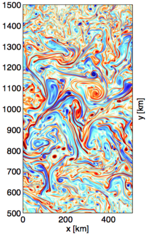
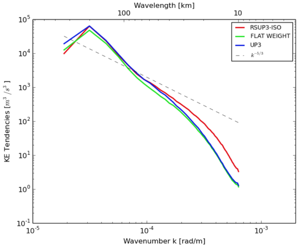


Shchepetkin and McWilliams (2005)



Baroclinic Jet



KE spectrum

- Internal Gravity Wave propagation

Other sources of artificial diffusion (and dispersion)

- Amount of numerical viscosity and vertical advection schemes
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$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{1}{\rho_0}\dfrac{\partial p}{\partial x} = 0 \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial w}{\partial z} = 0 \\ \dfrac{\partial p}{\partial z} = -\rho g \\ \dfrac{\partial \rho}{\partial t} + w\dfrac{\partial \bar{\rho}}{\partial z} = 0 \end{cases}$$

Very basic numerics:

- Second order discretization
- Computational mode of the Lorenz grid
- No dissipation

Decomposition into vertical modes and characteristic variables

$$u(x,z,t) = \sum u_n(x,t) M_n(z)$$
$$\rho(x,z,t) = -\rho_0 \sum h_n(x,t) \frac{\mathrm{d}M_n(z)}{\mathrm{d}z}$$

$$\frac{\partial}{\partial t}(u_n \pm \frac{g}{c_n} h_n) \pm c_n \frac{\partial}{\partial x}(u_n \pm \frac{g}{c_n} h_n) = 0$$

Other sources of artificial diffusion (and dispersion)

- Amount of numerical viscosity and vertical advection schemes
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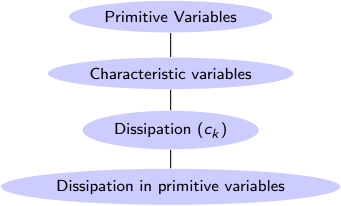
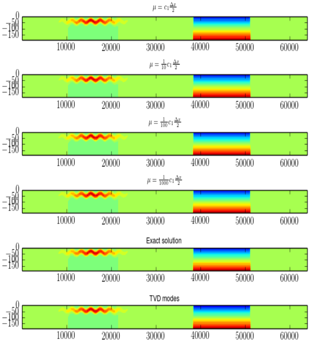
Monotonicity of characteristic variables in the primitive variables formulation ?

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{1}{\rho_0}\dfrac{\partial p}{\partial x} = \dfrac{c_1 \Delta x}{2}\dfrac{\partial^2 u}{\partial x^2} \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial w}{\partial z} = 0 \\ \dfrac{\partial p}{\partial z} = -\rho g \\ \dfrac{\partial \rho}{\partial t} + w\dfrac{\partial \bar{\rho}}{\partial z} = \dfrac{c_1 \Delta x}{2}\dfrac{\partial^2 \rho}{\partial x^2} \end{cases}$$



Internal Wave propagation



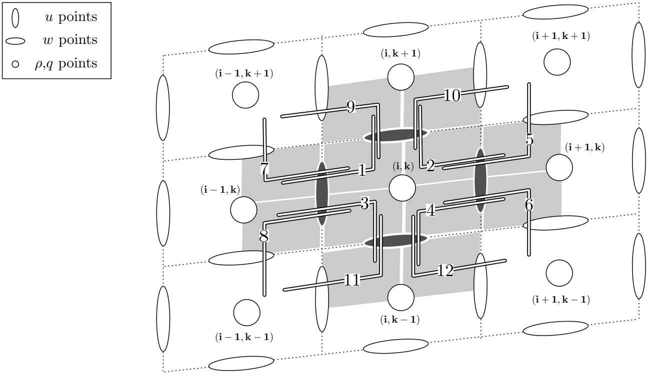
References

- Marchesiello, P., Debreu, L., Couvelard, X., 2009. Spurious diapycnal mixing in terrain-following coordinate models: The problem and a solution. Ocean Modell. 26, 159-169.
- Lemarié, F., Debreu, L., Shchepetkin, A.F., McWilliams, J.C., 2012. On the stability and accuracy of the harmonic and biharmonic isoneutral mixing operators in ocean models, Ocean Modell. , 52-53, 9-35.
- Demange J., Debreu L., Marchesiello P., 2014: On the use of a depth-dependent barotropic mode for free surface ocean models, to be submitted
- Demange J., Debreu L., Marchesiello P., 2014: Vertical mode decomposition and dissipation, to be submitted

Properties of the discretized operator ?

$$\mathcal{D}_2(q) = \partial_x \left(\kappa_x \frac{\mathcal{J}_x(q,\rho)}{\partial_z \rho} \right) - \partial_z \left(\kappa_x \frac{\partial_x \rho}{\partial_z \rho} \frac{\mathcal{J}_x(q,\rho)}{\partial_z \rho} \right)$$

$$\mathcal{J}_x^u = (\partial_x q)\,\widetilde{(\partial_z \rho)} - (\partial_x \rho)\,\widetilde{(\partial_z q)}, \qquad \mathcal{J}_x^w = \widetilde{(\partial_x q)}\,(\partial_z \rho) - \widetilde{(\partial_x \rho)}\,(\partial_z q)\,.$$



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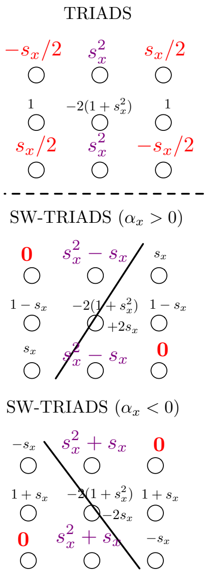
$$\mathcal{J}_x^u = (\partial_x q)\,\widetilde{(\partial_z \rho)} - (\partial_x \rho)\,\widetilde{(\partial_z q)}, \qquad \mathcal{J}_x^w = \widetilde{(\partial_x q)}\,(\partial_z \rho) - \widetilde{(\partial_x \rho)}\,(\partial_z q)\,.$$

	ref.	min-max	TVD[1]	$\mathbf{F}(\rho) = 0$	misc.
COX	[Cox, 1987]	no	no	no	$2\Delta x$ mode
TRIADS	[Griffies et al., 1998]	no	yes	yes	
SW-TRIADS	[Lemarié et al., 2012]	no	yes	yes	most compact stencil

→ all linear schemes produce over/under shootings [Beckers et al., 2000]

[1] Tracer Variance Dissipation

Spatial discretization and stability limits



	TRIADS	SW-TRIADS
Laplacian		
Rotated	$\sigma_x(1+s_x^2) \leq 1/2$	$\sigma_x \max(1, s_x^2) \leq 1/2$
Non-rotated	$\sigma_x \leq 1/2$	
Biharmonic		
Rotated	$\left(\sigma_x^{(4)}(1+s_x^2)\right)^2 \leq 1/8$	$\left(\sigma_x^{(4)} \max(1, s_x^2)\right)^2 \leq 1/8$
Non-rotated	$\left(\sigma_x^{(4)}\right)^2 \leq 1/8$	

with $\sigma_x = \kappa_x \dfrac{\Delta t}{\Delta x^2}$, $\quad s_x = \alpha_x \dfrac{\Delta x}{\Delta z}$, $\quad \sigma_x^{(4)} = \sqrt{B_x}\dfrac{\sqrt{\Delta t}}{\Delta x^2}$

Adaptive Mesh Refinement for ocean models

- Blayo and Debreu, 1999: Adaptive Mesh Refinement for Finite-Difference Ocean Models: First experiments, JPO

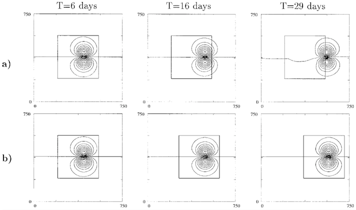


FIG. 3. Numerical solution of the modon problem at three different time steps (CI: 1000 m^2 s^{-1}) for the (a) nested-grid calculation and (b) AMR calculation. The location of the fine grid is indicated.

→ everything works fine !!!

- Debreu et al, 2005: A general adaptive multi-resolution approach to ocean modelling : experiments in a primitive equation model of the north atlantic. Adaptive Mesh Refinement - Theory and Applications, Lecture Notes in Computational Science and Engineering

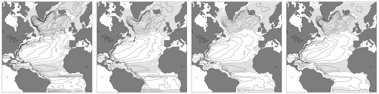


Fig. 3. Barotropic streamfunction : from left to right: uniform 1/3° simulation (reference simulation), uniform 1/2° simulation, uniform uniform 1° simulation, adaptive 1°-1/3° simulation. Contour interval is of 5 Sverdrup

→ refinement criterion ?
→ How to define the bathymetry across the grid hierarchy ?

~~Adaptive~~ Mesh Refinement for ocean models

Before doing adaptive mesh refinement, are we sure than fixed mesh refinement is doing well ?

- Debreu and Blayo, 2008: Two-way embedding algorithms : a review. Ocean Dynamics
- Debreu et al, 2012: Two-way nesting in split-explicit ocean models : Algorithms, implementation and validation. Ocean Modelling
  - High order update schemes / conservation and refluxing
  - Coupling at the fast modes (barotropic modes) level
- Debreu et al, 2008: AGRIF: Adaptive grid refinement in fortran. Comput. Geosci.
  A library implementing Berger and Oliger algorithm and a source-to-source (monogrid to multigrid) translator

Current developments in ocean models:
Different vertical grids and potentially different vertical coordinate systems between coarse and fine grids