A consistent framework for discrete integrations of soundproof and compressible PDEs of all-scale atmospheric dynamics

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The developed framework combines anelastic, pseudo-incompressible and compressible PDEs into a single generalised system, written in a perturbation form about a particular, compatible solution of the system.

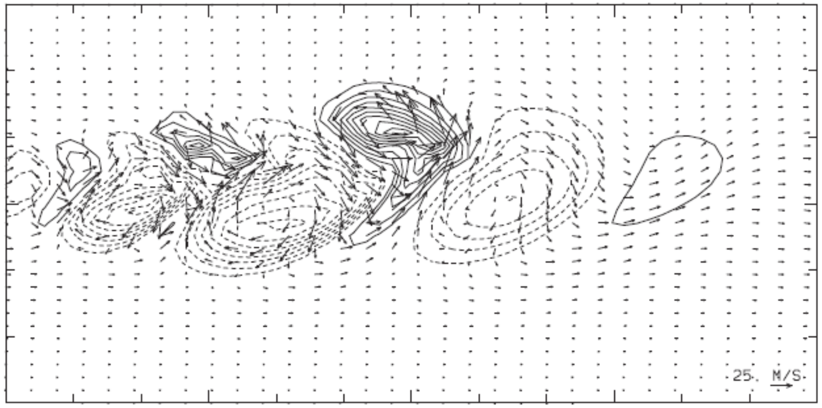
The generalised system is cast in curvilinear coordinates, admitting two congruent forms --- the Eulerian conservation-law form and the Lagrangian evolutionary form --- to facilitate design of consistent control-volume and semi-Lagrangian integrations.

The resulting equations are effectively integrable with proven semi-implicit Eulerian/semi-Lagrangian non-oscillatory forward-in-time (NFT) differencing approach, admitting schemes implicit with respect to buoyant, rotational and acoustic modes (in various combinations). In particular, the compressible Euler equations can be integrated with acoustic or soundproof time steps.

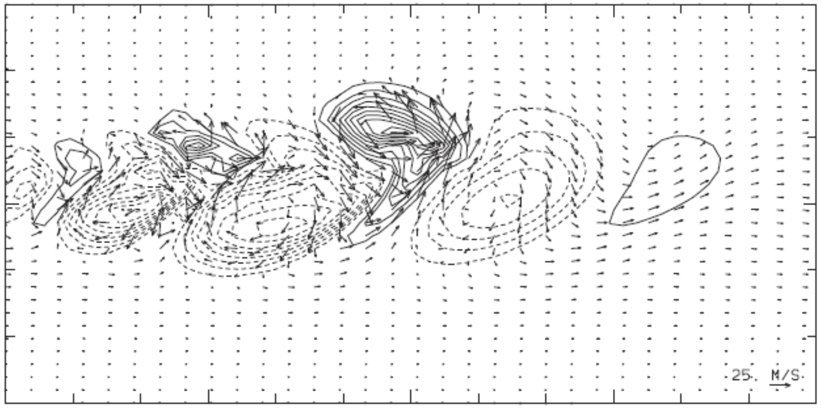
Technical advancements for the compressible NFT solvers include definition of the transporting momenta (for all specific dependent variables) by the mass continuity equation, and extension of the generalised Poisson solver to the corresponding Helmholtz solvers of two kinds.

Global baroclinic instability, compressible solutions:

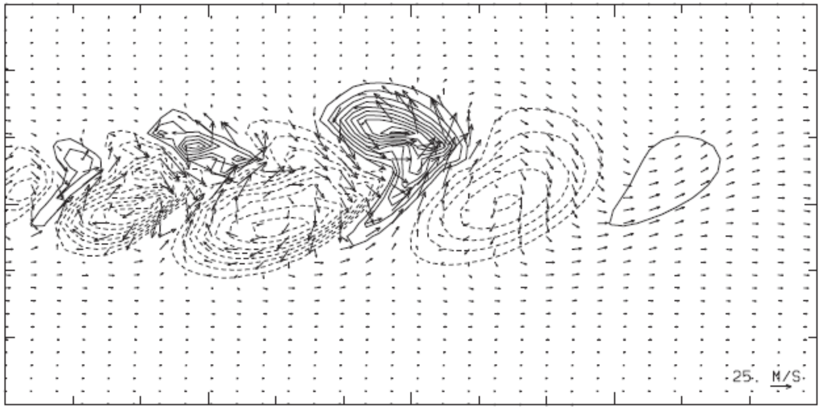
8 days, surface θ',
128x64x48 lon-lat grid,
128 PE of Power7 IBM



CPI2, 2880 dt=300 s,
wallclock time=2.0 mns,
wlt/dt=0.041 s

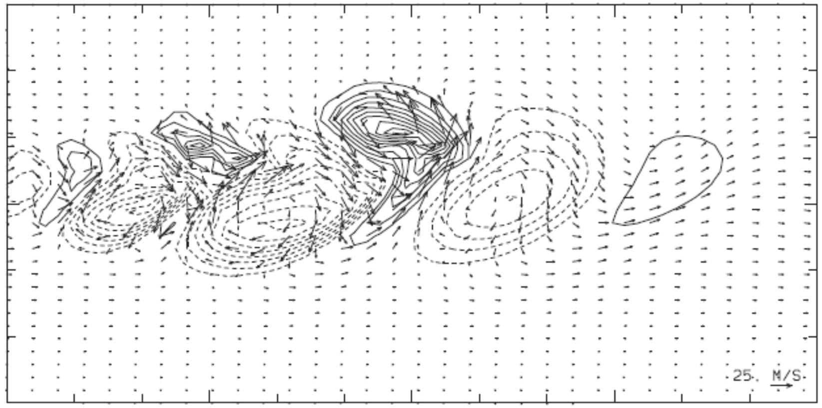


CPI1, 5760 dt=150 s,
wallclock time=3.7 mns,
wlt/dt=0.039 s

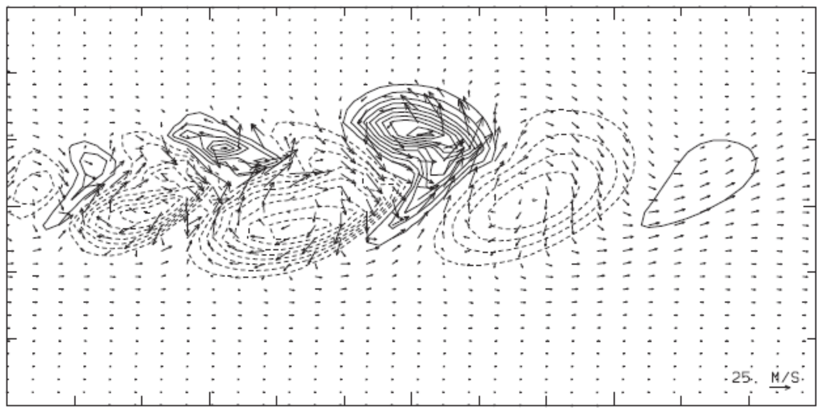


CPEX, 432000 dt=2 s,
wallclock time=178.9 mns,
wlt/dt=0.025 s

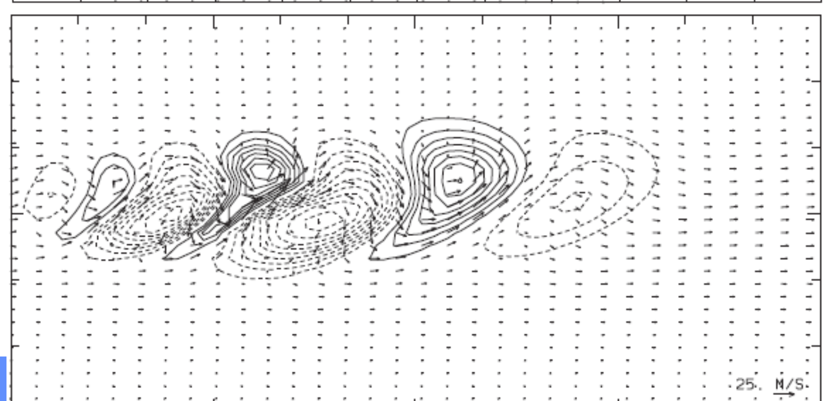
1.Global baroclinic instability;   large-time-step solutions for various PDEs



CPI2, 2880 dt=300 s,
wallclock time=2.0 mns,
wlt/dt=0.041 s

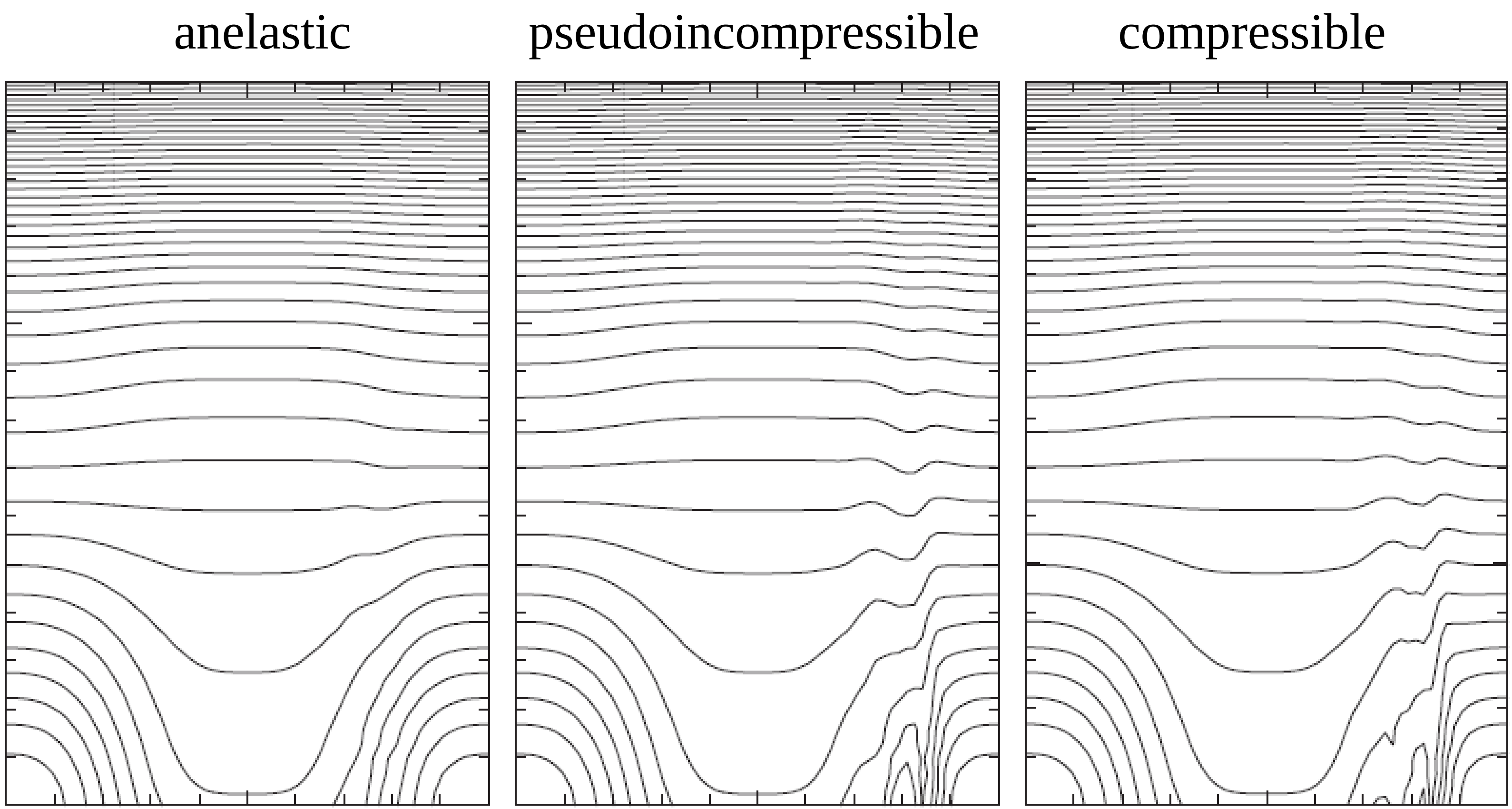


PSI, 2880 dt=300 s,
wallclock time=2.3 mns,
wlt/dt=0.048 s



ANL, 2880 dt=300 s,
wallclock time=2.1 mns,
wlt/dt=0.044 s

The role of (nonlinear) baroclinicity



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transiency of nonlinear effects

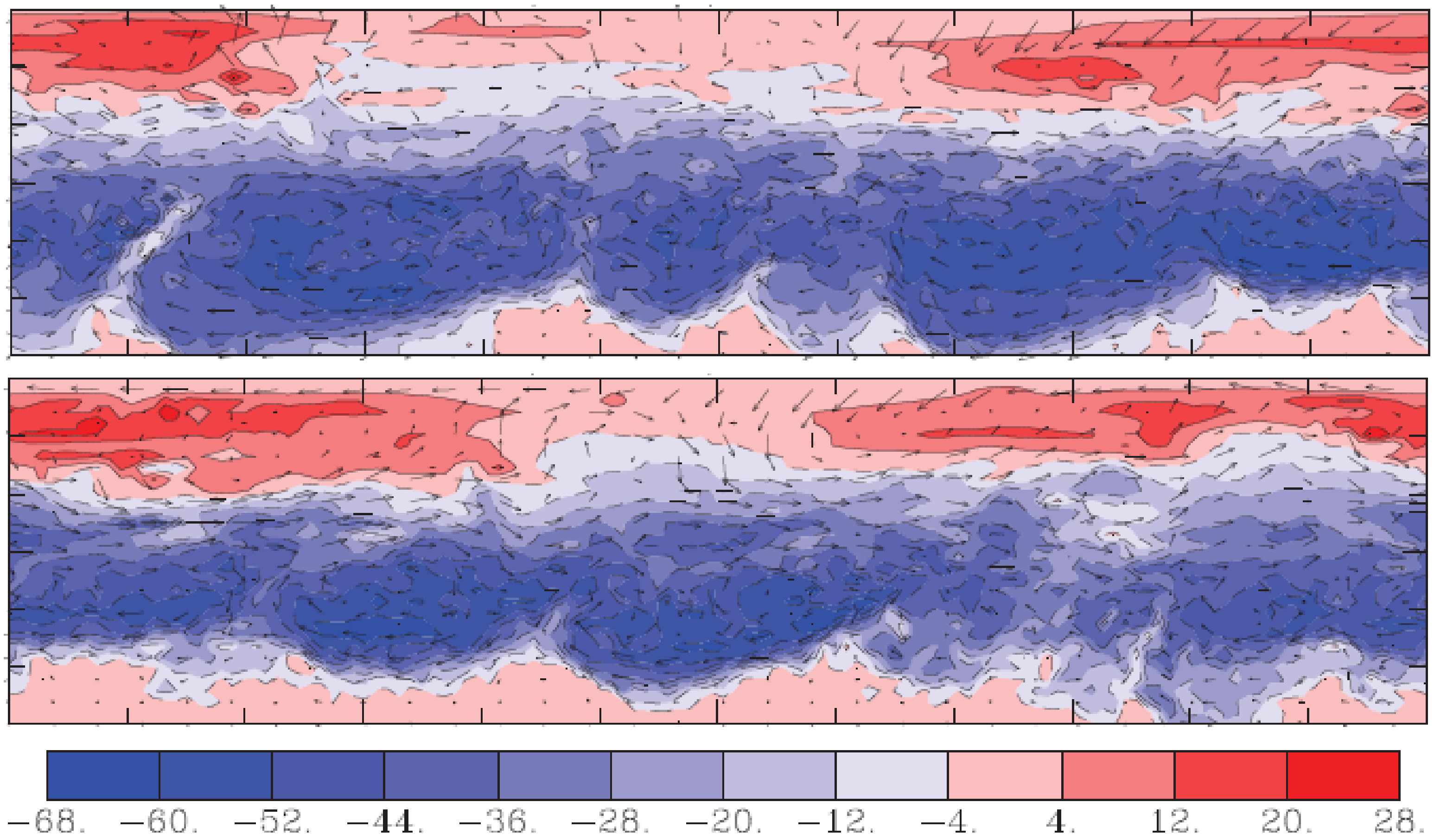
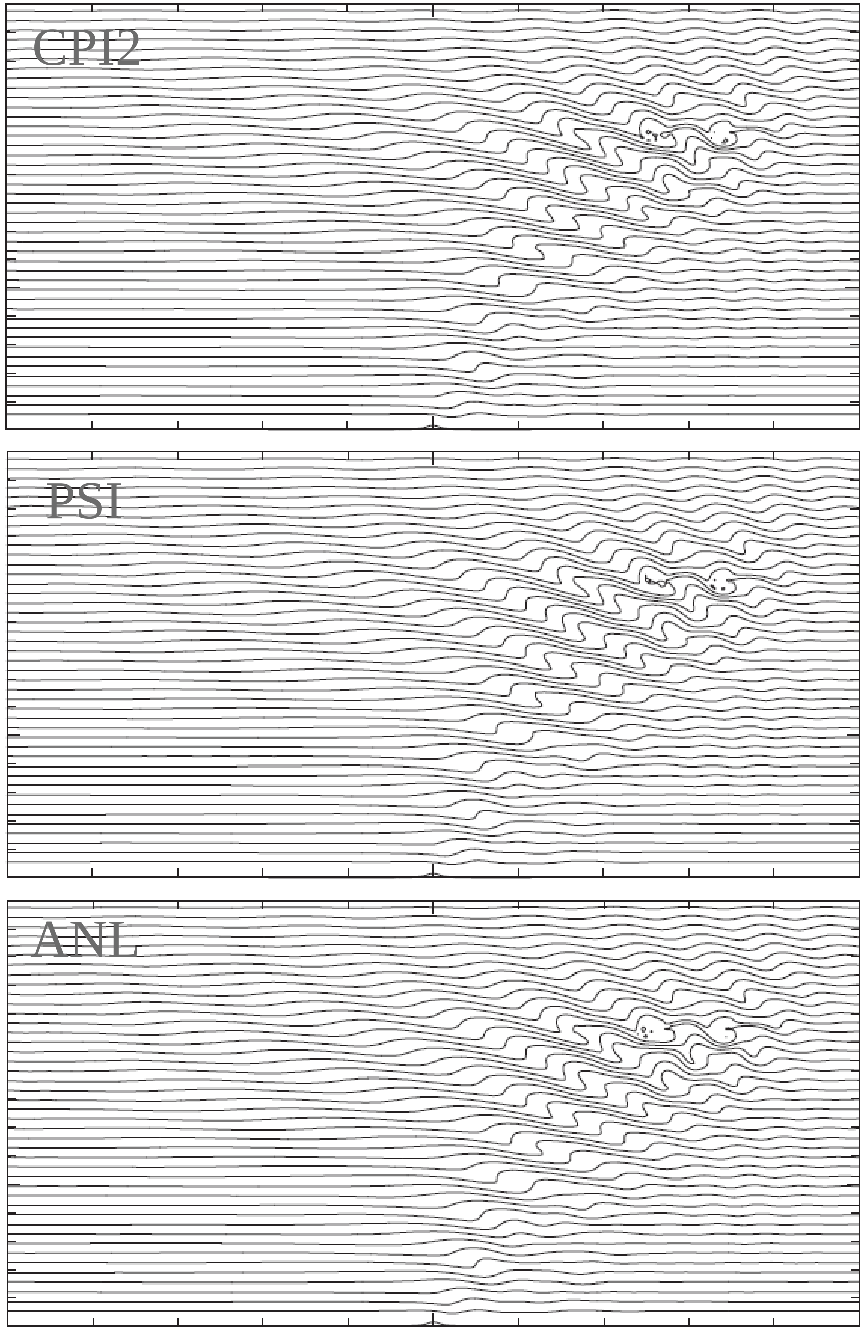


Figure 3: Baroclinic instability, day 30: surface potential temperature perturbations on the northern hemisphere's subdomain $[0, 360] \times [0, 90]$ *degrees, for compressible Euler equations integrated with the semi-implicit large-time-step algorithms of the second kind (top) and the anelastic (bottom). Maximal winds reach 60m* s^{-1} *in both solutions.*

breaking of deep stratospheric gravity wave



1.5h, lnθ, 320x160 grid,
domain 120 km x 60 km
``soundproof'' dt=5 s
``acoustic'' dt=0.5 s.

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Key results:

- The respective PDEs are integrated using essentially the same numerics
- Availability of compatible flux-form Eulerian and semi-Lagrangian options
- The flux-form solvers readily extend to unstructured-meshes
- Results are great: δt "convergence" and the adherence to linear and asymptotic predictions; revelation on the role of baroclinicity and its connection to the accuracy of elliptic solvers
- Soundproof and compressible models are not antitheses but they form complementary elements of a more general theoretical-numerical framework

Way forward:

- The new problem formulation (perturbation form and its associated accuracy) targets extended prognosis; cf. lessons learned from ocean solitary waves or solar MHD climate
- Technically, the consistent framework enables blending NWP and research models/codes

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