

# Non-hydrostatic sound-proof equations of motion for gravity-dominated compressible flows

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Compressible non-hydrostatic equations of motion with density diagnosed from potential temperature through hydrostatic balance are derived from Hamilton's principle of least action. The corresponding local budgets of energy, potential vorticity and momentum are obtained. Slaving density to potential temperature suppresses the degrees of freedom supporting the propagation of acoustic waves and results in a sound-proof system. The linear normal modes and dispersion relationship for small departures from an isothermal state of rest on  $f$ - and  $\beta$ - planes are studied and found to be accurate from hydrostatic to non-hydrostatic scales. Importantly the Lamb wave and long Rossby waves are not distorted, unlike with anelastic or pseudo-incompressible systems.

Compared to similar equations derived by Arakawa and Konor (2009), the unified system derived here possesses an additional term in the horizontal momentum budget. This apparent force is crucial to the derivation of a well-defined self-adjoint diagnostic equation from which non-hydrostatic pressure is deduced. Unlike with anelastic/pseudo-incompressible systems or the equations obtained by Arakawa and Konor (2009), this diagnostic equation is vertically fourth-order, reflecting the fact that the hydrostatic constraint satisfied by density involves a vertical derivative. As with hydrostatic equations, vertical velocity is diagnosed through Richardson's equation. Our unified system has therefore precisely the same degrees of freedom as the hydrostatic primitive equations, while retaining accuracy from hydrostatic to non-hydrostatic scales.

These equations may be useful as the basis of global non-hydrostatic numerical models. They also provide an accurate way to filter out the acoustic component from a given flow. Variational data assimilation systems may benefit from such a filter, restricting the optimization space to physically relevant motion. Similarly, filtering may be useful to prevent the transient emission of acoustic waves in a fully-compressible model at initialization or after physics parameterizations have acted.