# An Asymptotic Parallel-in-Time Method for Highly Oscillatory PDEs

Beth A. Wingate - University of Exeter
Terry Haut - Los Alamos National Laboratory
Jared Whitehead - Brigham Young University

It has been known since the earliest days of numerical weather prediction that the full system of equations used to describe the dynamics of the atmosphere and ocean have fast oscillations. In one of the most important breakthroughs in numerical weather prediction Charney (1948) derived reduced equations, called the Quasi-Geostrophic (QG) equations, that represented the large scales of interest to planetary scale dynamics and `filtered out' the fast waves that, at the time, could not be resolved numerically. These equations enabled the first realistic simulations of numerical weather prediction (Charney *et al.* 1950, Charney and Phillips 1953) which had previously been unstable due to the fast waves in the system (Richardson 1922). These papers sparked the well known debate on the existence of the slow manifold.

Contemporary models use more complex equation sets than QG to describe the dynamics of the atmosphere and ocean and it has been the work of numerical modelers to develop strategies to deal with the highly oscillatory nature of the equations and their impact on the timestep. This issue is particularly important now that computer architectures are changing: computer processor speeds are not expected to increase as quickly as they used to but they are expected to offer unprecedented parallelism.

In this talk we present a new algorithm for achieving parallel-in-time performance for highly oscillatory PDEs and show results with the shallow water equations. We show that the parallel speed-up increases as the time scale separation increases which results in an arbitrarily greater efficiency gain relative to standard numerical integrators. We also present numerical experiments for the doubly periodic shallow water equations that demonstrate the parallel speed up is more than 100 relative to exponential integrators such as ETDRK4 and more than 10 relative to the standard parareal method with a linearly exact coarse solver. Finally we show that the method also works in the absence of time scale separation, allowing for the method to work in different model regimes.