Variable Order Mixed Finite Elements on Quadrilateral Grids for the Shallow Water Equations

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Mixed finite elements methods, where different function spaces are used to represent the various fields, preserve a number of properties desirable for an atmospheric dynamical core including: good inherent conservation, compatibility with certain continuous vector calculus operators, no orthogonality condition and can be seen as extending the C-grid method into a finite element framework. Here, a particular family of mixed finite element spaces, characterized by use of the Raviart-Thomas (RT) space for vector fields, on arbitrary quadrilateral elements are investigated. At lowest order the RT0 elements are the finite element equivalent of the C-grid discretization. Investigating the dispersion properties of the RT elements it is found that the RT0 elements retain the desirable C-grid dispersion properties. With higher order elements a large amount of the spectrum is more accurately represented however, in common with other high order finite element methods, an increasing number of discontinuities appear in the dispersion relation. These discontinuities manifest as modes with zero group velocity. A method of removing a number of these modes, without effecting the underlying properties of the method, through modification of the mass matrix, is presented. Utilizing the tensor product formulation of the elements along with the Piola transform, a simple extension to the sphere is presented and a comparison of results for the shallow water equations will be given showing the impact of the mass lumping.

The added flexibility of the finite element method combined with the use of the Piola transform method to map from a reference element to the sphere allows simple switching between various grids, therefore results will also be compared for various quasi-uniform alternatives to the well used lat-long grid, namely the cubed-sphere, a diamond grid and kite based grid and the benefits and drawbacks of each choice will be discussed.