Implementing mixed finite elements on curved elements on the sphere

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This talk is about the application of mixed finite element methods to numerical weather prediction, in particular mixed finite element methods that are chosen to be *compatible* with the div, grad and curl operators of vector calculus. These compatible finite element methods can be interpreted as the extension of the C-grid staggered finite difference methods to mixed finite elements. As shown in (Cotter and Shipton, JCP (2012)), and (Cotter and Thuburn, JCP (2014)), they extend many of the desirable properties of the C-grid methods whilst introducing extra flexibility to (1) use arbitrary non-orthogonal grids with various element shapes, (2) alter the ratio of velocity DOFs to pressure DOFs in order to avoid spurious mode branches, and (3) obtain higher-order discretisations. These methods are currently being developed as part of the "Gung Ho" UK dynamical core project.

In this talk we describe the efficient and practical construction of compatible finite element spaces on a mesh that approximates the surface of sphere. In the case of flat elements, the Piola transformation is used to ensure that the velocity fields remain tangential to the mesh, as well as ensuring the compatibility conditions which are required for all the various desirable properties. When the element shapes are constructed using polynomial approximations to the surface of the sphere, this appears to lead to a break in the compatibility conditions. We show how this can be resolved by redefining the div operator; this depends crucially on geometric properties of the construction that were described in (Cotter and Thuburn, JCP (2014)). We demonstrate how this approach can restore the correct order of convergence for high-order compatible finite element schemes on the sphere, where it becomes crucial that the representation of the sphere is sufficiently high-order. This modification turns out to require very minimal modifications of the code; this will be illustrated by showing example code written using the Fenics package. We will finally explain some of our recent work on extending this approach to three dimensional terrain-following meshes on the sphere.