Towards a forced-dissipative shallow water test case with physics-dynamics coupling

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With thanks to the GungHo team

PDEs on the sphere, 7-11 April 2014

Motivation

• Need to test prototype models on flows of realistic complexity

Conservation; cascades; handling of small scales and shallow spectra; grid imprinting; relevance of high order ...

$\Rightarrow$ SW analogue of Held-Suarez test

• Need to test dynamical cores with sufficiently challenging physics-dynamics coupling

Small-scale in space and time; on-off behaviour; local positive feedbacks

Proposal 1: (dry dynamics)

$$\frac{D\Phi}{Dt} + \Phi\nabla\cdot\mathbf{u} = \frac{\Phi_{\mathrm{eqm}} - \Phi}{\tau_\Phi}; \qquad \frac{D\mathbf{u}}{Dt} + f\mathbf{k}\times\mathbf{u} + \nabla\Phi = \frac{\mathbf{u}_{\mathrm{eqm}} - \mathbf{u}}{\tau_u}$$

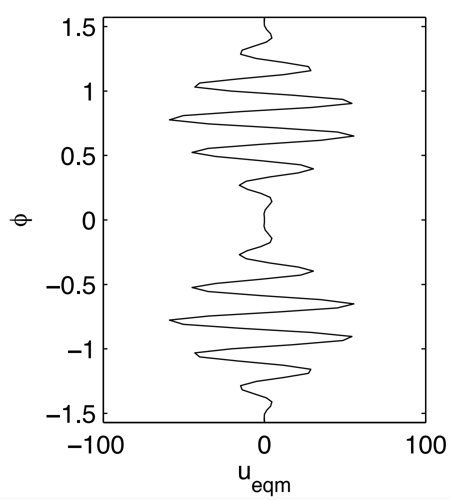
where

$$\Phi_{\mathrm{eqm}} = \Delta\Phi(1 - \sin^2\phi) + \Phi_0; \qquad u_{\mathrm{eqm}} = u_0\sin^2(2\phi)(\sin^2(m\phi) - 1/2)$$

with

$$g = 9.80616\,\mathrm{ms}^{-2}$$
$$2\Omega = 1.4584\times 10^{-4}\,\mathrm{s}^{-1}$$
$$a = 6731220\,\mathrm{m}$$
$$\Delta\Phi = 8000\,\mathrm{m}^2\mathrm{s}^{-2}$$
$$\Phi_0 = 10^3 g - 2\Delta\Phi/3$$

$$u_0 = 120\,\mathrm{ms}^{-1}$$
$$m = 12$$
$$\tau_\Phi = 100\ \mathrm{days}$$
$$\tau_u = 100\ \mathrm{days}$$



- Easy to set up
- Complex mid-latitude “weather”
- Zonally symmetric statistics

Diagnostics

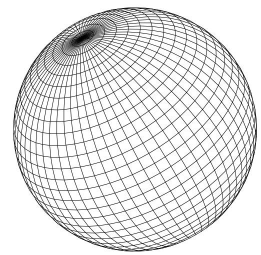
Time mean and temporal standard deviation of scalars: Φ, δ, ξ;

Zonal means and zonal standard deviations of these.

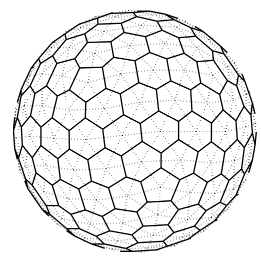
200 day spin up; 1000 day statistics.

Three shallow water models

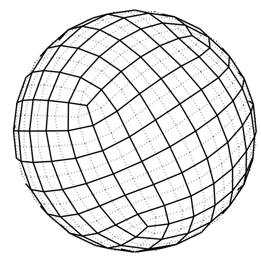
• ENDGame: lat-long; C-grid; SISL;
$320 \times 160 = 51200$ cells; 153280 dofs



Mimetic FEM: C-grid; SI; FV advection of Φ and PV



• hex-icosahedral grid
40962 cells; 163842 dofs



• cubed sphere
55296 cells; 165888 dofs

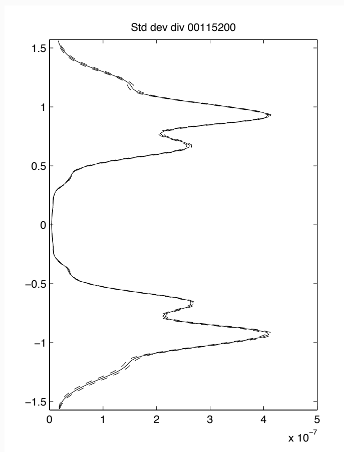
$\Delta t = 900\,\mathrm{s}; \quad \Delta x \sim 100\,\mathrm{km}$

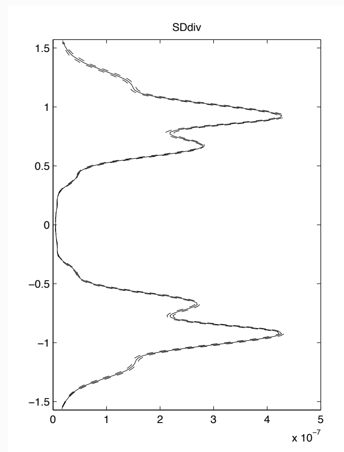
Example results

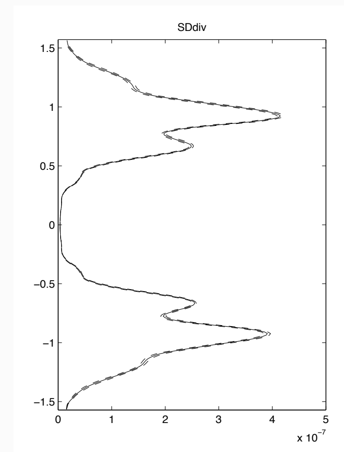
Most statistics are extremely similar for the three models...

EG Hex Cube

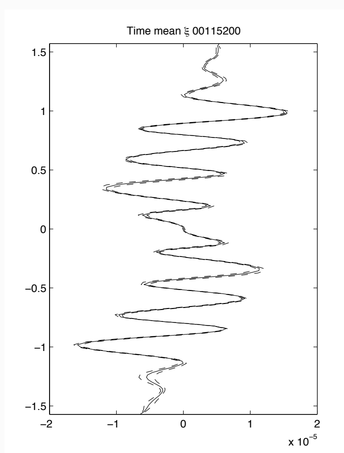
σ_δ^t

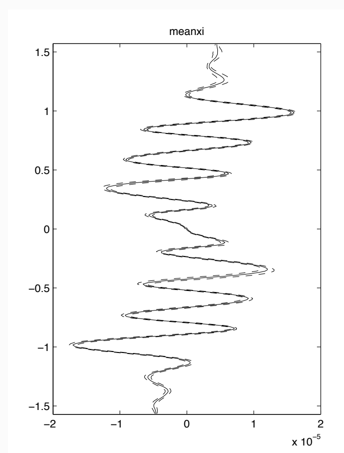


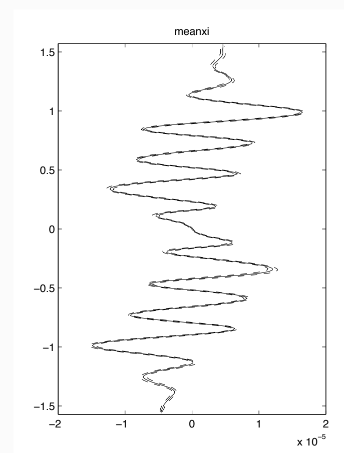




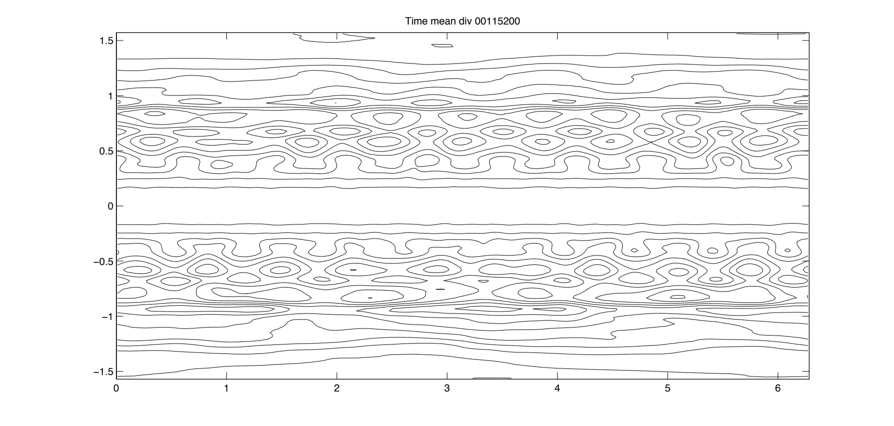
$\overline{\xi}^t$

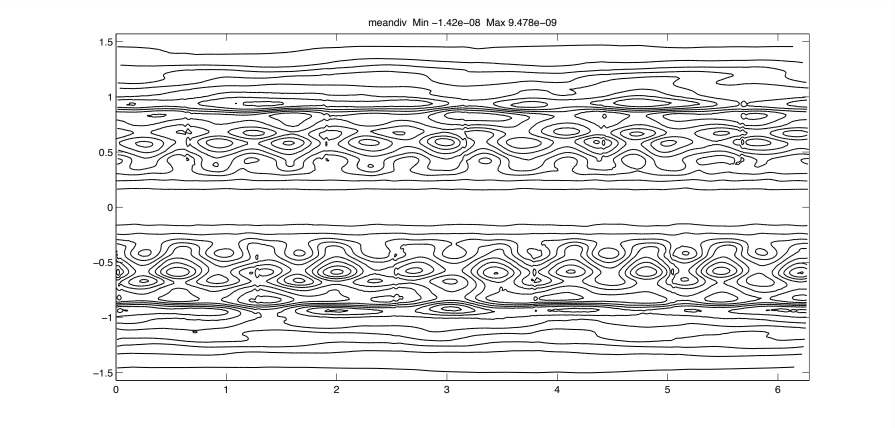


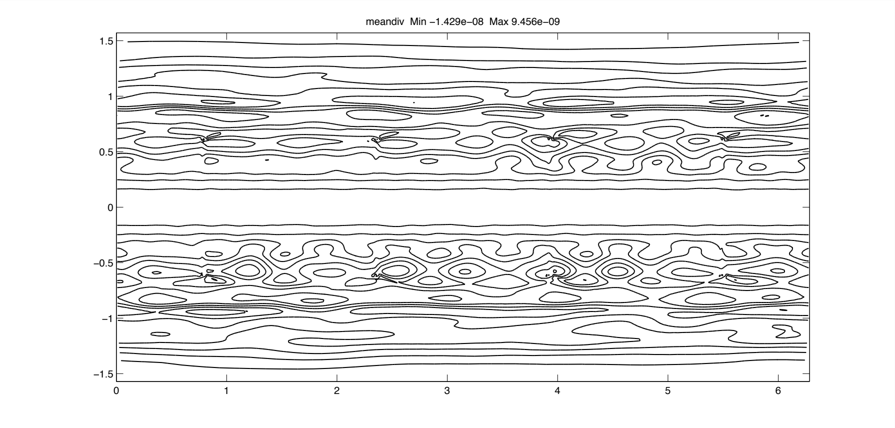




...except time mean divergence   contour $0.25 \times 10^{-8}\,\mathrm{s}^{-1}$







Proposal 2: (with "physics")

See Würsch and Craig, MZ, 2014

$$\frac{D\Phi}{Dt} + \Phi\nabla\cdot\mathbf{u} = \frac{\Phi_{\mathrm{eqm}} - \Phi}{\tau_\Phi}; \qquad \frac{D\mathbf{u}}{Dt} + f\mathbf{k}\times\mathbf{u} + \nabla(\Phi + \Phi_{\mathrm{phys}}) = \frac{\mathbf{u}_{\mathrm{eqm}} - \mathbf{u}}{\tau_u}$$

$$\frac{D\Phi_m}{Dt} = \frac{\Phi - \Phi_m}{\tau_m}; \qquad \frac{D\Phi_l}{Dt} = \frac{\Phi_{leq} - \Phi_l}{\tau_l} + S;$$

where

IF $\Phi - \Phi_m > T_c$ and $\Phi_l > 0$ then

$\Phi_{\mathrm{phys}} = -\beta\Phi_l - (\Phi - \Phi_m - T_c)$

$S = -(\Phi_{\mathrm{phys}}/\Phi)\nabla\cdot(\mathbf{u}\Phi)$

ELSE

$\Phi_{\mathrm{phys}} = 0$

$S = 0$

$T_c = 350\,\mathrm{m}^2\mathrm{s}^{-2}$

$\Phi_{leq} = 4\,\mathrm{m}^2\mathrm{s}^{-2}$

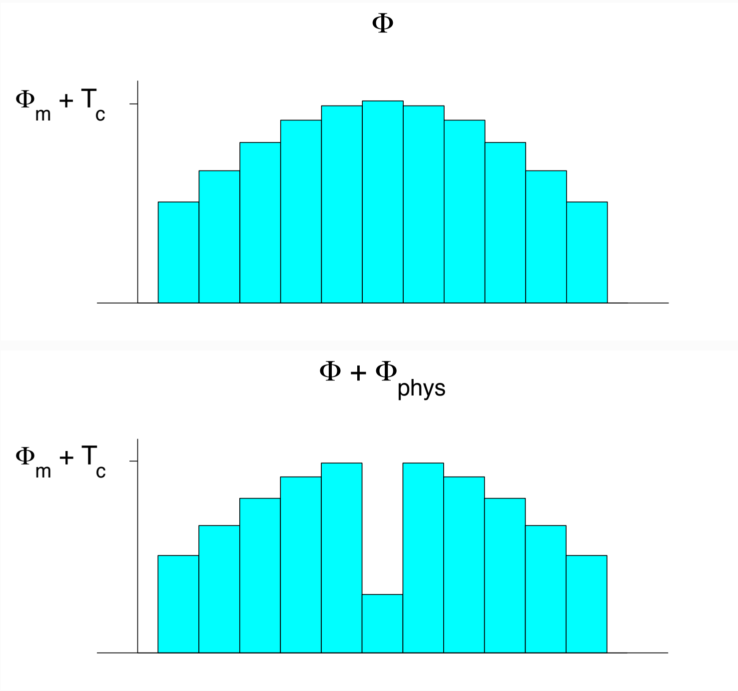
$\beta = 0.1$

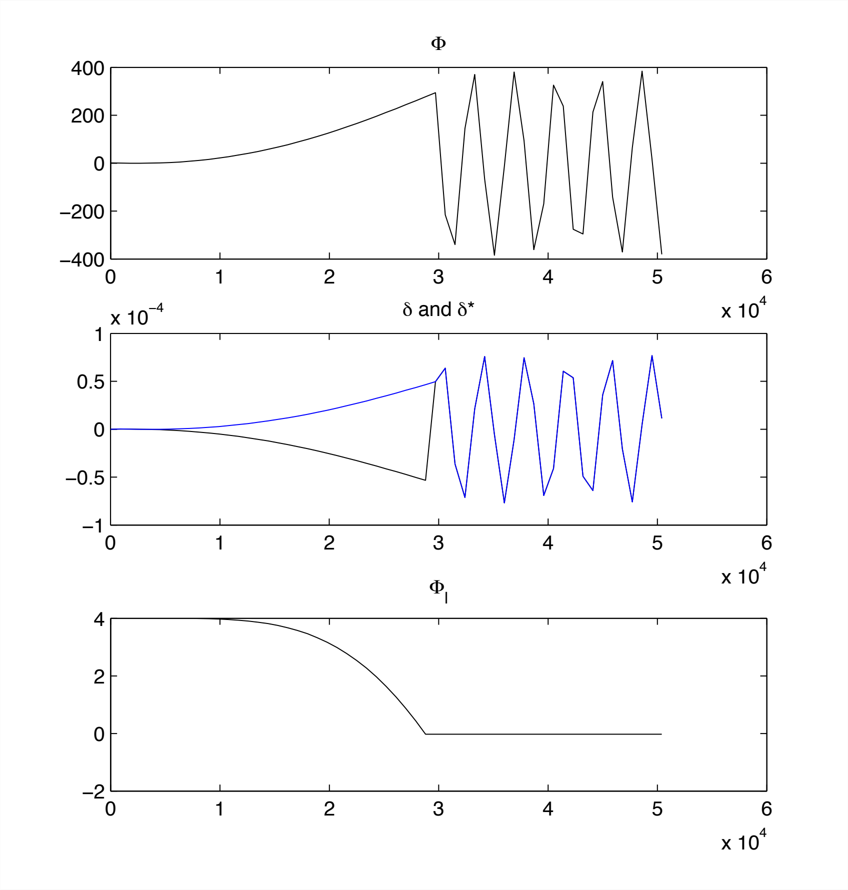
$\tau_m = 10\,\mathrm{days}$

$\tau_l = 5\,\mathrm{days}$

Idealized life cycle

Effect of Φ_{phys}





Note: current implementation is time split

What makes this case challenging?

- Fast dynamical and physical processes: Care needed to ensure balance/cancellation between Φ and Φ_{phys}; Need appropriate $\nabla \cdot (\mathbf{u}\Phi)$.

- Energy conservation.

- Grid-scale forcing of dynamics: Adjustment processes at end of convection ($\mathbf{c}_g$); Contamination of PV; grid imprinting.

- How can a grid-scale convective event propagate?

- Convective sink of Φ_l can undershoot zero. Advection of Φ_l can undershoot zero.

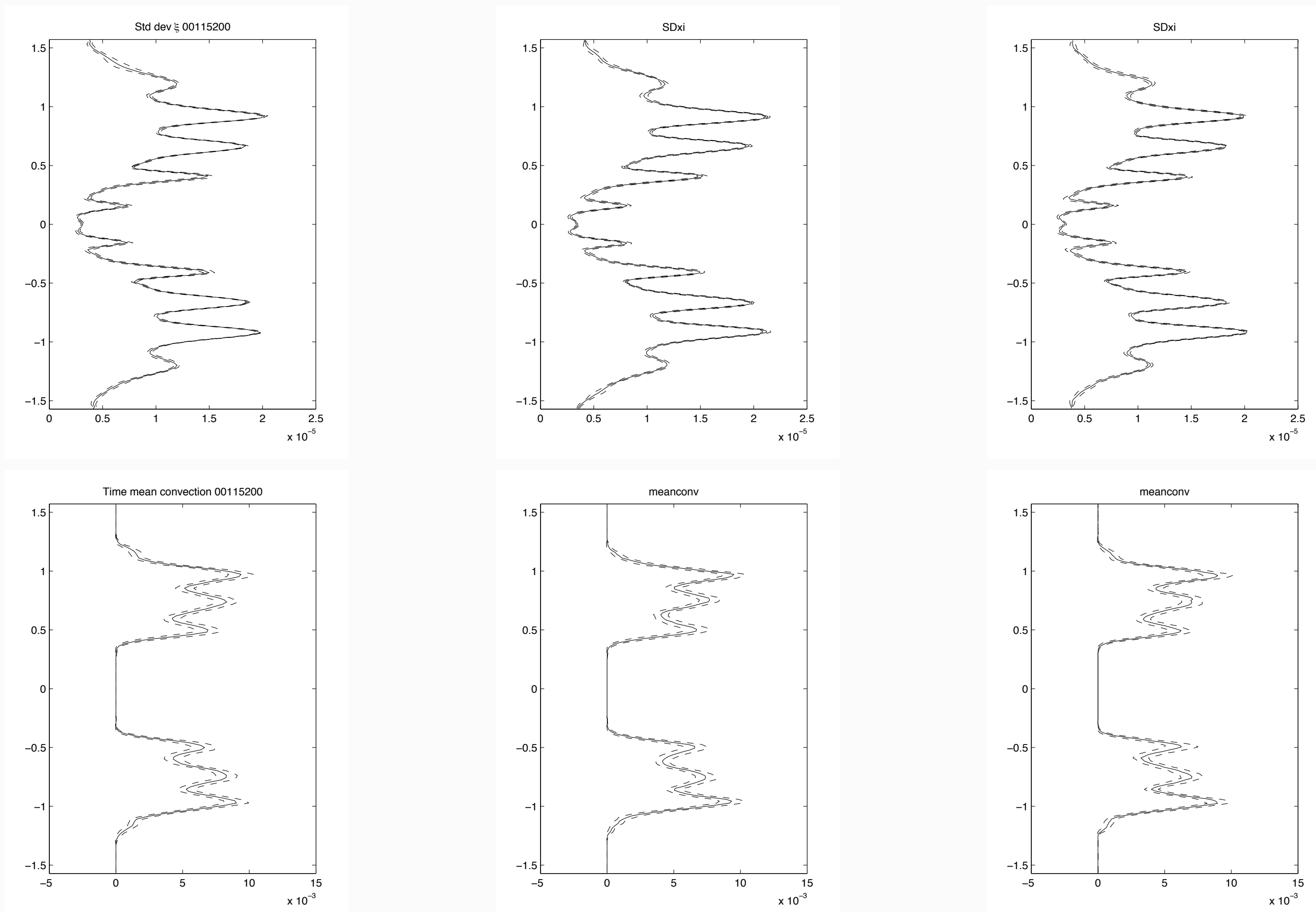
Example results

Again most statistics are extremely similar for the three models...

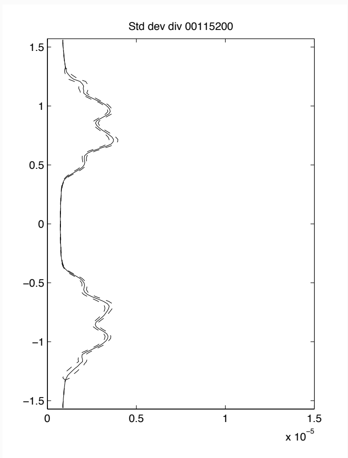
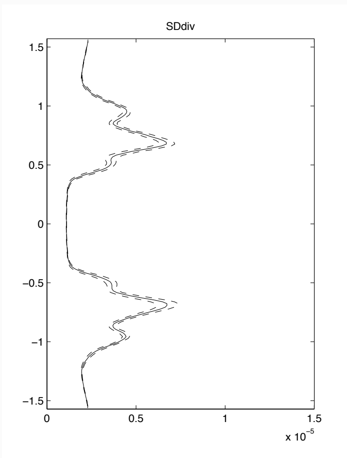
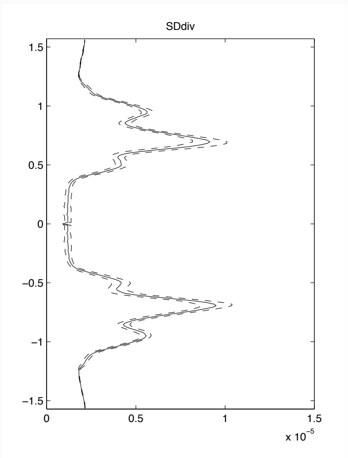
EG   Hex   Cube

σ_ξ^t

$\overline{\mathrm{conv}}^t$



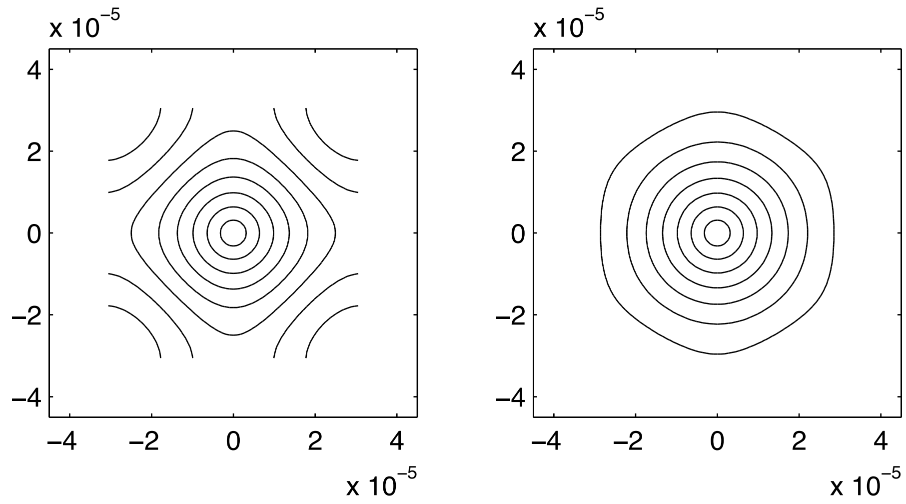
Amplitude of gravity waves

	EG	Hex	Cube
σ_δ^t			

- SL advection of $\mathbf{u}$ vs upwind FV advection of PV?
- Variations in mid-latitude cell size, and discrete Laplacian of δ-function?

Coherence of gravity waves

- More isotropic dispersion relation on hexagonal grid?



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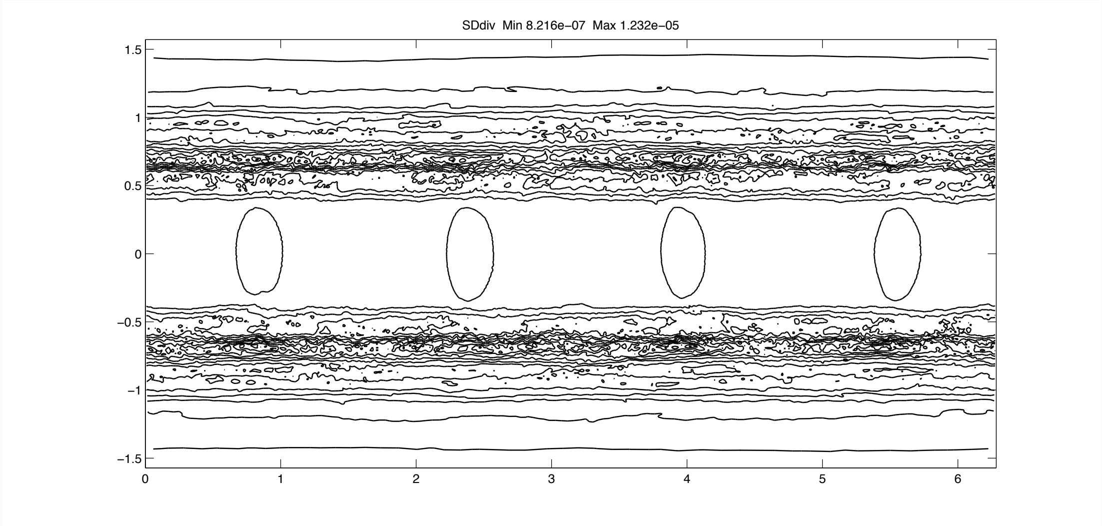
Grid imprinting signal in σ_δ^t



- Numerical wave refraction?

Discussion / Issues

Is this test too challenging? Not challenging enough?

Are these the most useful diagnostics? (Spectra?)

Is the test well posed? (How should the parameters depend on spatial resolution? Convergence? Is there a *right answer*?)

Would others find this test useful? Feedback welcome!

Summary

• A shallow water analogue of the Held-Suarez test case has been proposed. Simple zonally symmetric forcing leads to complex mid-latitude 'weather'.

• A convection-like parameterization has been included, giving physics-dynamics coupling on small space and time scales.

• A tool to probe aspects of model performance not covered by other tests.

• Obliges developers to consider physics-dynamics coupling at an early stage.

• Reveals some interesting differences among three models tested.