

Generalization of Arakawa's Jacobian

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There are several well known methods to discretize, by finite differences, the vorticity equation for two-dimensional incompressible flow, but Arakawa's Jacobian is one of the most renowned solutions because it satisfies special integral constraints such as conservation of mean kinetic energy and mean square vorticity. We considered a general finite differences discretization of a general operator which uses a 9x9 grid and we asked it to satisfy the physical properties on energy and enstrophy, written in terms of coefficients for the general scheme. It turns out that all these properties can be written by skew-symmetric equations. Then we forced our general operator to be consistent (order two) with the Jacobian operator, and we found out that there exists a whole set of solutions which satisfy the integral constraints mentioned before and this set depends on one parameter (when the parameter is zero, we recover Arakawa's solution). We will show the scheme by different points of view: algebraic, analytic and geometric; we will explore the parameter behaviour both in physical and Fourier spaces in order to study the truncation error and the modified wave number and how to handle the parameter to optimize the scheme. We will also present some numerical tests to show these results. Furthermore, in the future, this idea could be generalized (albeit with technical difficulties) for any order scheme, for any number of grid points, making it very suitable for applications.