PDE on the Sphere 2014

Development of a non-hydrostatic vertical slice model based on the spectral element method and mass-based vertical coordinate

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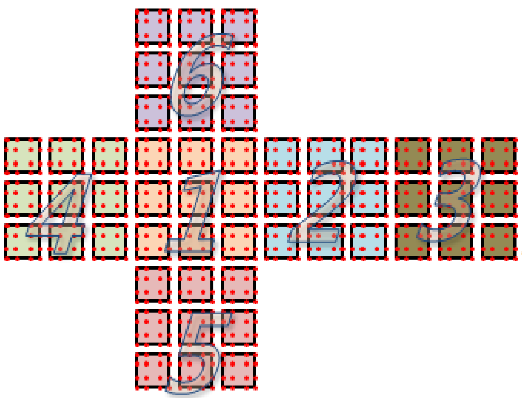
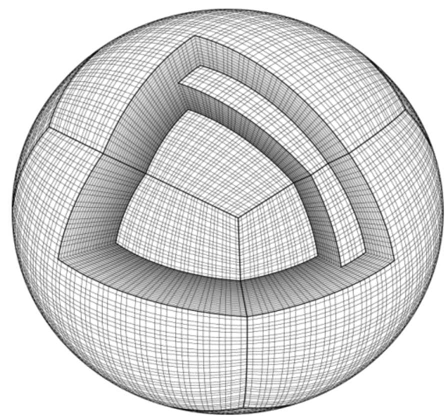
- Modeling Framework and Components
- Slice Model A
- Slice Model B
- Experiments: Benchmark tests
- Experiments: Moist simulations with a microphysics
- Plans for further research and development

Background of the Development

KIAPSGM: 3D Hydrostatic KIAPS-Global Model for short-range forecasts



Quadrature points on the cube



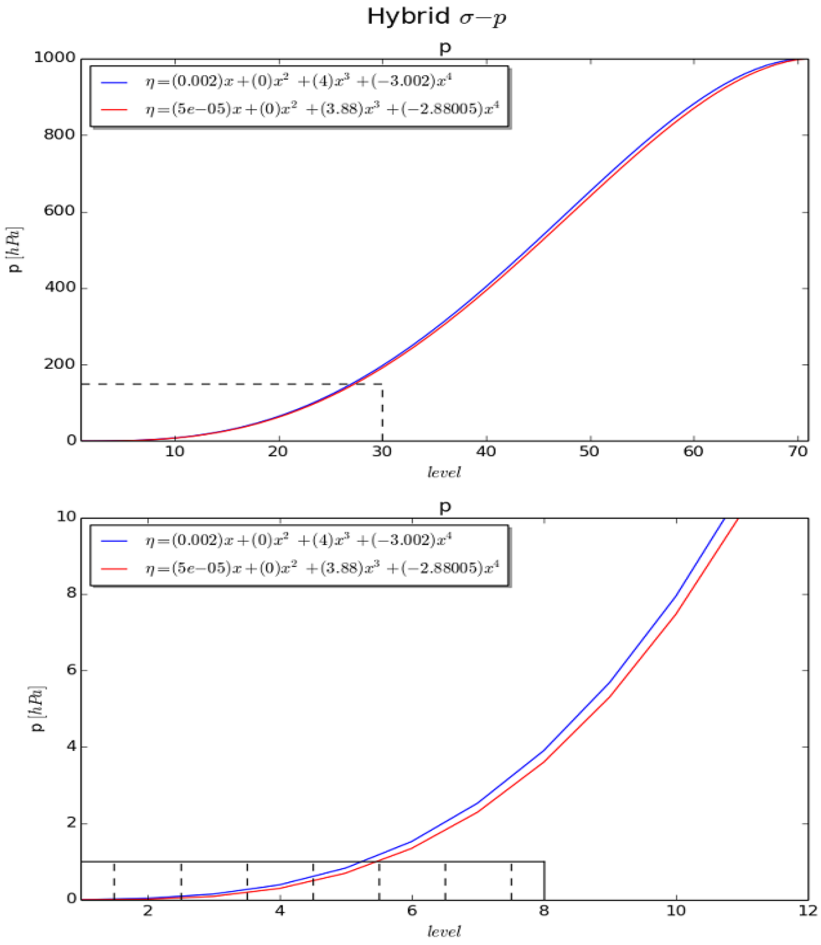
Gnomonic projection on the sphere

Dynamic core:

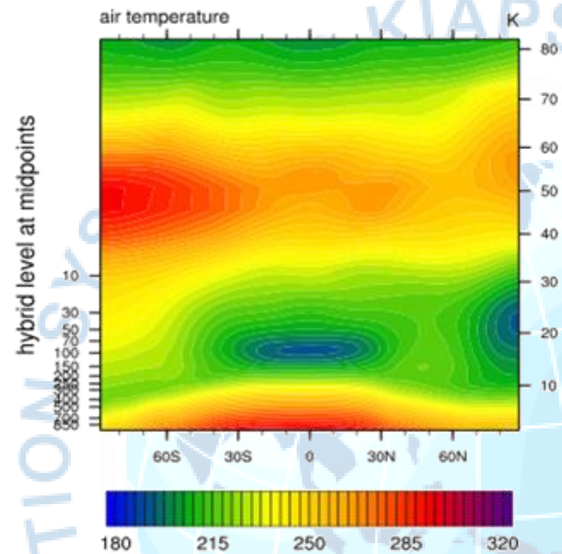
A **Spectral Element Method** dynamical core of the High-Order Method Modeling Environment (**HOMME**)

$$
\begin{aligned}
\frac{\partial \mathbf{V_k}}{\partial t} &= -(\zeta_k + f_k)\hat{k} \times \mathbf{V_k} - \nabla_\eta \left(\frac{1}{2}\mathbf{V_k}\cdot\mathbf{V_k} + \Phi_k\right) - \left(\dot{\eta}\frac{\partial \mathbf{V}}{\partial \eta}\right)_k - \frac{R_d (T_v)_k}{p_k}\nabla_\eta p_k \\
\frac{\partial T_k}{\partial t} &= -\mathbf{V_k}\cdot\nabla_\eta T_k - \left(\dot{\eta}\frac{\partial T}{\partial \eta}\right)_k + \frac{R_d (T_v)_k}{c_p^*}\frac{\omega_k}{p_k} \\
\frac{\partial p_S}{\partial t} &= -\int_0^1 \nabla_{\eta'}\cdot(\pi'\mathbf{V})\, d\eta' = -\sum_{i=1}^{L}\left(\nabla_\eta\cdot(\pi\mathbf{V})\right)_k \Delta\eta_i
\end{aligned}
$$

KIAPSGM: 3D Hydrostatic KIAPS-Global Model for short-range forecasts



Vertical Coordinate:
A **hybid sigma-pressure system with 70 Levels extended up to 0.02 hPa**



Zonal Mean temperature of the averaged value of 15 day forecasts

Laprise (1992) and two variations

A terrain-following, mass-based hybrid sigma-pressure coordinate

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= -u\nabla u - \dot{\eta}\frac{\partial u}{\partial \eta} - \frac{R_d T_v}{p}\nabla p - (\nabla_\eta \Phi)\frac{\partial p}{\partial p^h} \\
\frac{\partial w}{\partial t} &= -u\nabla w - \dot{\eta}\frac{\partial w}{\partial \eta} - g(1 - \frac{\partial p}{\partial p^h}) \\
\frac{\partial T}{\partial t} &= -u\nabla T - \dot{\eta}\frac{\partial T}{\partial \eta} - \frac{R_d T_v}{c_v^*} D_3 \\
\frac{\partial p}{\partial t} &= -u\nabla p - \dot{\eta}\frac{\partial p}{\partial \eta} - \frac{c_p^*}{c_v^*} p D_3 \\
\frac{\partial p_s}{\partial t} &= -\int_0^1 \nabla \cdot \left(u \frac{\partial p^h}{\partial \eta'}\right) d\eta'
\end{aligned}
$$

$$
D_3 = \nabla_\eta \cdot u + \frac{p}{R_d T_v} \frac{1}{\frac{\partial p^h}{\partial \eta}} \left[(\nabla_\eta \Phi)\frac{\partial u}{\partial \eta} - g\frac{\partial w}{\partial \eta} \right]
$$

Aladin

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= -u\nabla_\eta u - \dot{\eta}\frac{\partial u}{\partial \eta} - \frac{R_d T}{p^h(1+F)}\nabla_\eta \left[p^h(1+F)\right] - (\nabla_\eta \Phi)\left[(1+F)\frac{\partial p^h}{\partial \eta} + p^h \frac{\partial F}{\partial \eta}\right] / \frac{\partial p^h}{\partial \eta} + \mathcal{V} \\
\frac{\partial w}{\partial t} &= -u\nabla_\eta w - \dot{\eta}\frac{\partial w}{\partial \eta} + g\left[\frac{\partial (p^h F)}{\partial \eta} / \frac{\partial p^h}{\partial \eta}\right] + \mathcal{W} \\
\frac{\partial T}{\partial t} &= -u\nabla_\eta T - \dot{\eta}\frac{\partial T}{\partial \eta} - \frac{R_d T_v}{c_v} D_3 + \frac{Q}{c_v} \\
\frac{\partial F}{\partial t} &= -u\nabla_\eta F - \dot{\eta}\frac{\partial F}{\partial \eta} - (1+F)\frac{1}{p^h}\frac{\mathrm{d}p^h}{\mathrm{d}t} - \frac{c_p}{c_v}(1+F)D_3 \\
\frac{\partial p_s^h}{\partial t} &= -\int_0^1 \nabla_{\eta'} \cdot \left(\frac{\partial p^h}{\partial \eta'} u\right) \mathrm{d}\eta'
\end{aligned}
$$

$$
d = -\frac{gp}{\frac{\partial p^h}{\partial \eta} R_d T} \frac{\partial w}{\partial \eta}
$$

$$
F = \frac{p - p^h}{p^h}
$$

$$
\Phi = \Phi_s + \int_\eta^{1(\eta_s)} \frac{R_d T}{p^h(1+F)} \frac{\partial p^h}{\partial \eta'} \mathrm{d}\eta'
$$

$$
D_3 = \nabla_\eta \cdot u + \frac{p^h(1+F)}{R_d T} / \frac{\partial p^h}{\partial \eta} (\nabla_\eta \Phi) \cdot \frac{\partial u}{\partial \eta} - g \frac{p^h(1+F)}{R_d T} / \frac{\partial p^h}{\partial \eta} \frac{\partial w}{\partial \eta}
$$

WRF

$$
\mathrm{V} = \mu \mathrm{v} = (U, V, W), \quad \Omega = \mu \dot{\eta}, \quad \Theta = \mu \theta.
$$

$$
\phi = gz, \quad p, \quad \alpha = 1/\rho
$$

For dry atmosphere, the flux-form Euler Equations

$$
\begin{aligned}
&\partial_t U + (\nabla \cdot \mathrm{V}u) + \mu_d \alpha_d \partial_x p + \partial_\eta p \partial_x \phi = F_{U\,coriolis+curvature+\ldots} \\
&\partial_t V + (\nabla \cdot \mathrm{V}v) + \mu_d \alpha_d \partial_y p + \partial_\eta p \partial_y \phi = F_{V\,coriolis+curvature+\ldots} \\
&\partial_t W + (\nabla \cdot \mathrm{V}w) - g[\partial_\eta p - \mu_d] = F_{W\,coriolis+curvature+\ldots} \\
&\partial_t \Theta + (\nabla \cdot \mathrm{V}\theta) = F_\Theta \\
&\partial_t \mu_d + (\nabla \cdot \mathrm{V}) = 0 \\
&\partial_t \phi + \frac{1}{\mu_d}[(\mathrm{V} \cdot \nabla \phi) - gW] = 0
\end{aligned}
$$

$$
\partial_\eta \phi = -\alpha_d \mu_d
$$

$$
p = p_0(R_d \theta_d / p_0 \alpha_d)^\gamma, \quad \text{where } \gamma = c_p / c_v
$$

Two formulations

Common features

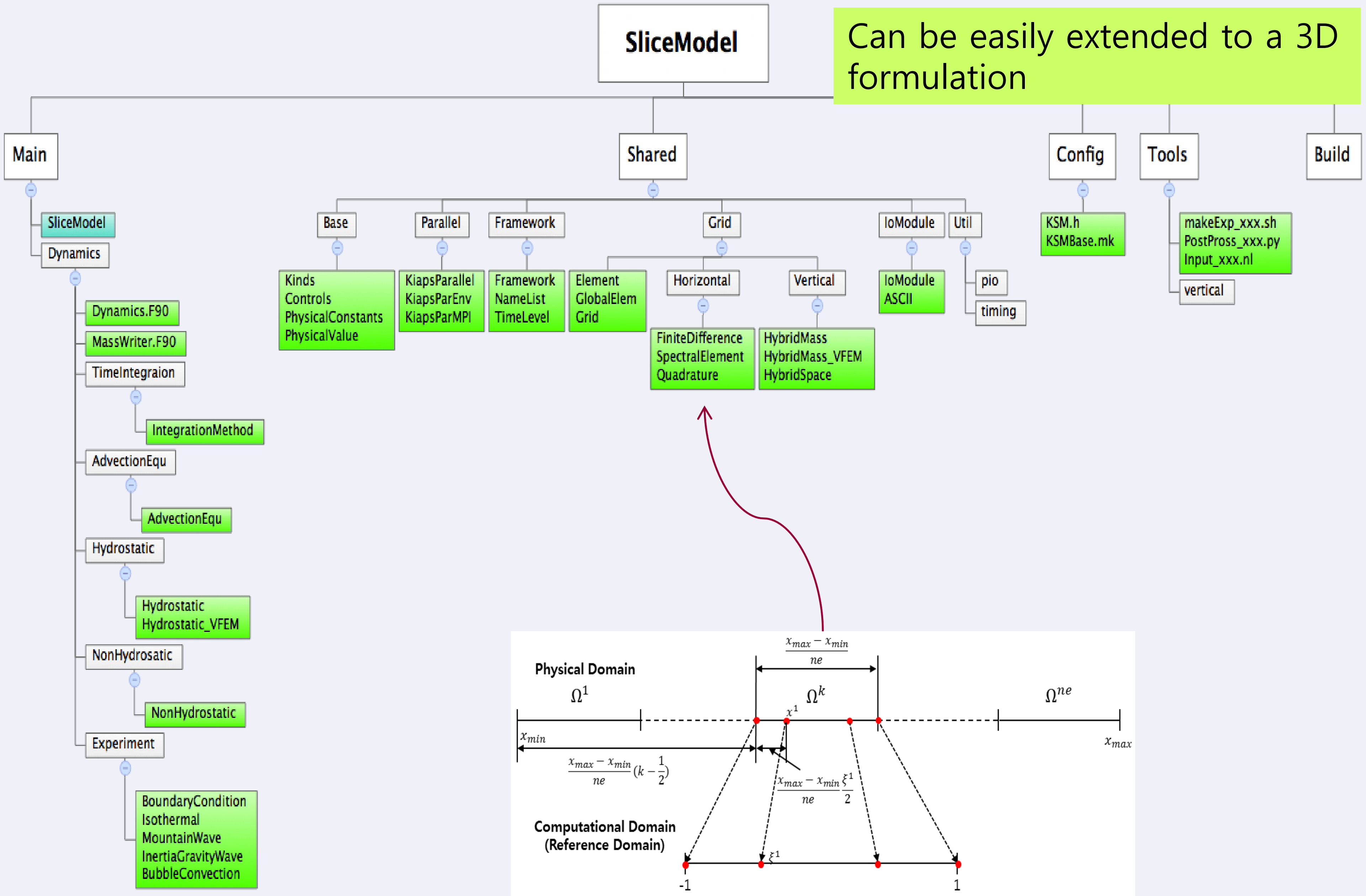
- Spectral Element method (horizontal discretization)
- Finite Difference method (vertical discretization)
- Hybrid sigma-pressure coordinate

Differences

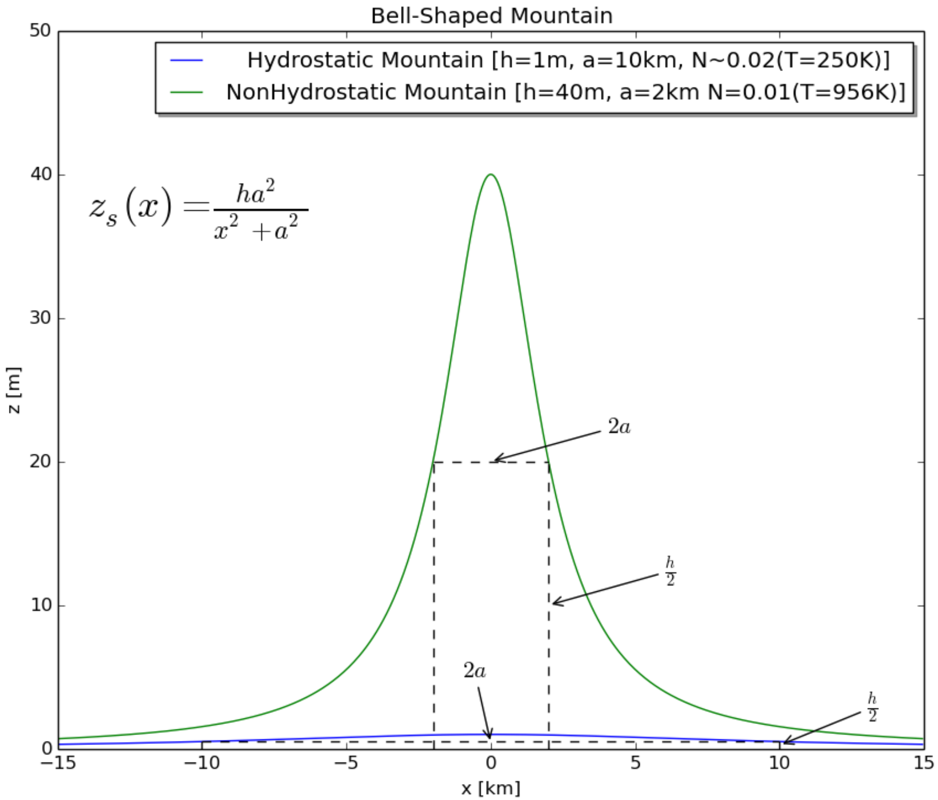
- Prognostic variables (F, T / θ, Φ)
- Time integration (Explicit / HEVI)
- Total and perturbation form

Modeling Framework and Components

Can be easily extended to a 3D formulation



Slice Model A



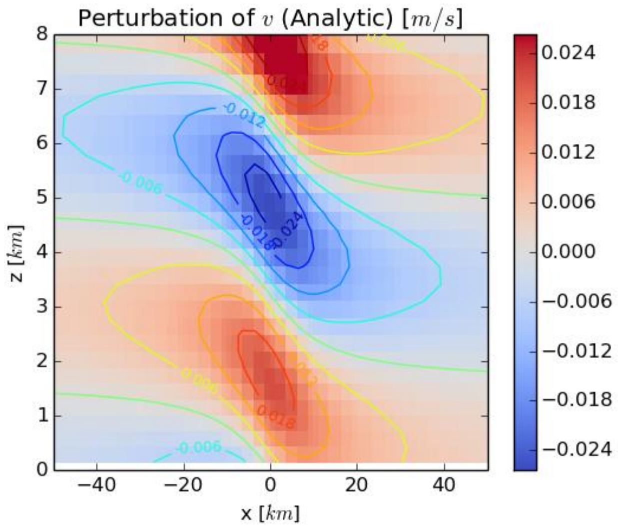
		Linearity Scale Factor ($C_l = \frac{Nh}{\bar{u}}$)
		Linear ($C_l \ll 1$)
Hydrostatic Scale Factor ($C_h = \frac{\bar{u}}{Na}$)	Hydrostatic ($C_h \ll 1$)	$C_l = 0.000978$ $C_h = 0.102$
	NonHydrostatic	$C_l = 0.04$ $C_h = 0.5$

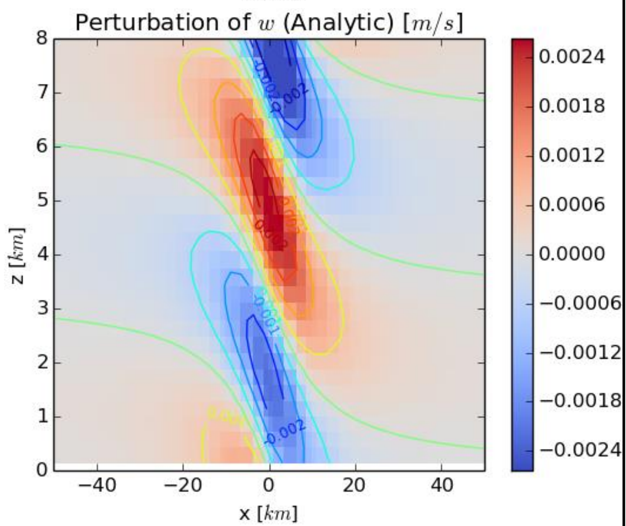
	Durran and Klemp (1983)	**Experiment**
xmin, xmax	-90 km, 90 km	-200 km, 200 km
zmin, zmax	?	0 km, ~83 km
ne	FDM	40
np		6
Δx	2 km	~ 2 km
Δz	200 m	~ 220 m
Δt	20 s (Leapfrog) 4 s (Split-Explicit)	2 s (Hydrostatic, Leapfrog) 0.2 s (nonhydrostatic Leapfrog)
h	1 m	
a	10 km	
$\bar{u}$	20 m/s	
$\bar{T}$	250 K	
Nh/ū, Na/ū	0.000978, 9.78	
integral time	500 minute	
boundary condition	radiation b.c.	absorber b.c.

Slice Model A

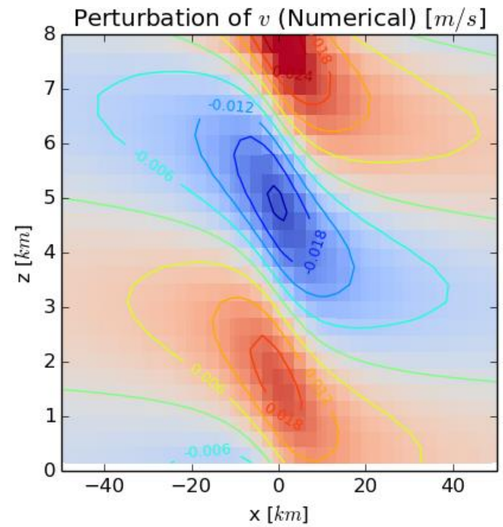
T = 500 min

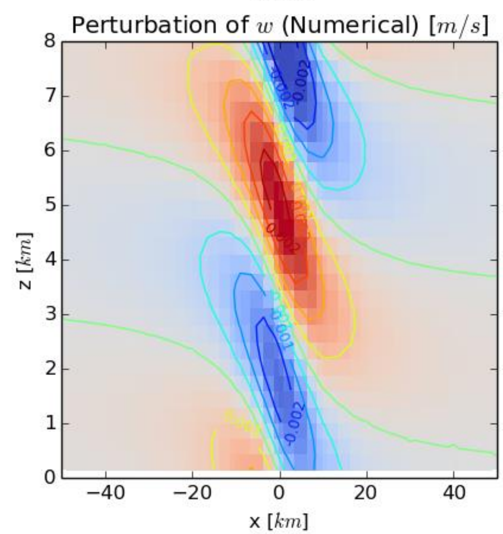
Reference steady solution



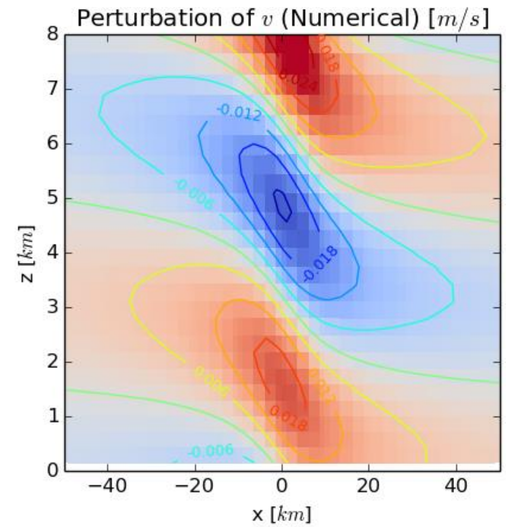


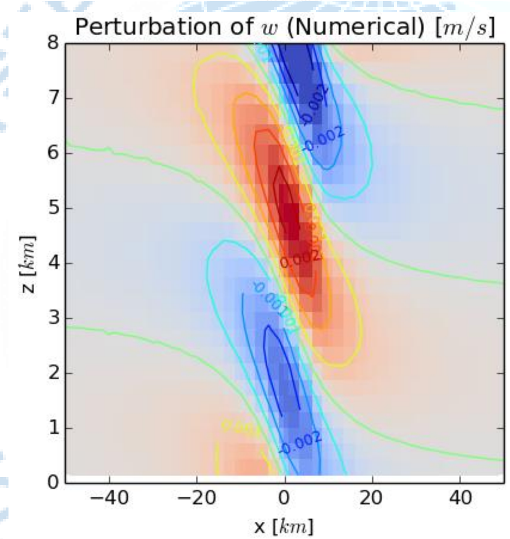
Hydrostatic model





Nonhydrostatic model





Slice Model A

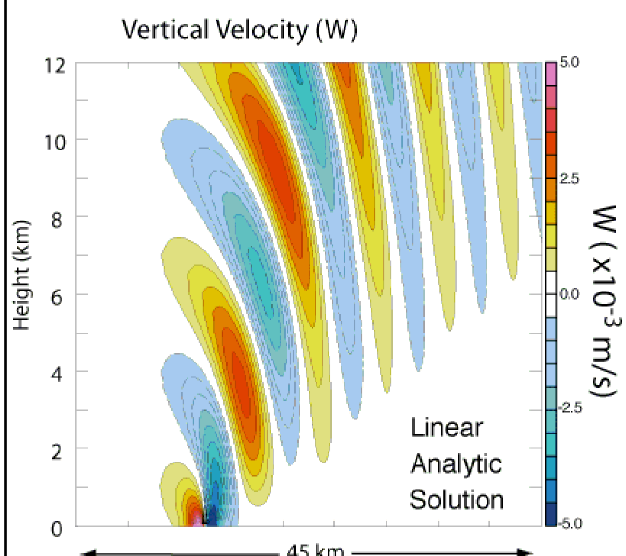
Linear nonhydrostatic mountain wave: T = 300 min

Reference solution	Hydrostatic model	Nonhydrostatic model

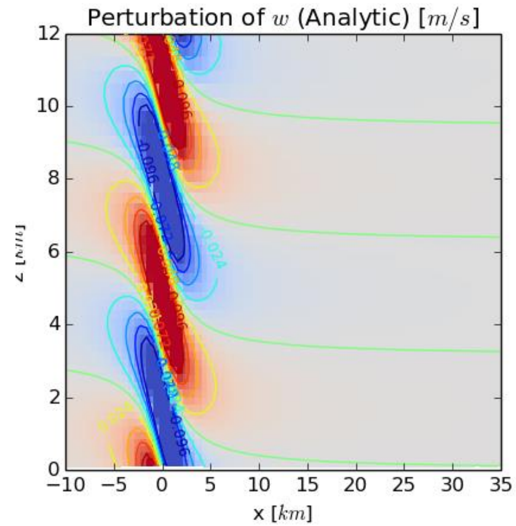
Reference solution

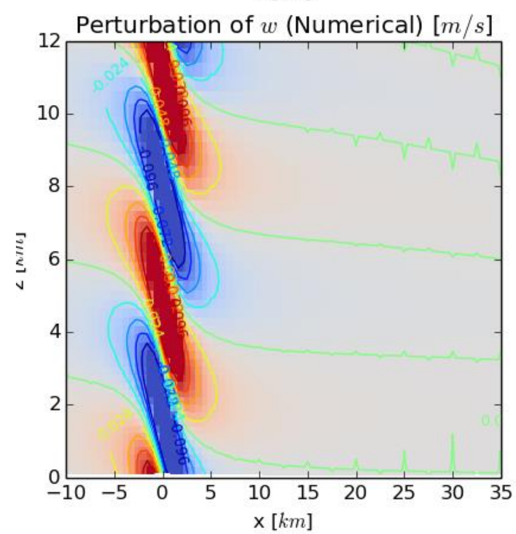
http://www.mmm.ucar.edu/projects/srnwp_tests/2d_mountain_waves/2d_mountain_waves.html

$\Delta x \sim 500m$
$\Delta z \sim 250m$
$\Delta t = 0.1s/0.05s$
$\mathrm{N} = 0.01\ s^{-1}$
$\mathrm{h} = 40m$
$\mathrm{a} = 2km$
$\bar{\mathrm{u}} = 10ms^{-1}$

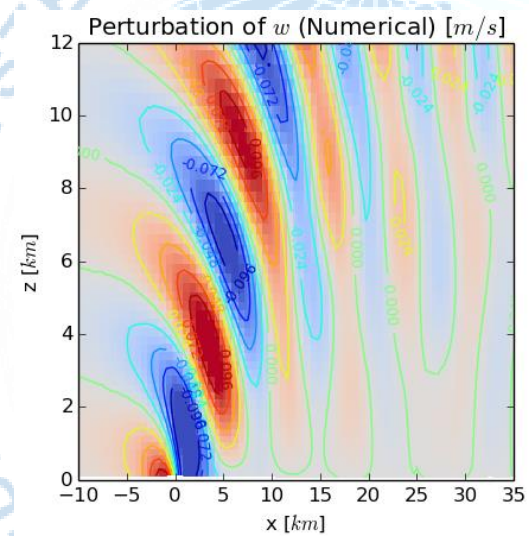


Hydrostatic model

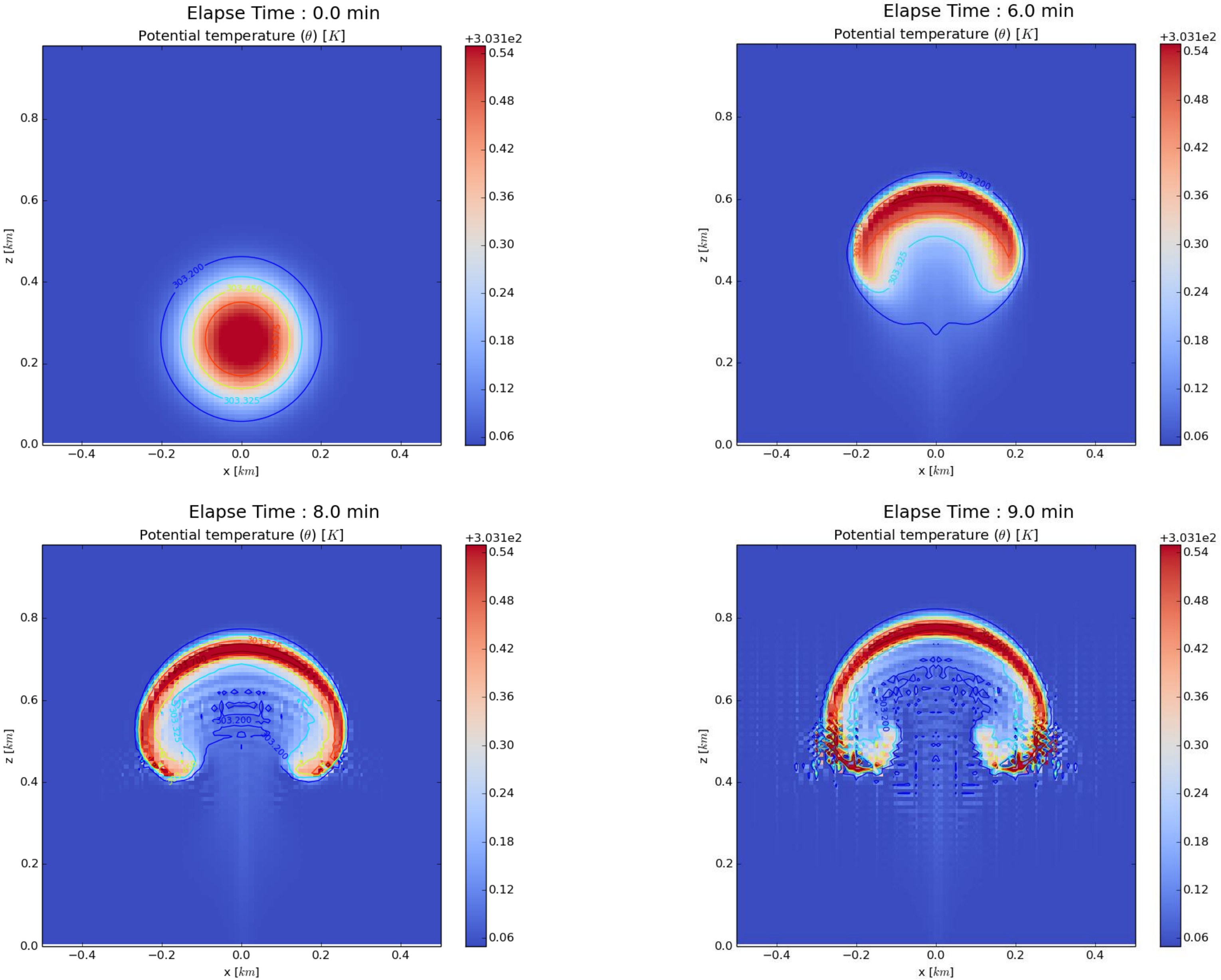




Nonhydrostatic model



Slice Model A



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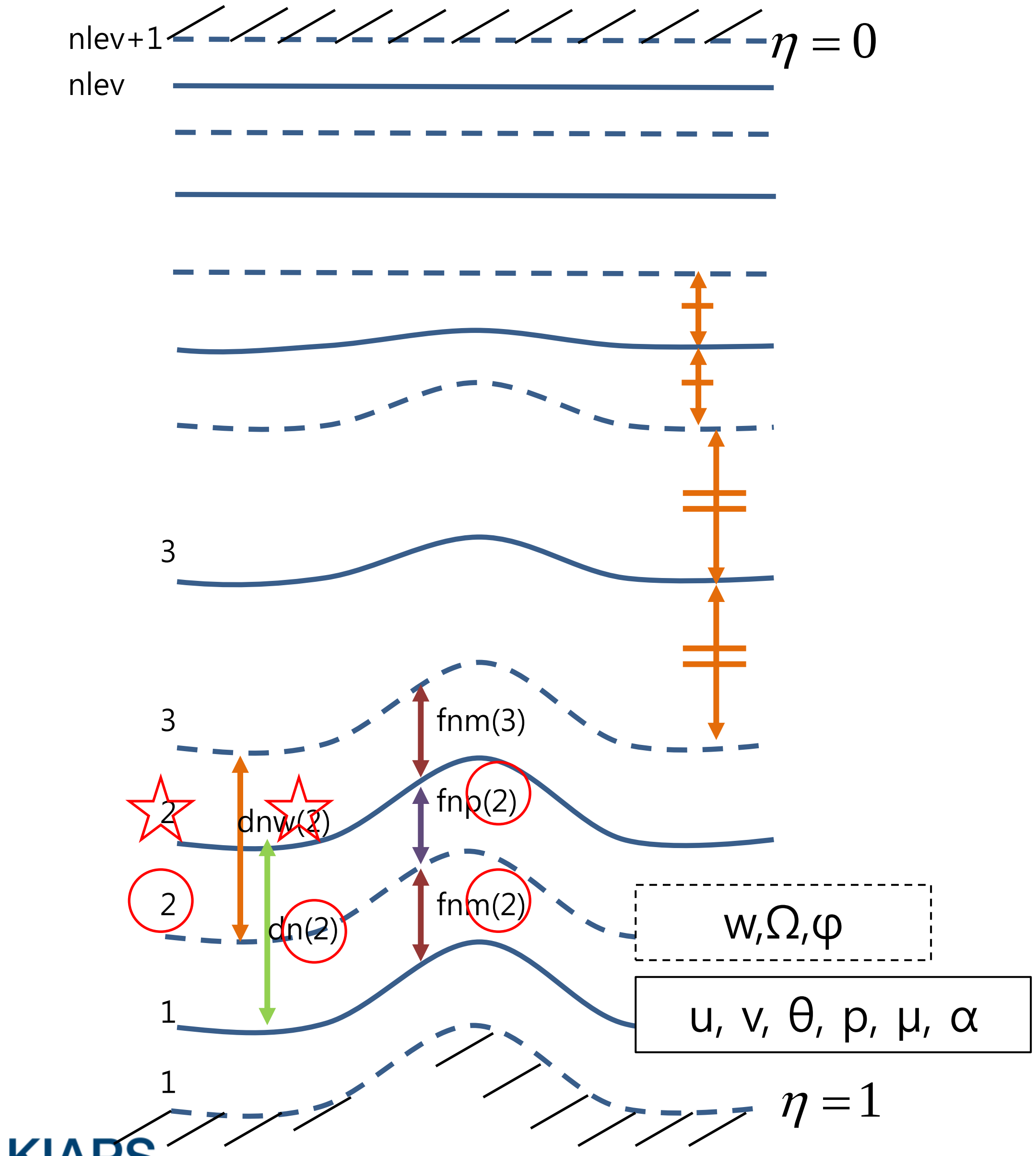
Slice Model B



1) Sigma coordinate

$$p_h = \eta(p_{hs} - p_{ht}) + p_{ht}$$
where $\eta = [0,1]$

$$\mu = \frac{\partial p_h}{\partial \eta} = p_{hs} - p_{ht}$$

2) hybrid sigma coordinate

$$p_h = A(\eta)(p_{h0} - p_{ht}) + B(\eta)(p_{hs} - p_{ht}) + p_{ht}$$
where $A(\eta) = \eta - B(\eta)$,
$\eta = [0,1]$, $B(\eta) = [0,1]$

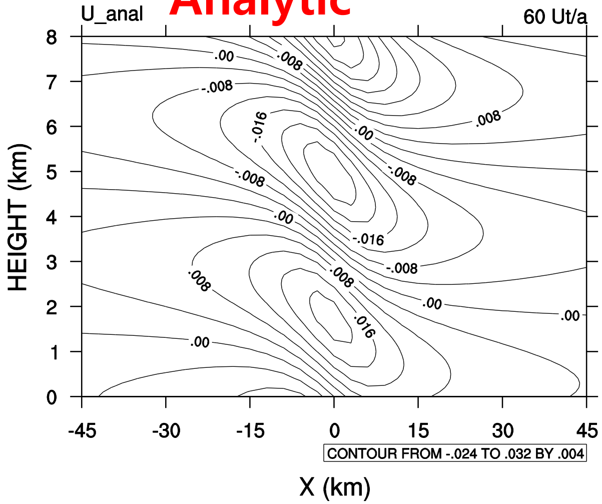
$$\mu = \frac{\partial p_h}{\partial \eta} = (1 - \frac{\partial B}{\partial \eta})(p_{h0} - p_{ht}) + \frac{\partial B}{\partial \eta}(p_{hs} - p_{ht})$$

If A(η)=0, i.e, B(η)= η,
Hybrid-sigma coordinate
became sigma coordinate.

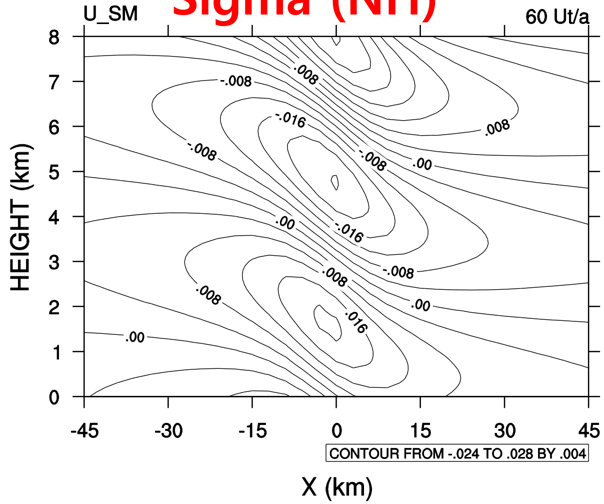
V_full(k) = fnm(k)*V_half(k)+fnp(k)*V_half(k-1)

Slice Model B

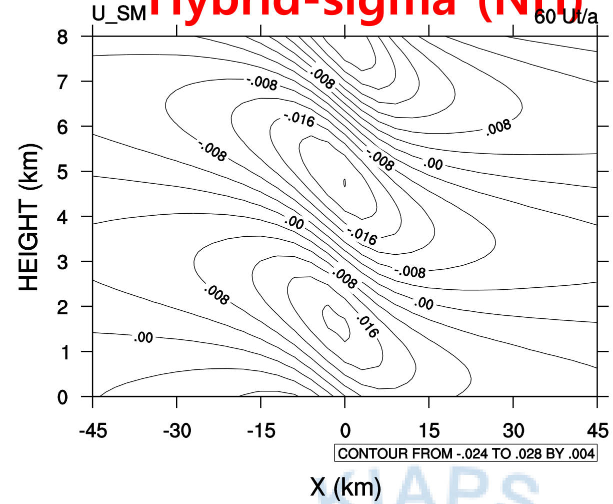
Analytic



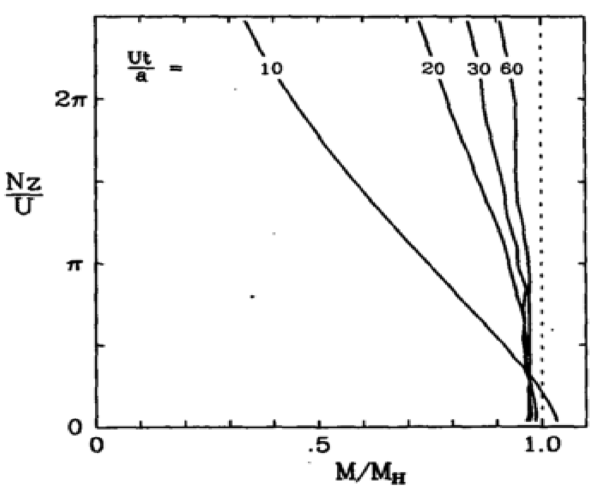
Sigma (NH)



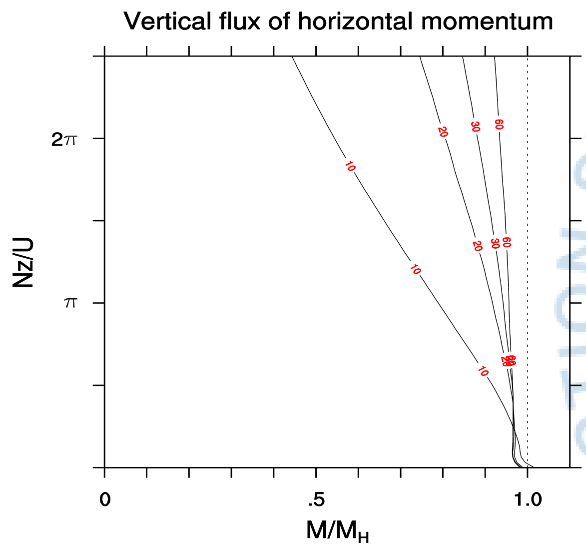
Hybrid-sigma (NH)



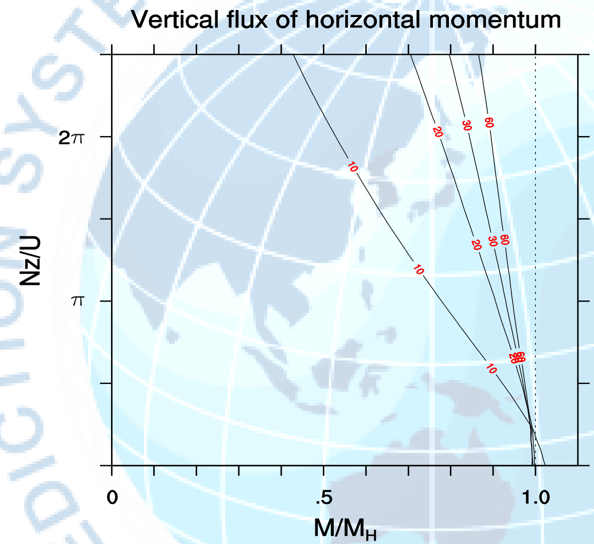
Durran & Klemp (1983)



Sigma (NH)



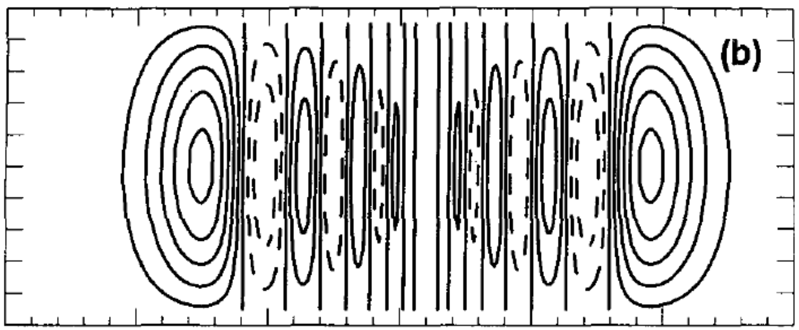
Hybrid-sigma (NH)



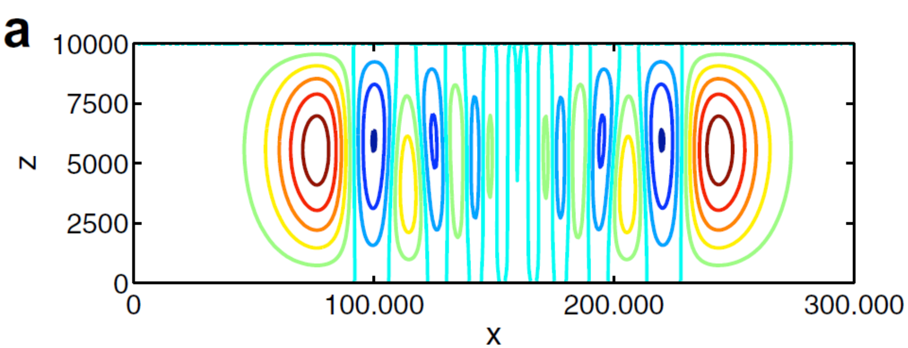
Slice Model B

Analytic solution of Boussinesq equation set (Skamarock and Klemp, 1994)

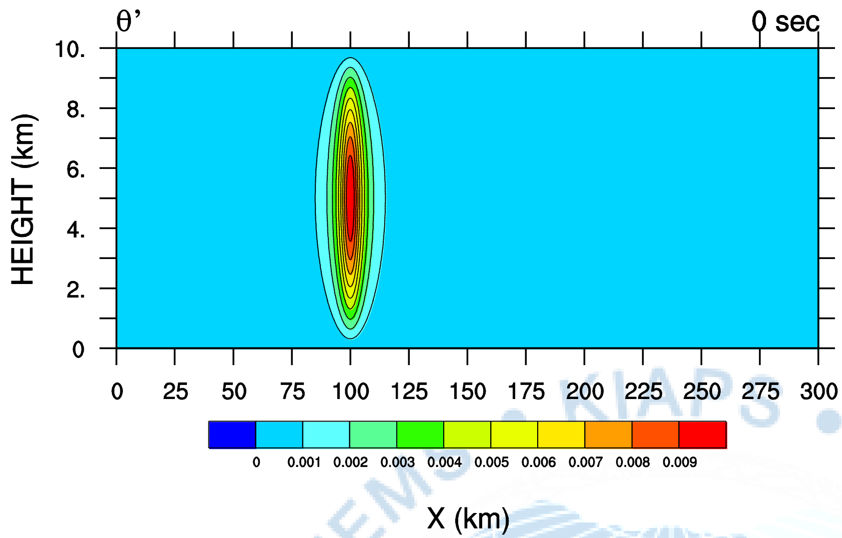
3000 sec

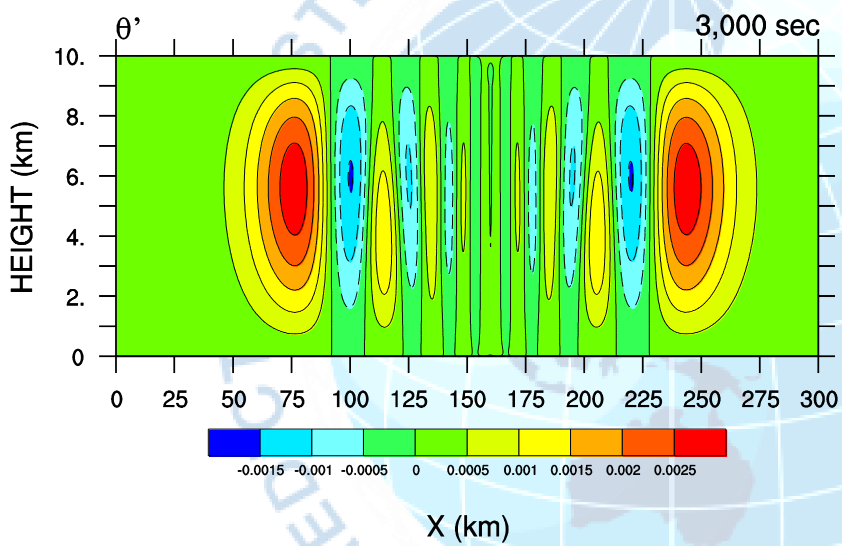


Reference solution (Giraldo and Restelli, 2008)



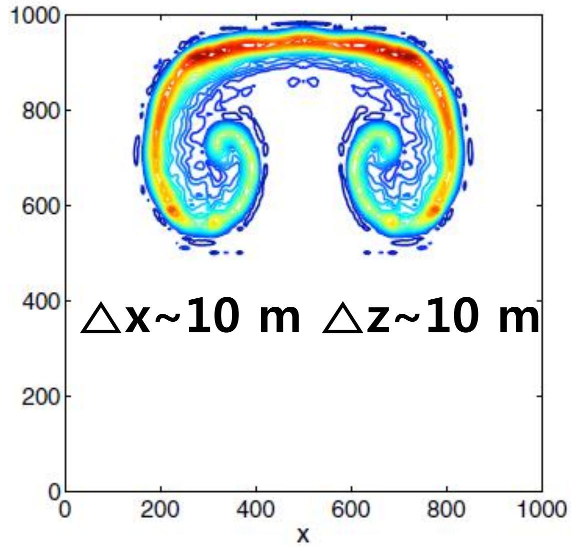
△x~250 m (np=9,ne=150)
△z~250 m (nlev=40)

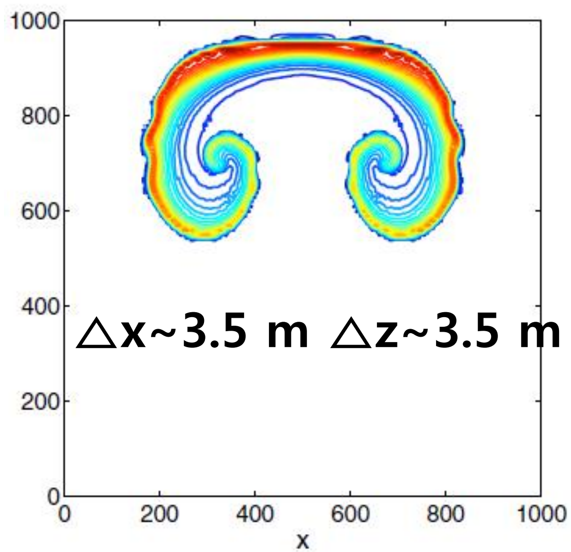




Slice Model B

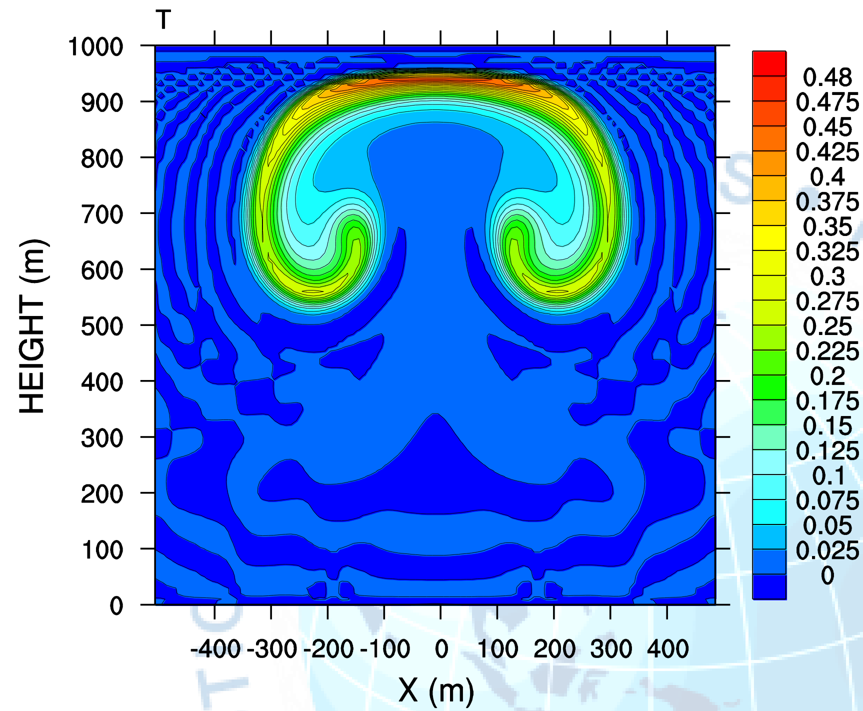
Reference solution
(Giraldo and Restelli, 2008)





- **After 700 sec.**

△x~10 m (np=11,ne=10)
△z~10 m (nlev=100)



Slice Model B

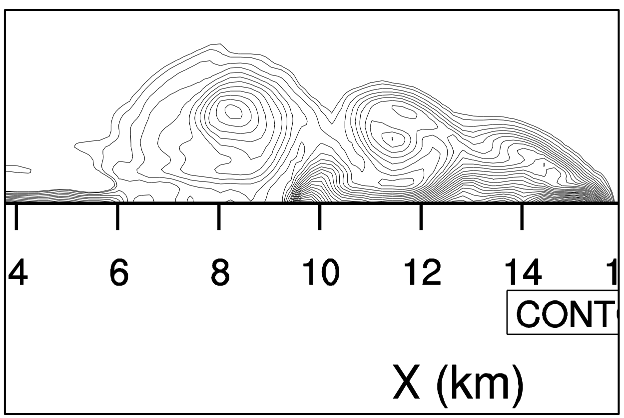
The density current test shows the evolution of a cold bubble dropped in a constant potential temperature environment ($\theta_0 = 300$ K).

- Base Temperature = 300 K, U=0 m/s
- Domain size [x][z] = [-25.6km, 25.6km][0, 6.4km]
- perturbation potential temperature(θ)
- hyperviscosity terms are included

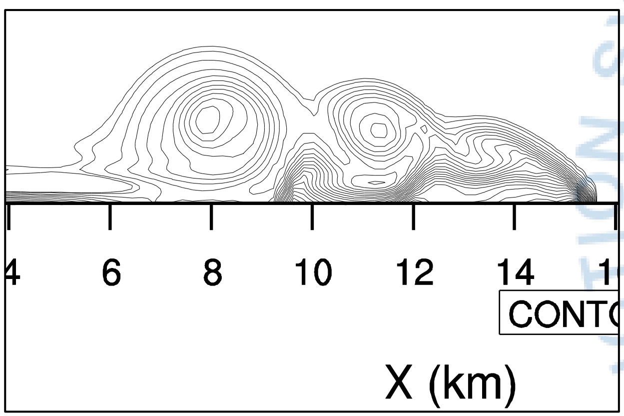
$$\theta(x,z,0) = \theta_c\left[1 + \cos(\pi r)\right]$$

$$\theta_c = -15.0$$

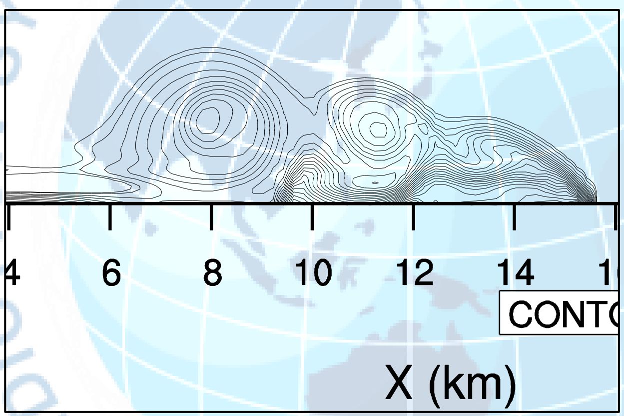
△x~320m



△x~160 m



△x~80 m



Slice Model B- Moist Simulations

- **Moist Simulation with Plug-in of a Microphysics Scheme**
  - Microphysics scheme (WSM3)
  - Diabatic heating
  - Moisture (Tracer) advection

- TEST : Idealized thunderstorm experiment. (Hong et al. 2004)
  - The grid in the *x* direction is 201 points with a 250-m grid spacing.
  - The number of vertical layers is 80.
  - The model is integrated for 30 min with a time step of 3 s. The initial condition has a warm bubble that has a radius of 4 km and a maximum perturbation of 3 K at the center of the domain.
  - A 12 m/ s wind is applied in the positive *x* direction at the surface, decreasing to zero at 2.5 km above the ground, with no wind above.
  - Periodic boundary conditions are applied
  - There is no Coriolis force or friction.
  - The only physical parameterization is the microphysical scheme.

Slice Model B- Moist Simulations

$$\mathbf{V} = \mu_d \mathbf{v} = (U,V,W),\ \Omega = \mu_d \dot{\eta},\ \Theta = \mu_d \theta,\ Q_m = \mu_d q_m$$

$$\phi = gz,\quad p,\quad \alpha_{(d)} = 1/\rho_{(d)}$$

For moist atmosphere, the flux-form Euler Equations

(3 Momentums)

$$\partial_t U + (\nabla \cdot \mathbf{V}u) + \mu_d \alpha \partial_x p + (\alpha / \alpha_d)\partial_\eta p \partial_x \phi = F_{U\,coriolis+curvature+\ldots}$$

$$\partial_t V + (\nabla \cdot \mathbf{V}v) + \mu_d \alpha \partial_y p + (\alpha / \alpha_d)\partial_\eta p \partial_y \phi = F_{V\,coriolis+curvature+\ldots}$$

$$\partial_t W + (\nabla \cdot \mathbf{V}w) - g[(\alpha / \alpha_d)\partial_\eta p - \mu_d] = F_{W\,coriolis+curvature+\ldots}$$

(Energy) $\partial_t \Theta + (\nabla \cdot \mathbf{V}\theta) = F_\Theta$

(Continuity) $\partial_t \mu_d + (\nabla \cdot \mathbf{V}) = 0$

(pressure eq) $\partial_t \phi + \mu_d^{-1}[(\mathbf{V} \cdot \nabla \phi) - gW] = 0$

(moisture eq) $\partial_t Q_m + (\nabla \cdot \mathbf{V}q_m) = F_{Q_m}$

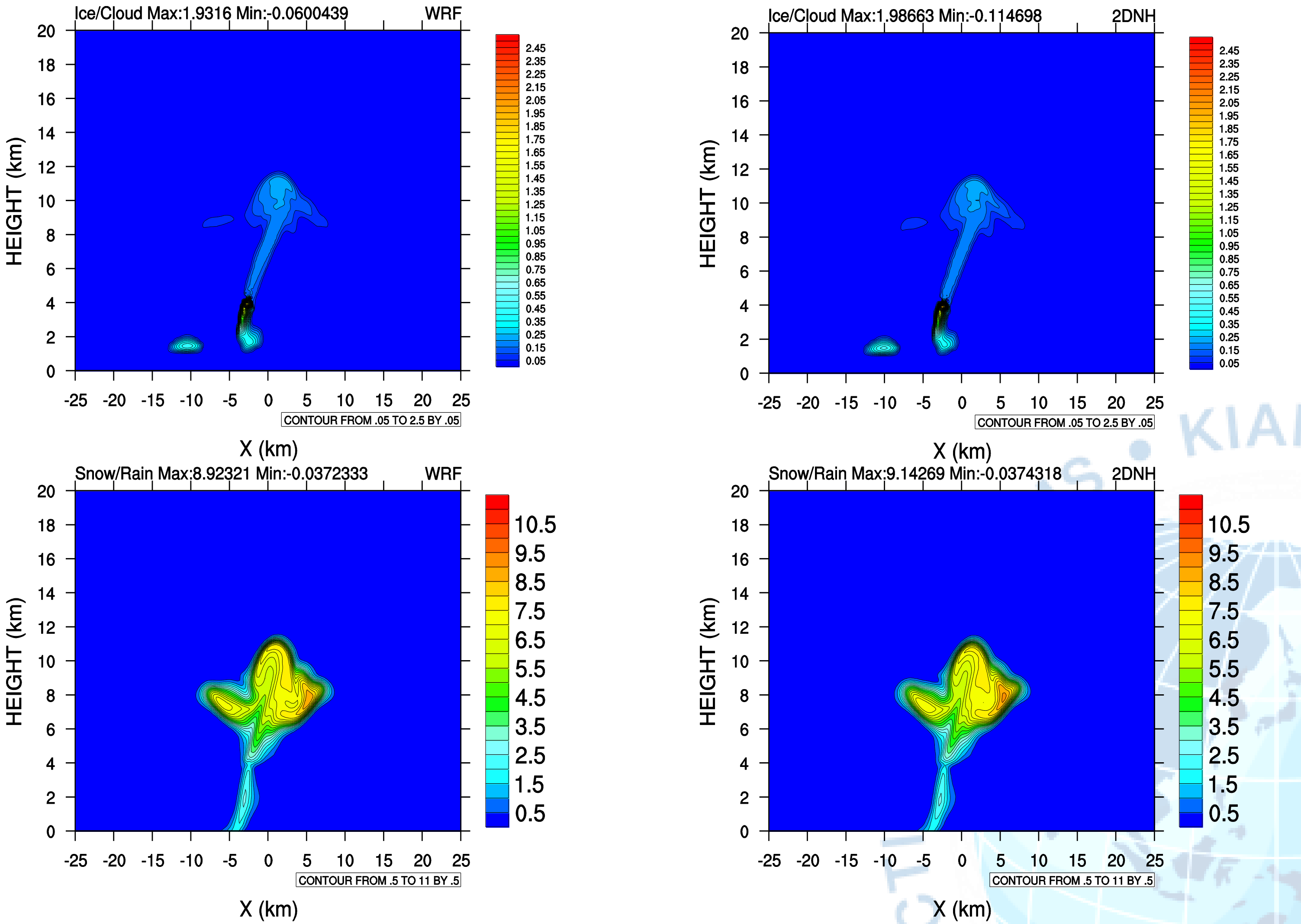
$$\alpha = \frac{\alpha_d}{(1 + q_v + q_c + q_r + \cdots)}$$

(for mu) vertical coor. def. $\mu_d = \partial_\eta p_{hd}$

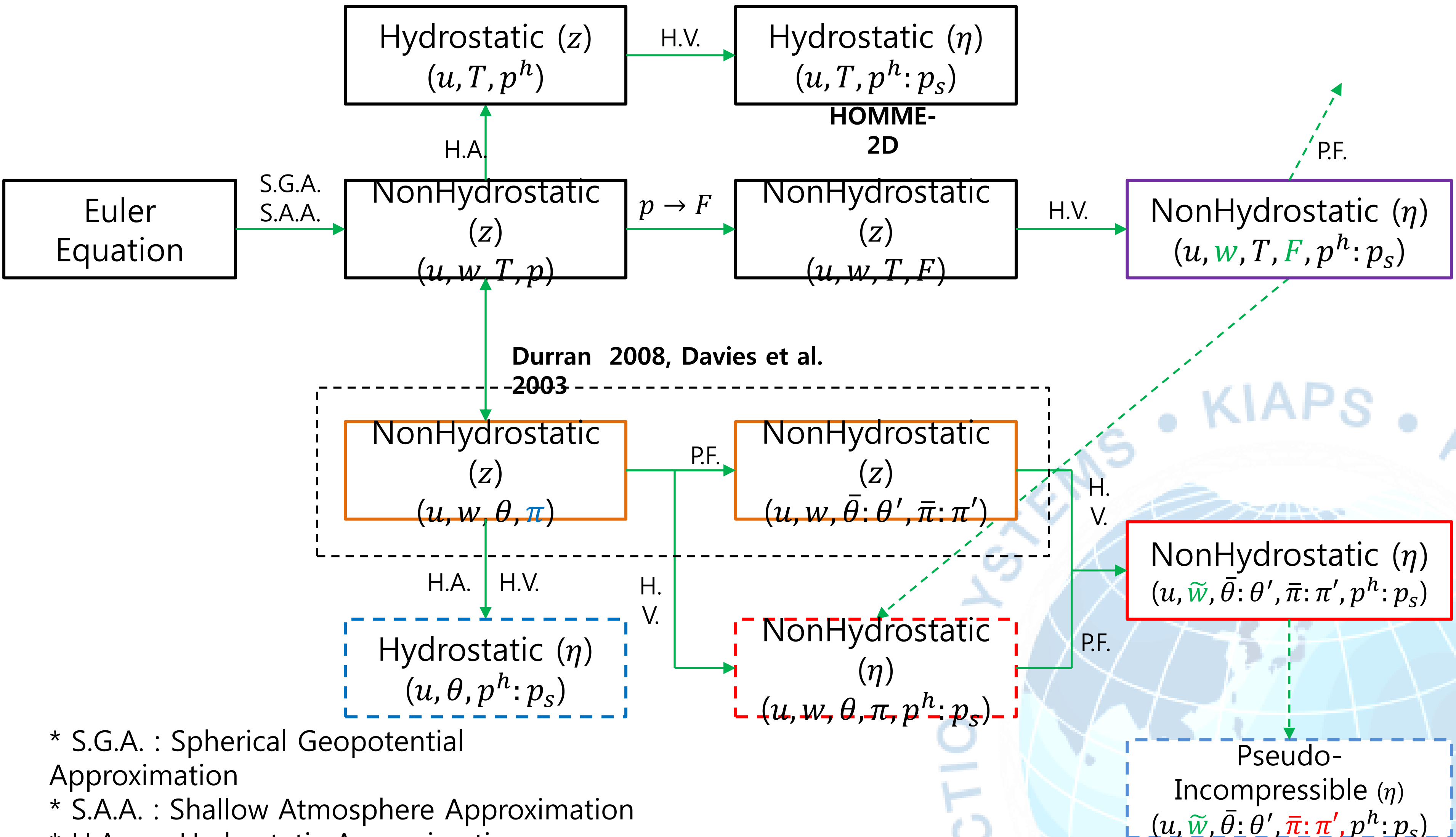
(for full pressure) equation of state

$$p = p_0 (R_d \theta_m / p_0 \alpha_d)^\gamma,\quad \gamma = c_p / c_v \quad \theta_m = \theta\left(1 + (R_v / R_d) q_v\right) \approx \theta\left(1 + 1.61 q_v\right)$$

Slice Model B- Moist Simulations



Further Research and Development



* S.G.A. : Spherical Geopotential Approximation
* S.A.A. : Shallow Atmosphere Approximation
* H.A. : Hydrostatic Approximation
* H.V. : Hybrid Vertical
* P.F. : Perturbation Form

Further Research and Development

- 2014/2015 Tentative Plan

Year	Horizontal Discretization	Vertical Discretization	Time integration	Evaluation	+
2013 (Slice Model)	Spectral Element -1D	Finite Difference	Explicit/HEVI	Benchmark tests	Prognostic variables
2014 (3D on the cubed sphere)	Spectral Element -2D	Finite Difference (Finite Element)	HEVI	Comparison with the hydrostatic version	Sound-Proof equation set
2015 (3D on the Cubed sphere)	Spectral Element -2D	Finite Difference (Finite Element)	Semi-implicit	Comparison with the hydrostatic version	Realistic topography

Challenging Tasks:

Implementation of realistic topography

Coupling of moist air

Time-Stepping....etc

Thank You!

KIAPS