Hermite Functions as a basis of spectral global scale Shallow Water models

Nathan Paldor

Fredy and Nadine Institute of Earth Sciences, The Hebrew University of Jerusalem, Jerusalem, Israel. Email: Nathan.paldor@huji.ac.il

Currently, all global scale spectral models are based on the Spherical Harmonics functions and this tradition has its origin in the non-divergent dynamics where the velocity is derived by applying the Laplacian operator on the streamfunction and the well know fact that Spherical Harmonics are the eigenfunctions of the Laplacian operator on a sphere. Recent advances made in the theory of wave solutions on the rotating spherical earth have shown that the eigenfunctions of the Linearized Shallow Water Equations (LSWE) at gravity waves' speed of a few m/s (i.e. a "thin" or baroclinic ocean) are the Hermite Functions (HH) i.e. Hermite polynomials multiplied by a fast decaying Gaussian. These finding have motivated a revisitation of the bases used for numerical solutions of the LSWE over a sphere and, in particular, a comparison between the Hermite Harmonics based solver and the, widely used, Spherical Harmonics (SH) based solver. The comparison is carried out by initializing each of these solvers by a zonally propagating wave solution of LSWE and examining the errors in each solver's calculations as a function of time. Our results shown in Figure 1 demonstrate that for Rossby waves with meridional mode n=5 and zonal wavenumber k=5 the HH solver has a negligible error (as judged by the propagation speed and the change in amplitude) after 100 days. In contrast, the results of Figure 2 demonstrate that for the same initial field the SH solver destroys the initial Rossby wave beyond recognition in only 4 days. Similar results apply to the faster Inertia-Gravity waves and other meridional mode (n) numbers and zonal wavenumbers (k). Our analysis demonstrates that the improved accuracy of the HH solver results from its unique fit to the LSWE numerical matrix propagator. The new set of highly accurate zonally propagating wave solutions can be used to construct a set of quantitative test cases that will determine the performance of a given numerical model by examining the accuracy with which it simulates the phase propagation of these solutions and the change in the amplitudes of the resulting fields as function of time. Initial results in a barotropic (i.e. "thick") ocean where the speed of gravity waves is O(200 m/s) show that the eigenfunctions of the LSWE equations are Gegenbauer polynomials multiplied by an envelope function which is a high power of cosine(latitude) that plays the role of the Gaussian in the HH basis.

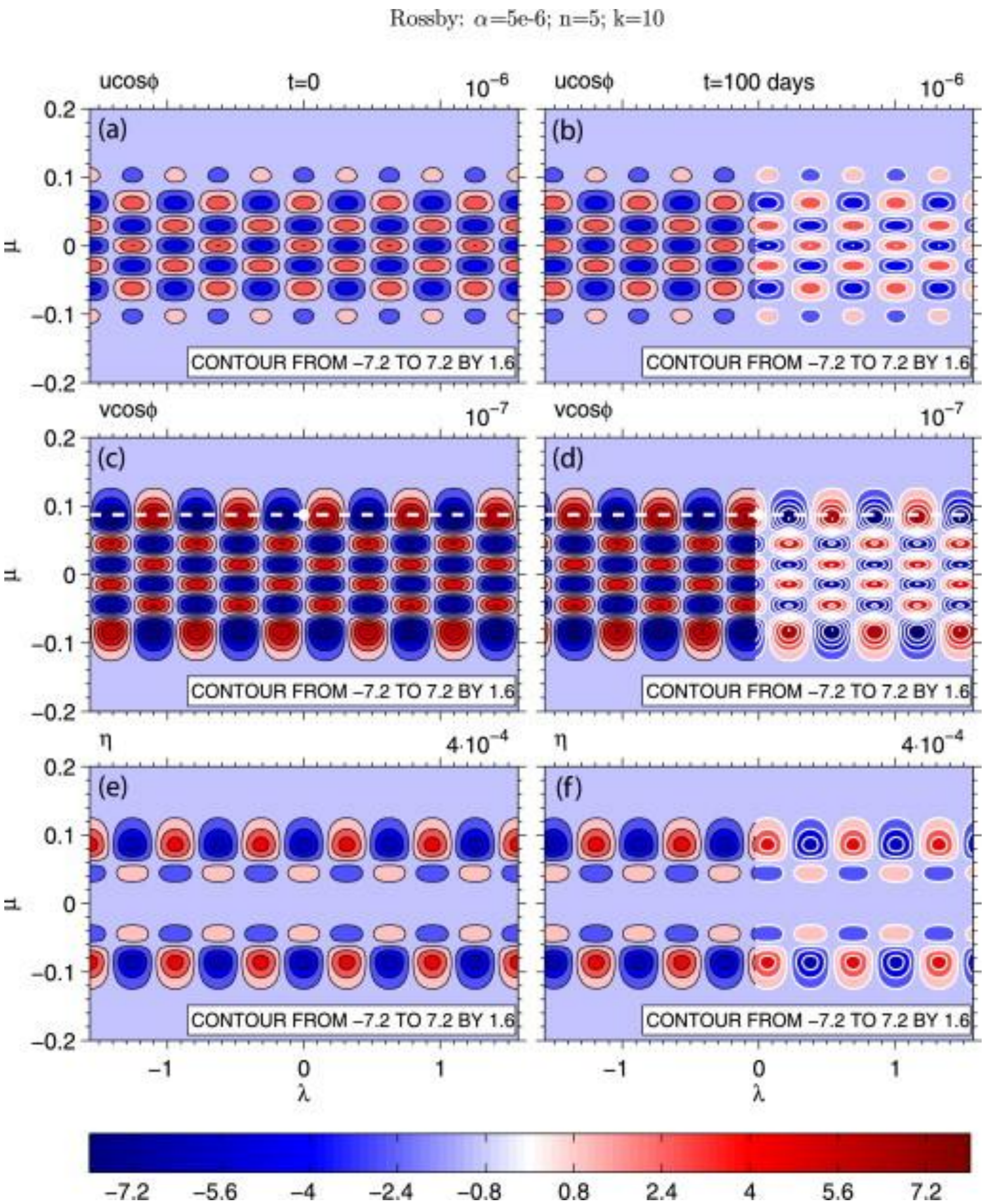

Figure 1: Fields of U , V and η at t =0 (left column) and at t =100 days (right column) for Rossby wave with$(\alpha, n, k) = (5 \cdot 10^{-6}, 5, 10)$. Though k=10 the zonal domain covers only 5 wavelengths in order to allow the details of the fields to be examined. The analytical solutions at t=100 are shown by the white contours on the rightmost half of the fields on the right column.

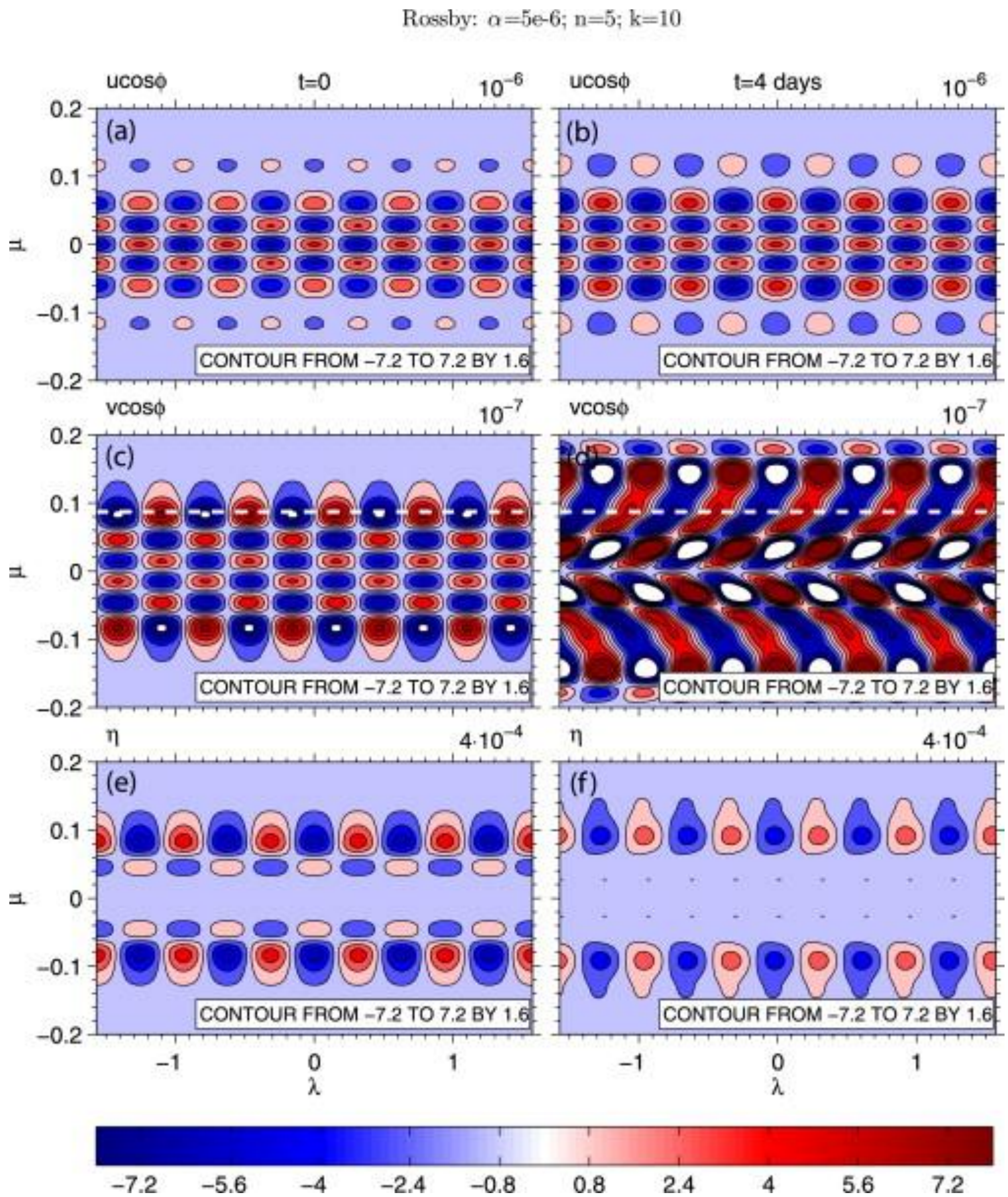

Figure 2: Same as Fig. 1 but with the Spherical Harmonics basis functions at t=0 and t=4 days.