Free and convectively coupled equatorial waves diagnosis using 3-D Normal Modes

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Convectively coupled equatorial-wave diagnosis using three-dimensional normal modes

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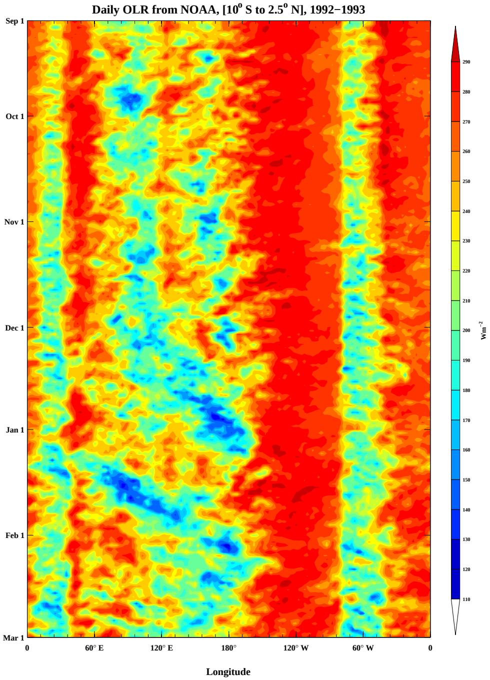
A new methodology for the diagnosis of convectively coupled equatorial waves (CCEWs) is presented. It is based on a pre-filtering of the geopotential and horizontal wind, using three-dimensional (3D) normal mode functions of the adiabatic linearized equations of a resting atmosphere, followed by a space–time spectral analysis to identify the spectral regions of coherence.

The methodology permits a direct detection of various types of equatorial wave, compares the dispersion characteristics of the coupled waves with the theoretical dispersion curves and allows an identification of which vertical modes are more involved in the convection. Moreover, the proposed methodology is able to show the existence of free dry waves and moist coupled waves with a common vertical structure, which is in conformity with the

Outline

CCEW



Daily OLR from NOAA, [10° S to 2.5° N], 1992−1993
Sep 1
Oct 1
Nov 1
Dec 1
Jan 1
Feb 1
Mar 1
0
60° E
120° E
180°
120° W
60° W
0
Longitude
290
280
270
260
250
240
230
220
210
200
190
180
170
160
150
140
130
120
110
Wm⁻²

Equatorial waves

- Linearized primitive equations:

$$\frac{\partial u}{\partial t} - f\,v + \frac{1}{a\cos\theta}\frac{\partial \phi}{\partial \lambda} = 0 \quad (1)$$

$$\frac{\partial v}{\partial t} + f\,u + \frac{1}{a}\frac{\partial \phi}{\partial \theta} = 0 \quad (2)$$

$$\nabla \cdot \mathbf{V} + \frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w) = 0 \quad (3)$$

$$\frac{1}{\rho_0}\frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial z}\right) + wN^2 = 0 \quad (4)$$

Equatorial waves

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$$\nabla \cdot \mathbf{V} + \frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w) = 0 \qquad (3)$$

$$\frac{1}{\rho_0}\frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial z}\right) + wN^2 = 0 \qquad (4)$$

- Assuming N constant and combining (3) and (4) to eliminate w:

Equatorial waves

- Linearized primitive equations:

$$\frac{\partial u}{\partial t} - f\,v + \frac{1}{a\cos\theta}\frac{\partial \phi}{\partial \lambda} = 0 \quad (5)$$

$$\frac{\partial v}{\partial t} + f\,u + \frac{1}{a}\frac{\partial \phi}{\partial \theta} = 0 \quad (6)$$

$$\frac{\partial}{\partial t}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\rho_0\frac{\partial \phi}{\partial z}\right)\right] - N^2\nabla\cdot\mathbf{V} = 0 \quad (7)$$

Equatorial waves

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$$\frac{\partial}{\partial t}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\rho_0\frac{\partial \phi}{\partial z}\right)\right] - N^2\nabla\cdot\mathbf{V} = 0 \qquad (7)$$

- Considering solutions with separable vertical structure:

$$[u, v, \phi] = G(z)\,\left[\tilde{u}(t,\theta,\lambda), \tilde{v}(t,\theta,\lambda), \tilde{\phi}(t,\theta,\lambda)\right]$$

Equatorial waves

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- Like in a wave

$$[u, v, \phi] = \left[\hat{u}(\theta), \hat{v}(\theta), \hat{\phi}(\theta)\right] e^{z/2H} e^{imz} e^{i(k\lambda+\omega t)}$$

Equatorial waves

- One obtains the horizontal structure equations:

$$\frac{\partial \tilde{u}}{\partial t} - f\,\tilde{v} + \frac{1}{a\cos\theta}\frac{\partial \tilde{\phi}}{\partial \lambda} = 0 \tag{8}$$

$$\frac{\partial \tilde{v}}{\partial t} + f\,\tilde{u} + \frac{1}{a}\frac{\partial \tilde{\phi}}{\partial \theta} = 0 \tag{9}$$

$$\frac{\tilde{\phi}}{\partial t} + gh_e \nabla \cdot \tilde{\mathbf{V}} = 0 \tag{10}$$

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$$\frac{\partial \tilde{v}}{\partial t} + f\,\tilde{u} + \frac{1}{a}\frac{\partial \tilde{\phi}}{\partial \theta} = 0 \tag{9}$$

$$\frac{\tilde{\phi}}{\partial t} + gh_e \nabla \cdot \tilde{\mathbf{V}} = 0 \tag{10}$$

- and the vertical structure equation:

$$\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\rho_0 \frac{\partial G}{\partial z}\right) + \frac{N^2}{gh_e} G = 0 \tag{11}$$

- where $-1/gh_e$ is the separation constant.

Equatorial waves

- For a wave with vertical wave number m:

$$[u, v, \phi] = \left[\hat{u}(\theta), \hat{v}(\theta), \hat{\phi}(\theta)\right] e^{z/2H} e^{imz} e^{i(k\lambda + \omega t)}$$

Equatorial waves

- For a wave with vertical wave number m:

$$[u, v, \phi] \quad = \quad \left[\hat{u}(\theta), \hat{v}(\theta), \hat{\phi}(\theta)\right] e^{z/2H} e^{imz} e^{i(k\lambda+\omega t)}$$

- the equivalent depth is given by

$$h_e = \frac{N^2}{g\left(m^2 + \frac{1}{4H^2}\right)} \tag{12}$$

Equatorial waves

- Using the equatorial β-plane approximation:

$$\frac{\partial \tilde{u}}{\partial t} - \beta y \, \tilde{v} + \frac{\partial \tilde{\phi}}{\partial x} = 0 \tag{13}$$

$$\frac{\partial \tilde{v}}{\partial t} + \beta y \, \tilde{u} + \frac{\partial \tilde{\phi}}{\partial y} = 0 \tag{14}$$

$$\frac{\tilde{\phi}}{\partial t} + g h_e \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \tag{15}$$

Equatorial waves

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$$\frac{\tilde{\phi}}{\partial t} + g h_e \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \quad (15)$$

- A complete set of zonal propagating wave solutions of these system of shallow water equations was found by *Matsuno* (1966).

3-D normal mode basis

- We solved the equations over the sphere in isobaric coordinates, with the vertical structure equation given by:

$$\frac{\partial}{\partial p}\left(\frac{1}{S_0}\frac{\partial G}{\partial p}\right)+\frac{1}{gh_e}G=0$$

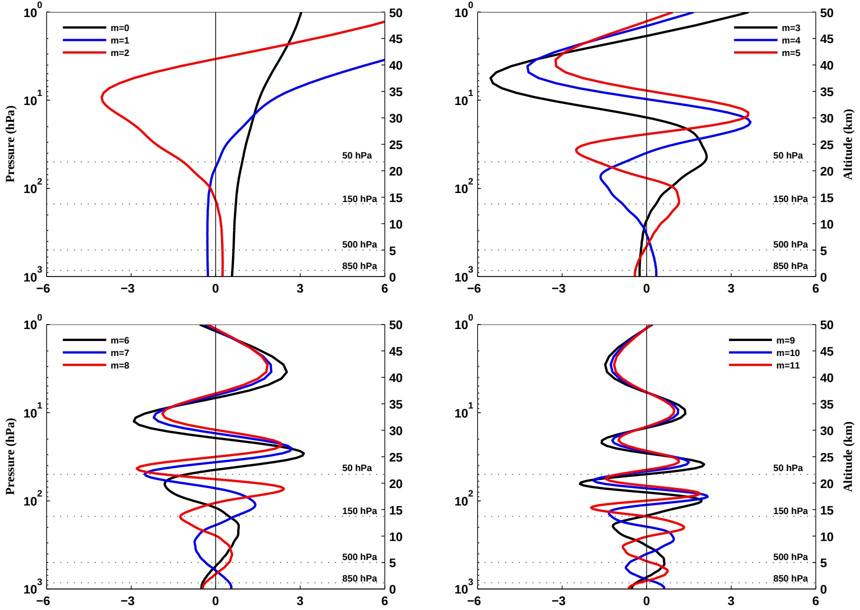
3-D normal mode basis

- We solved the equations over the sphere in isobaric coordinates, with the vertical structure equation given by:

$$\frac{\partial}{\partial p}\left(\frac{1}{S_0}\frac{\partial G}{\partial p}\right)+\frac{1}{gh_e}G=0$$

- The VSE was solved numerically with a spectral method as in *Kasahara* (1984) and *Castanheira et al.* (1999).

Vertical Structure Functions



3-D normal mode basis

- The horizontal structure equations over the sphere

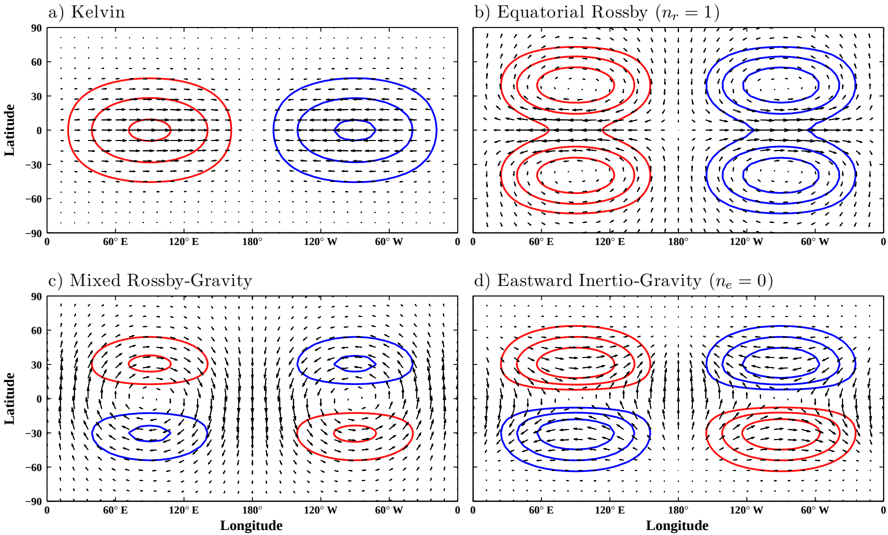
$$\frac{\partial \tilde{u}}{\partial t} - f\,\tilde{v} + \frac{1}{a\cos\theta}\frac{\partial \tilde{\phi}}{\partial \lambda} = 0 \quad (16)$$

$$\frac{\partial \tilde{v}}{\partial t} + f\,\tilde{u} + \frac{1}{a}\frac{\partial \tilde{\phi}}{\partial \theta} = 0 \quad (17)$$

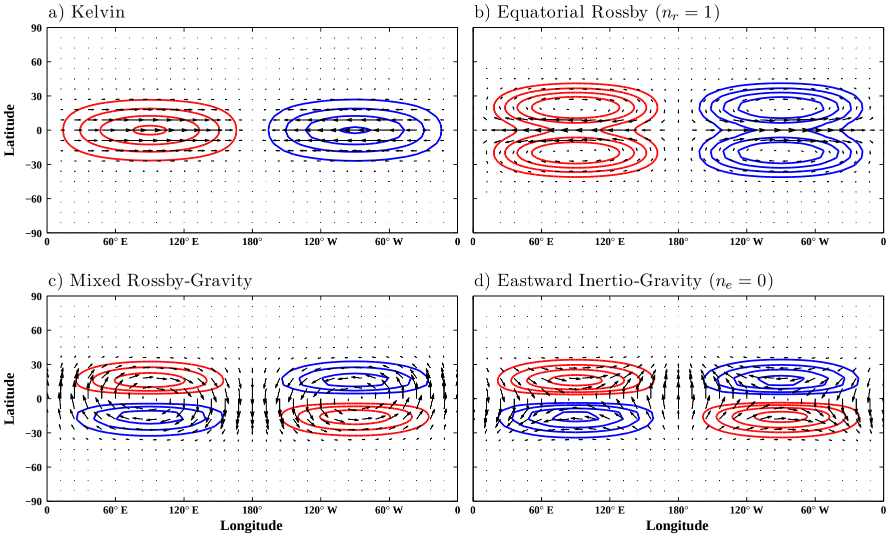
$$\frac{\tilde{\phi}}{\partial t} + gh_e \nabla \cdot \tilde{\mathbf{V}} = 0 \quad (18)$$

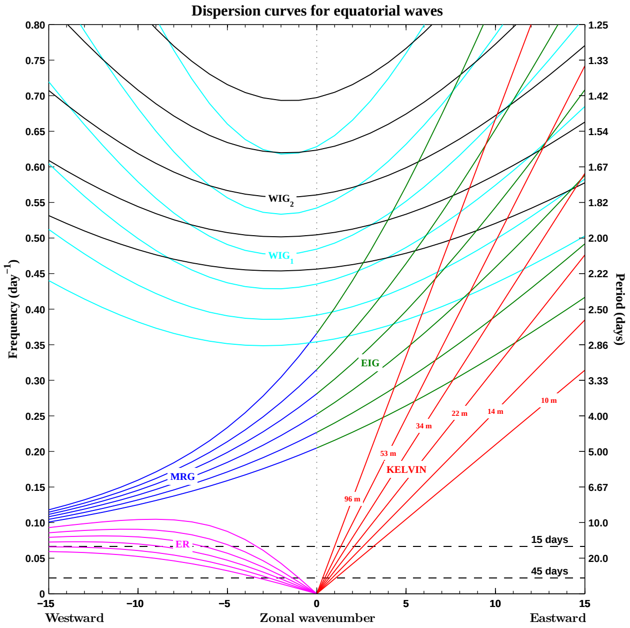
- were solved using the methodology of *Swarztrauber and Kasahara* (1985).

Shallow water waves over the sphere ($h_1 = 5591\,\mathrm{m}$)



Shallow water waves over the sphere ($h_4 = 474\,\mathrm{m}$)





Dispersion curves for equatorial waves
Frequency (day^{-1})
Period (days)
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
1.25
1.33
1.42
1.54
1.67
1.82
2.00
2.22
2.50
2.86
3.33
4.00
5.00
6.67
10.0
20.0
WIG$_2$
WIG$_1$
EIG
MRG
ER
KELVIN
96 m
53 m
34 m
22 m
14 m
10 m
15 days
45 days
−15
−10
−5
0
5
10
15
Westward
Zonal wavenumber
Eastward

Data

- Outgoing Longwave Radiation (OLR) from the National Oceanic and Atmospheric Administration (NOAA) for the period 1979-2012.

- Horizontal wind (u, v) and geopotencial (ϕ) from the ERA-Interim reanalysis (1979-2012).

Method: filtering the dynamical fields

- Projection of the horizontal wind (u, v) and geopotential (ϕ) onto the normal modes of the linearized primitive equations on the sphere.

$$\begin{bmatrix} u \\ v \\ \phi \end{bmatrix} = \sum_{m,k,n} w_{mkn}(t)\; G_m(p)\, e^{ik\lambda}\, \mathbf{C}_m \cdot \begin{bmatrix} U(\theta) \\ iV(\theta) \\ Z(\theta) \end{bmatrix}_{mkn}$$

Method: filtering the dynamical fields

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$$\begin{bmatrix} u \\ v \\ \phi \end{bmatrix} = \sum_{m,k,n} w_{mkn}(t)\ G_m(p)\, e^{ik\lambda}\, \mathbf{C}_m \cdot \begin{bmatrix} U(\theta) \\ iV(\theta) \\ Z(\theta) \end{bmatrix}_{mkn}$$

- Reconstitution of the horizontal wind (u, v) and geopotential (ϕ) with a given subset of modes (filtering):

$$\begin{bmatrix} u_n \\ v_n \\ \phi_n \end{bmatrix} = \sum_{m,k} w_{mkn}(t)\ G_m(p)\, e^{ik\lambda}\, \mathbf{C}_m \cdot \begin{bmatrix} U(\theta) \\ iV(\theta) \\ Z(\theta) \end{bmatrix}_{mkn}$$

Method: space-time cross-spectral analysis (Hayashi, 1982)

- Considering a space-time series

$$Y(\lambda, t) = \sum_k \left[C_k(t) \cos(k\lambda) + S_k(t) \sin(k\lambda) \right]$$

Method: space-time cross-spectral analysis (Hayashi, 1982)

- Considering a space-time series

$$Y(\lambda, t) = \sum_k [C_k(t)\cos(k\lambda) + S_k(t)\sin(k\lambda)]$$

- The space-time power spectra is given by

$$4\,P_{k,\pm\omega}(Y) \quad = \quad P_\omega(C_k) + P_\omega(S_k) \pm 2\,Q_\omega(C_k, S_k),$$

where P_ω and Q_ω are the time power and quadrature spectra, respectively.

Method: space-time cross-spectral analysis (Hayashi, 1982)

- Spectral coherence and phase difference between two fields $Y(\lambda, t)$ and $Y'(\lambda, t)$ are given by

$$\mathrm{Coh}^2_{k,\pm\omega}(Y, Y') = \frac{K^2_{k,\pm\omega}(Y, Y') + Q^2_{k,\pm\omega}(Y, Y')}{P_{k,\pm\omega}(Y) P_{k,\pm\omega}(Y')}$$

- and

$$\mathrm{Ph}_{k,\pm\omega}(Y, Y') = \tan^{-1}\left[Q_{k,\pm\omega}(Y, Y')/K_{k,\pm\omega}(Y, Y')\right]$$

- where

Method: space-time cross-spectral analysis (Hayashi, 1982)

- where

$$
\begin{aligned}
4\,K_{k,\pm\omega}(Y,Y') &= K_\omega(C_k,{C_k}') + K_\omega(S_k,{S_k}') \\
&\quad \pm Q_\omega(C_k,{S_k}') \mp Q_\omega(S_k,{C_k}')
\end{aligned}
$$

- and

$$
\begin{aligned}
4\,Q_{k,\pm\omega}(Y,Y') &= \pm Q_\omega(C_k,{C_k}') \pm Q_\omega(S_k,{S_k}') \\
&\quad - K_\omega(C_k,{S_k}') + K_\omega(S_k,{C_k}')
\end{aligned}
$$

- are cospectra and quadrature spectra, respectively.

Testing the methodology

- Reconstructing the circulation field

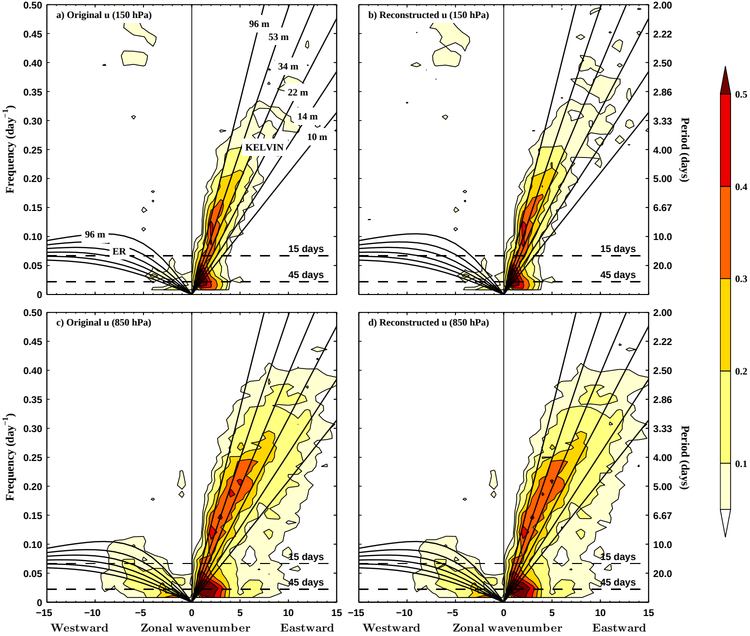
$$\begin{bmatrix} u \\ v \\ \phi \end{bmatrix} = \sum_{m,k,n} w_{mkn}(t)\ G_m(p)\, e^{ik\lambda}\, \mathbf{C}_m \cdot \begin{bmatrix} U(\theta) \\ iV(\theta) \\ Z(\theta) \end{bmatrix}_{mkn}$$

- Symmetric and antisymmetric components of a variable Y

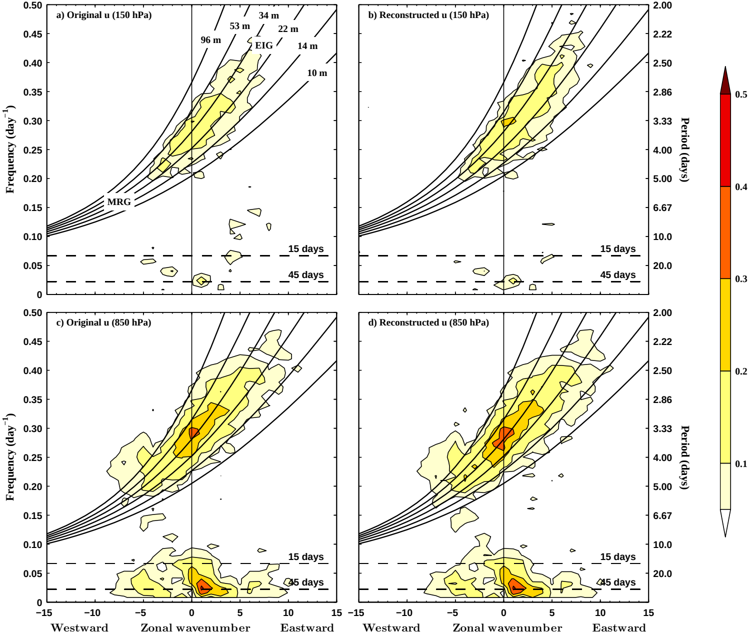
$$Y_S(\theta) = \frac{Y(\theta) + Y(-\theta)}{2}$$

$$Y_A(\theta) = \frac{Y(\theta) - Y(-\theta)}{2}$$

Squared coherence between symmetric components of zonal wind and OLR



Squared coherence between anti−symmetric components of zonal wind and OLR



Selecting waves

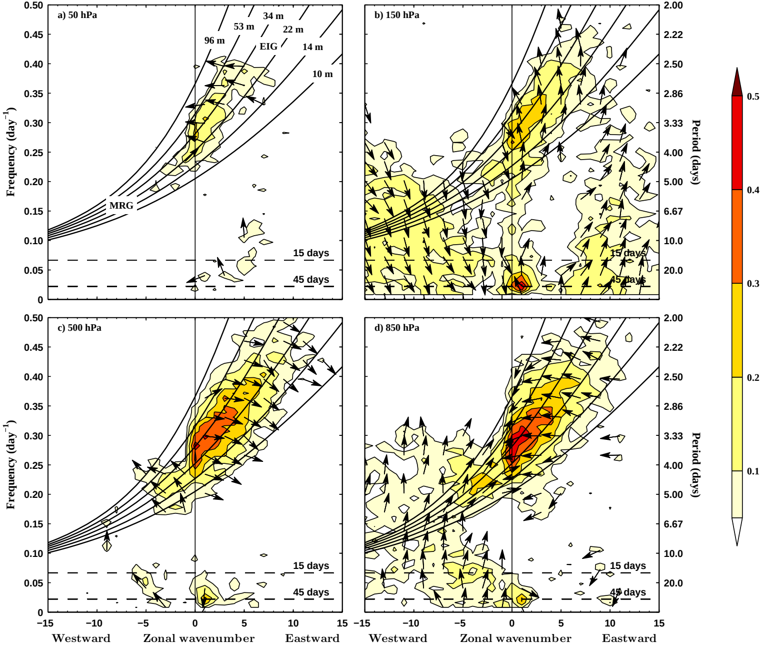
- Fixing a given pair of zonal, k and meridional n indices, waves of a given type are selected

$$\begin{bmatrix} u_{kn} \\ v_{kn} \\ \phi_{kn} \end{bmatrix} = \sum_m w_{mkn}(t)\; G_m(p)\, e^{ik\lambda}\, \mathbf{C}_m \cdot \begin{bmatrix} U(\theta) \\ iV(\theta) \\ Z(\theta) \end{bmatrix}_{mkn}$$

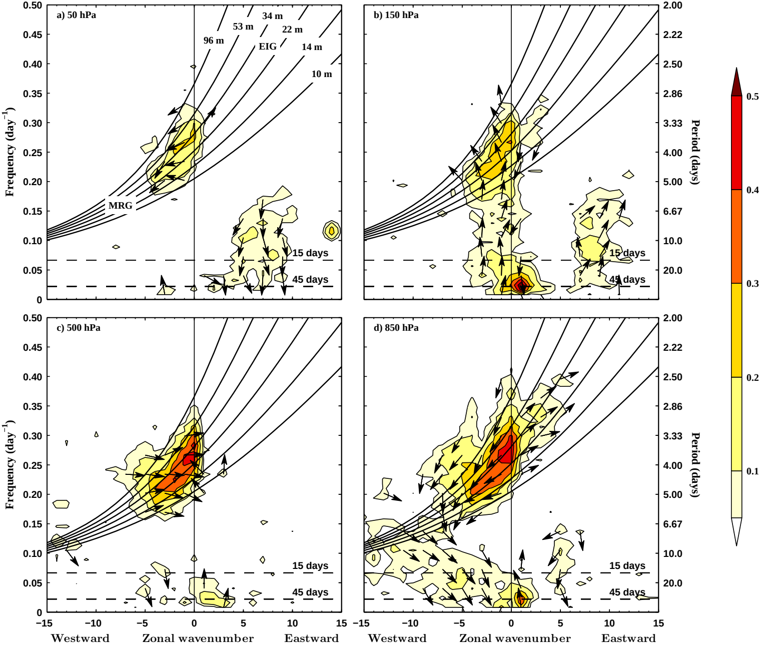
Results

- Space-time coherence spectra for eastward inertio-gravity (EIG), mixed Rossby-gravity (MRG), Kelvin (Kel) and Equatorial Rossby (ER) waves.

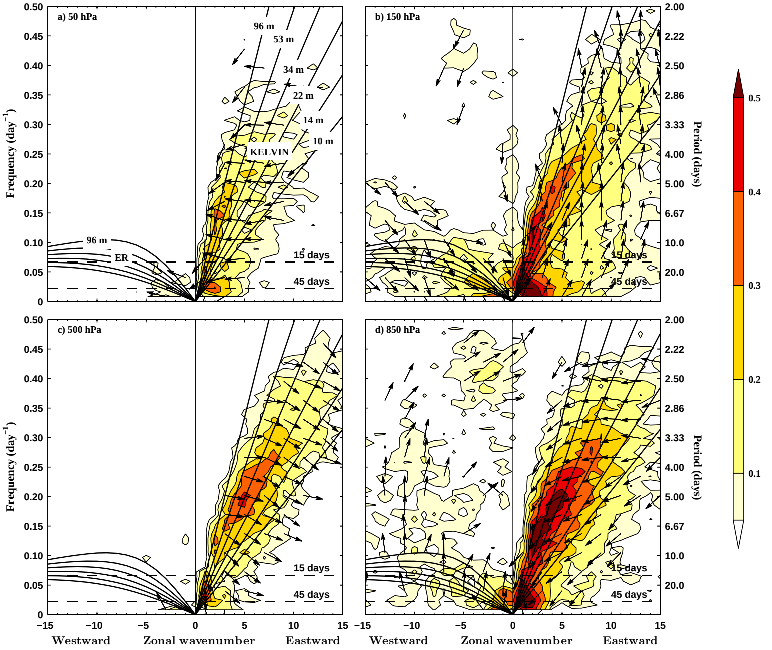
Squared coherence between u_{EIG} and anti−symmetric OLR

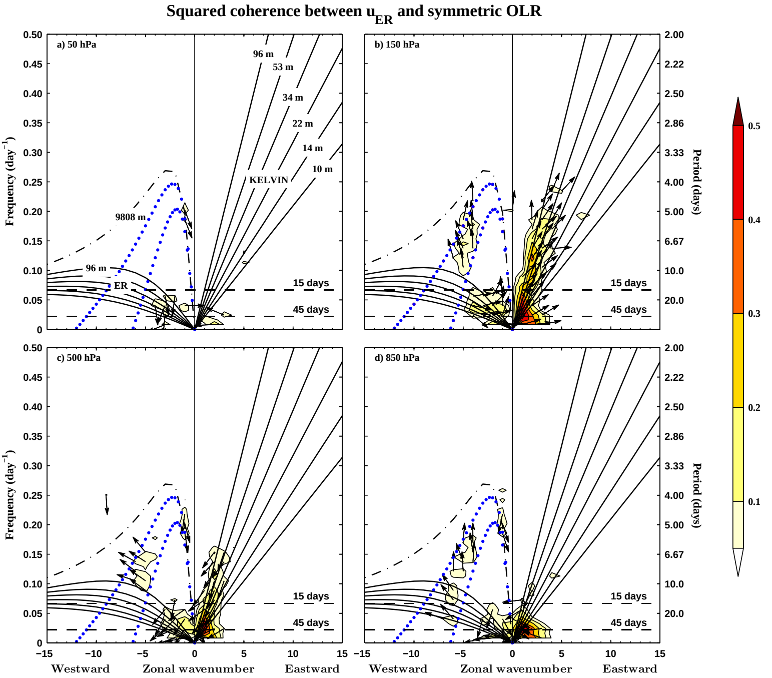


Squared coherence between u_{MRG} and anti−symmetric OLR



Squared coherence between u_{KEL} and symmetric OLR




Squared coherence between u_{ER} and symmetric OLR
a) 50 hPa
b) 150 hPa
c) 500 hPa
d) 850 hPa
96 m
53 m
34 m
22 m
14 m
10 m
KELVIN
9808 m
96 m
ER
15 days
45 days
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
0.5
0.4
0.3
0.2
0.1

Vertical mode decomposition of waves

- Fixing the vertical, zonal,and meridional indices, *mkn*, respectively, we obtain a vertical modal decomposition of the waves

$$\begin{bmatrix} u_{mkn} \\ v_{mkn} \\ \phi_{mkn} \end{bmatrix} = w_{mkn}(t)\ G_m(p)\, e^{ik\lambda}\, \mathbf{C} \cdot \begin{bmatrix} U(\theta) \\ iV(\theta) \\ Z(\theta) \end{bmatrix}_{mkn}$$

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- Now for, the cosine and sine coefficients are given by

$$C_k^{mn} \propto \Re\left[w_{mkn}(t)\right] \qquad S_k^{mn} \propto \Im\left[w_{mkn}(t)\right]$$

Vertical mode decomposition of waves

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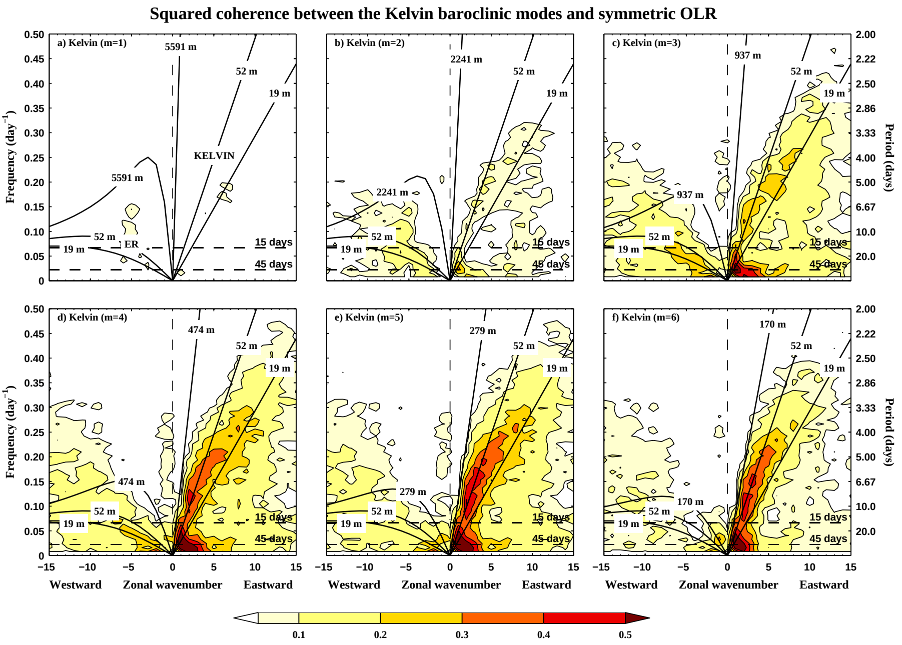
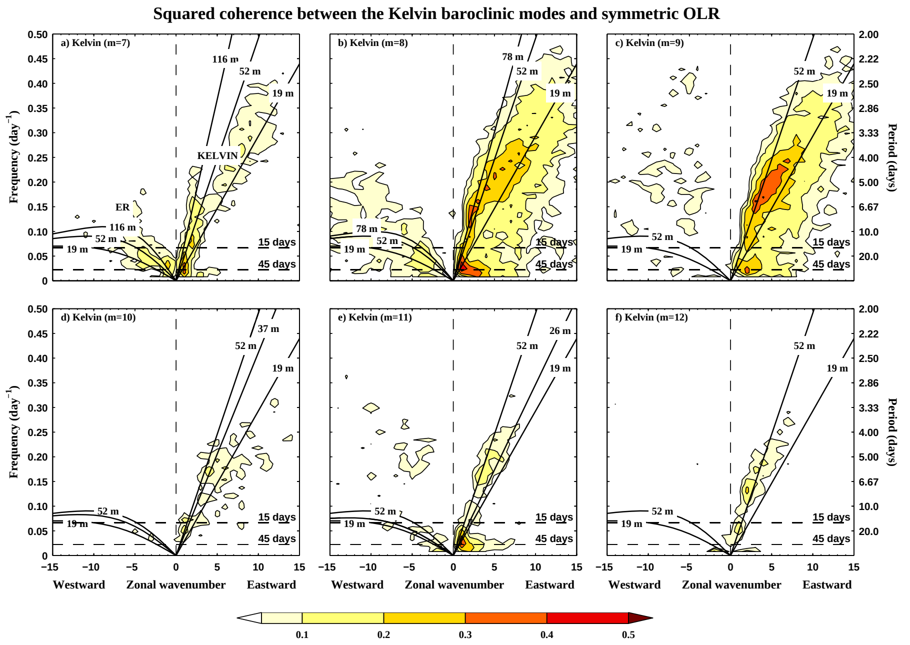
$$C_k^{mn} \propto \Re\left[w_{mkn}(t)\right] \qquad S_k^{mn} \propto \Im\left[w_{mkn}(t)\right]$$

- and we may analyze the coherence of the 3-D waves with the OLR and interpret their power spectra as the total (Kinetic + Available Potential) energy spectra.

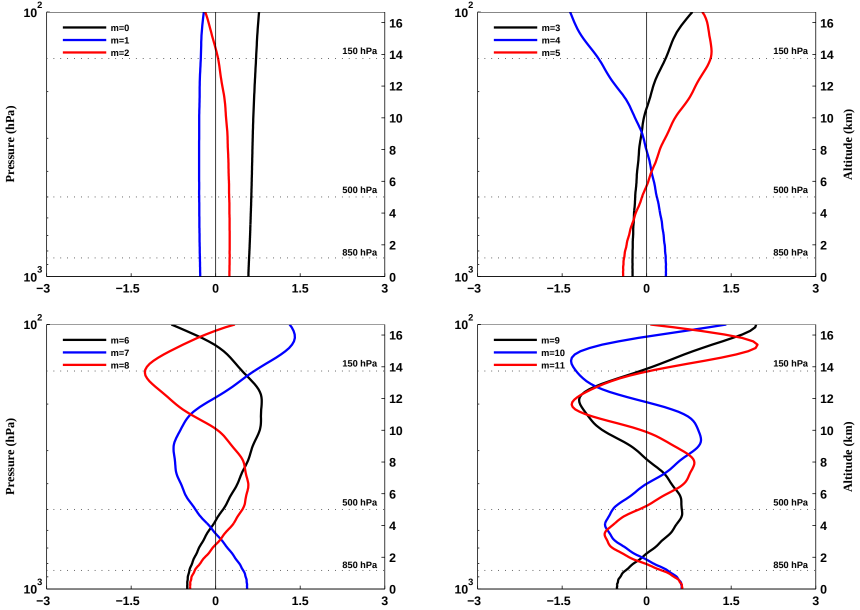


Squared coherence between the Kelvin baroclinic modes and symmetric OLR
a) Kelvin (m=1)
b) Kelvin (m=2)
c) Kelvin (m=3)
d) Kelvin (m=4)
e) Kelvin (m=5)
f) Kelvin (m=6)
5591 m
2241 m
937 m
474 m
279 m
170 m
52 m
19 m
KELVIN
ER
15 days
45 days
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
0.1
0.2
0.3
0.4
0.5

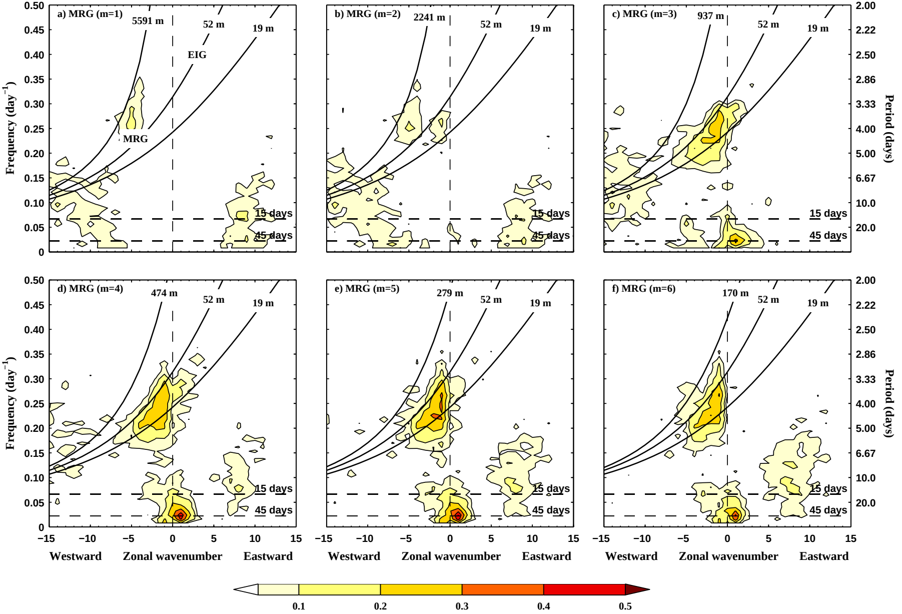


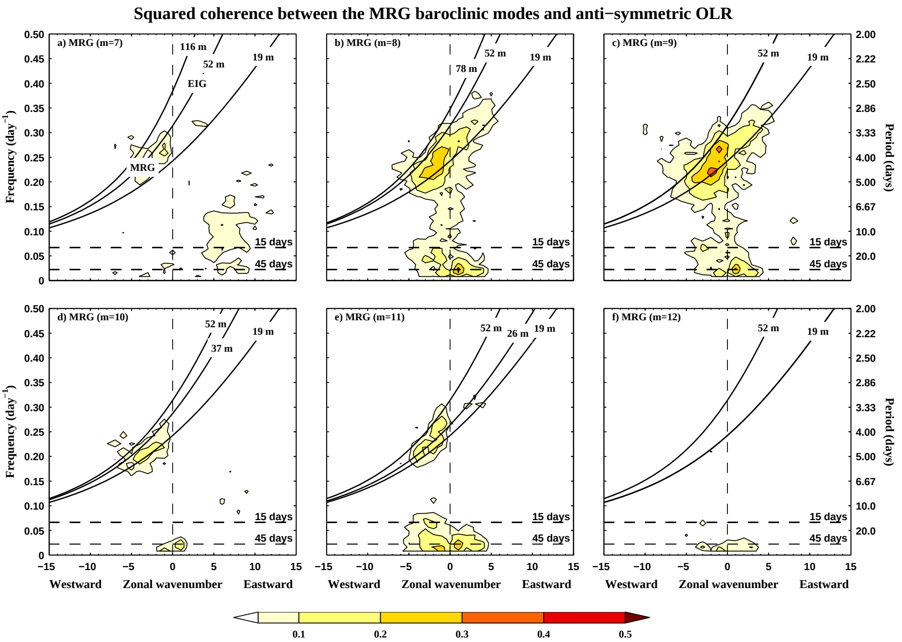
Squared coherence between the Kelvin baroclinic modes and symmetric OLR
a) Kelvin (m=7)
b) Kelvin (m=8)
c) Kelvin (m=9)
d) Kelvin (m=10)
e) Kelvin (m=11)
f) Kelvin (m=12)
KELVIN
ER
116 m
78 m
52 m
37 m
26 m
19 m
15 days
45 days
Frequency (day−1)
Period (days)
Westward
Zonal wavenumber
Eastward
0.1
0.2
0.3
0.4
0.5

Vertical Structure Functions

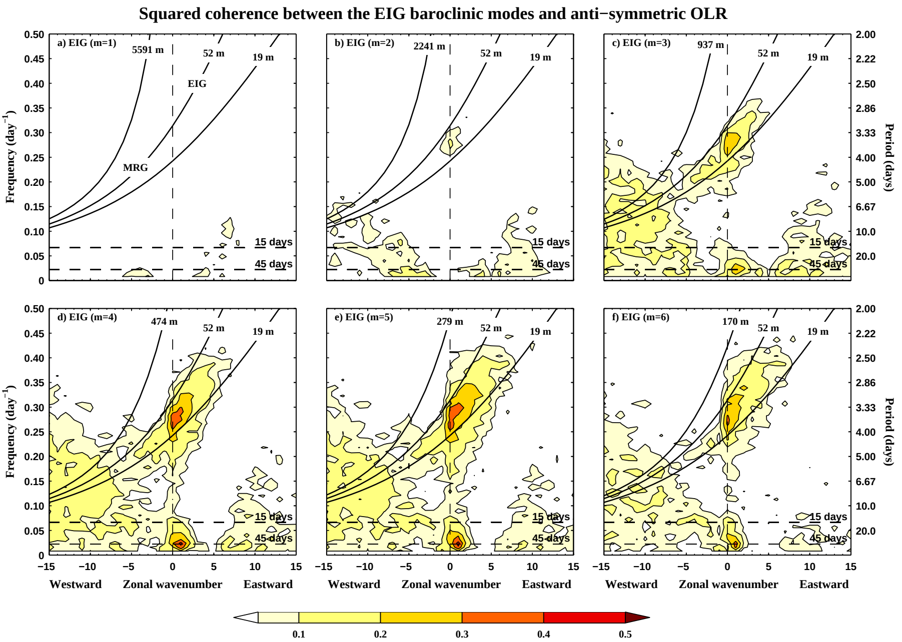


Squared coherence between the MRG baroclinic modes and anti−symmetric OLR

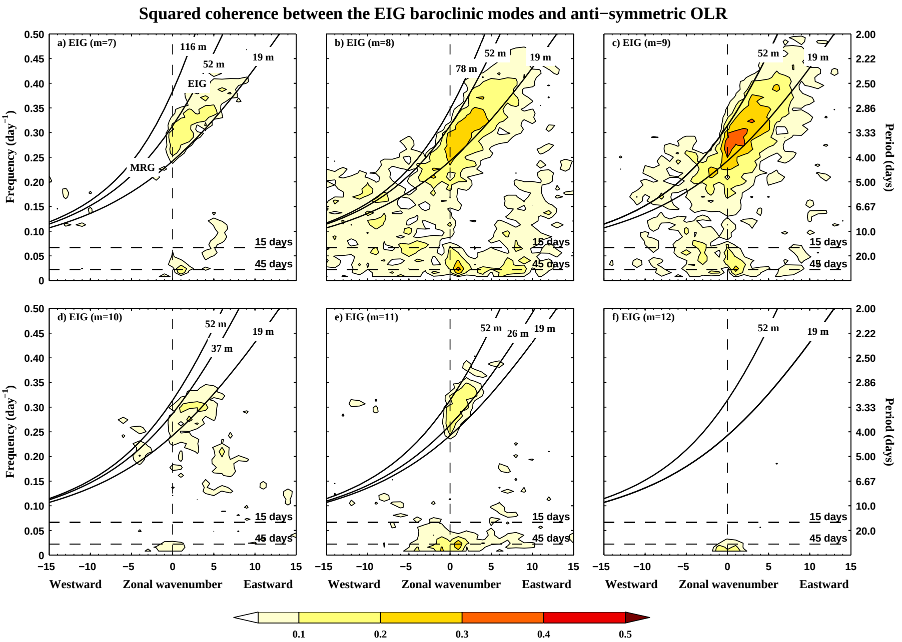




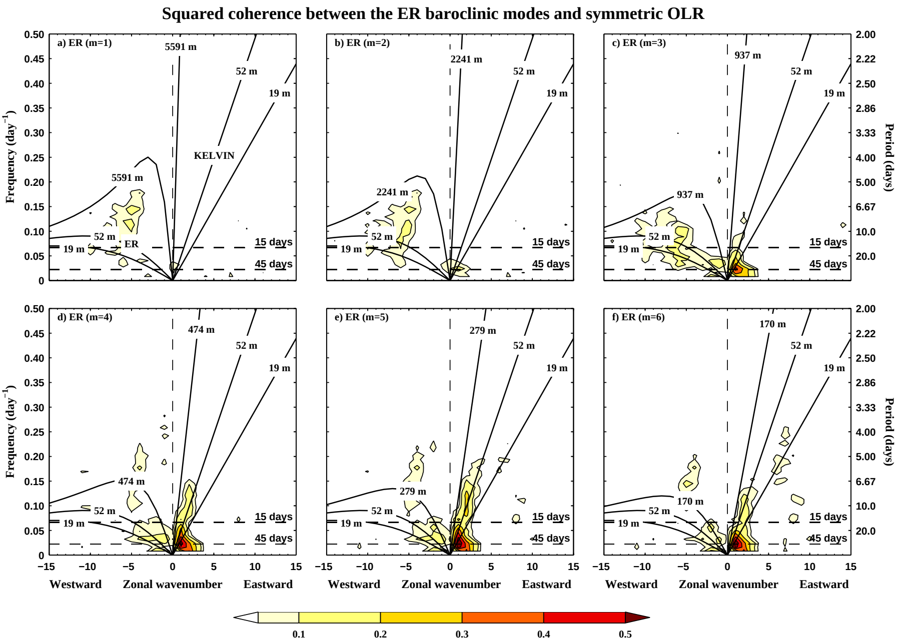
Squared coherence between the MRG baroclinic modes and anti−symmetric OLR
a) MRG (m=7)
b) MRG (m=8)
c) MRG (m=9)
d) MRG (m=10)
e) MRG (m=11)
f) MRG (m=12)
116 m
52 m
78 m
37 m
26 m
19 m
EIG
MRG
15 days
45 days
Frequency (day−1)
Period (days)
Westward
Zonal wavenumber
Eastward
0.1
0.2
0.3
0.4
0.5



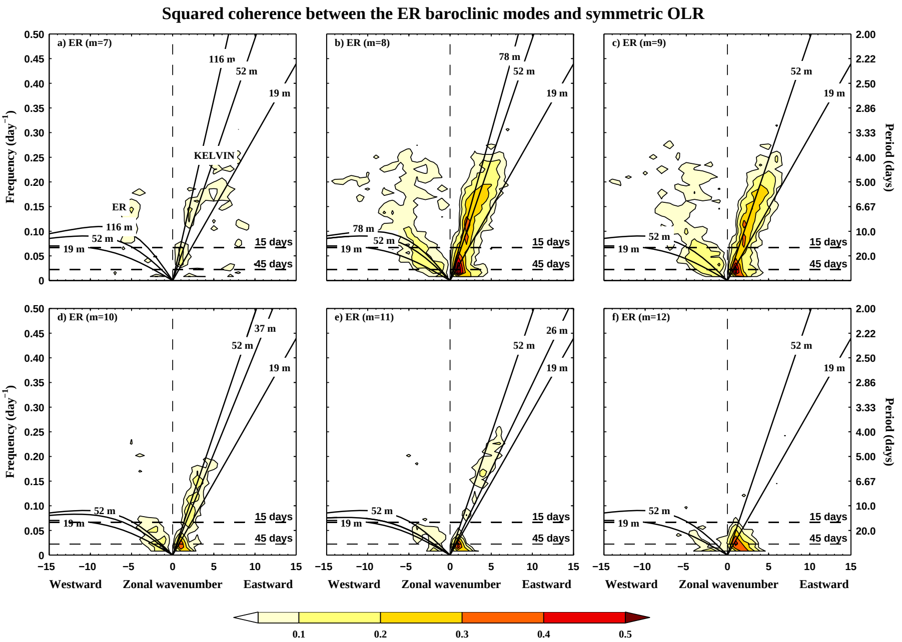
Squared coherence between the EIG baroclinic modes and anti−symmetric OLR
a) EIG (m=1)
b) EIG (m=2)
c) EIG (m=3)
d) EIG (m=4)
e) EIG (m=5)
f) EIG (m=6)
5591 m
2241 m
937 m
474 m
279 m
170 m
52 m
19 m
EIG
MRG
15 days
45 days
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
0.1
0.2
0.3
0.4
0.5



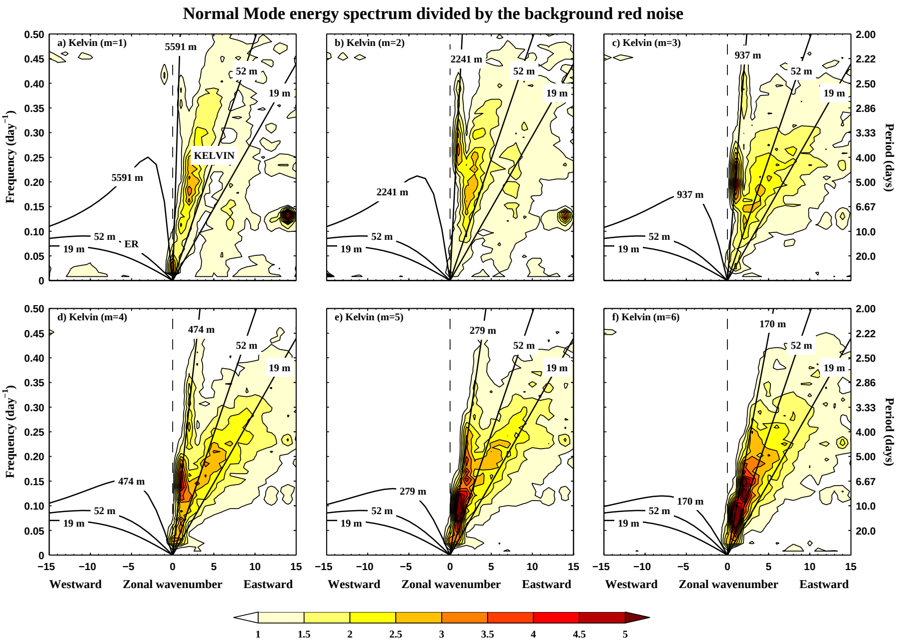
Squared coherence between the EIG baroclinic modes and anti−symmetric OLR
a) EIG (m=7)
b) EIG (m=8)
c) EIG (m=9)
d) EIG (m=10)
e) EIG (m=11)
f) EIG (m=12)
116 m
78 m
52 m
37 m
26 m
19 m
EIG
MRG
15 days
45 days
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
0.1
0.2
0.3
0.4
0.5



Squared coherence between the ER baroclinic modes and symmetric OLR
a) ER (m=1)
b) ER (m=2)
c) ER (m=3)
d) ER (m=4)
e) ER (m=5)
f) ER (m=6)
5591 m
2241 m
937 m
474 m
279 m
170 m
52 m
19 m
KELVIN
ER
15 days
45 days
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
0.1
0.2
0.3
0.4
0.5

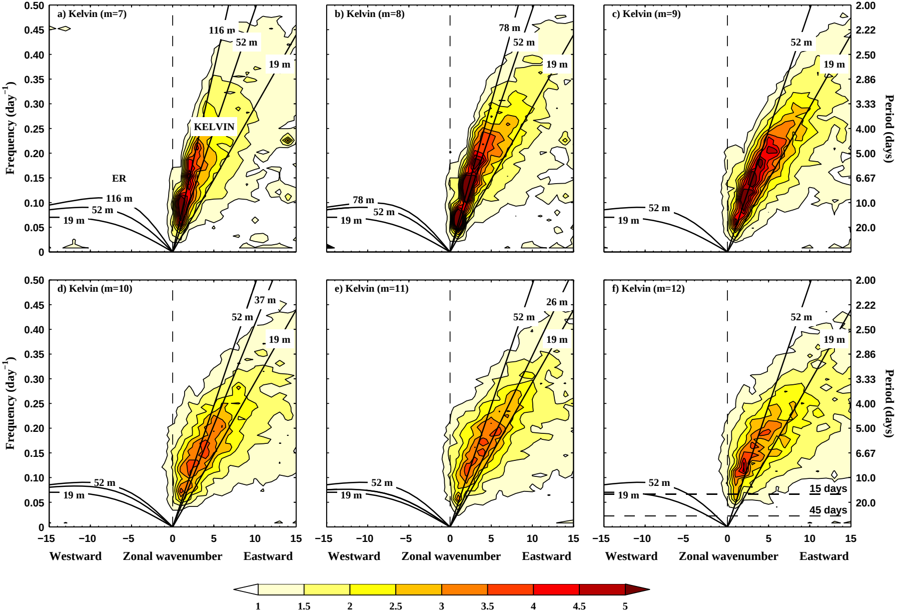


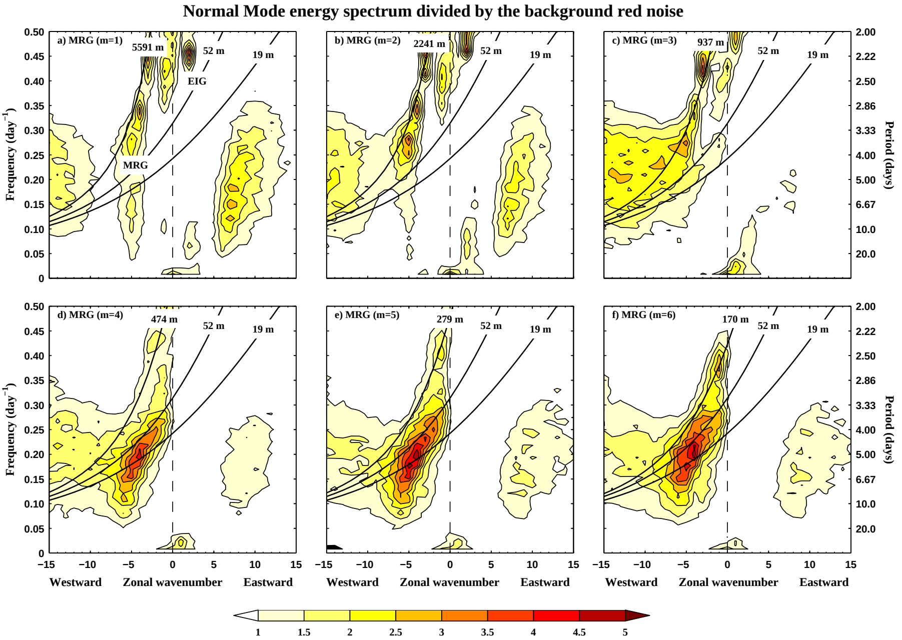
Squared coherence between the ER baroclinic modes and symmetric OLR
a) ER (m=7)
b) ER (m=8)
c) ER (m=9)
d) ER (m=10)
e) ER (m=11)
f) ER (m=12)
KELVIN
ER
116 m
78 m
52 m
37 m
26 m
19 m
15 days
45 days
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
0.1
0.2
0.3
0.4
0.5



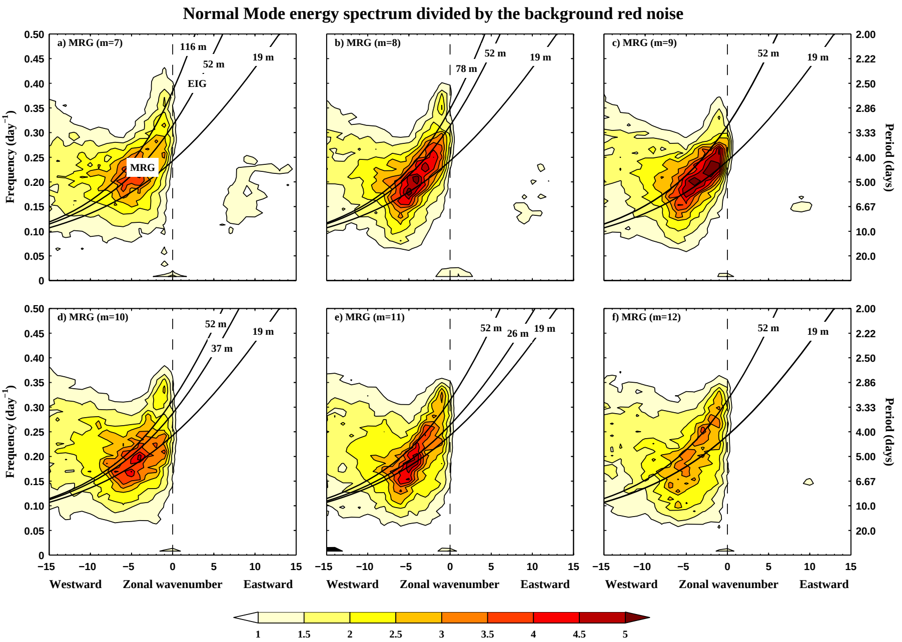
Normal Mode energy spectrum divided by the background red noise
a) Kelvin (m=1)
b) Kelvin (m=2)
c) Kelvin (m=3)
d) Kelvin (m=4)
e) Kelvin (m=5)
f) Kelvin (m=6)
5591 m
2241 m
937 m
474 m
279 m
170 m
52 m
19 m
KELVIN
ER
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward

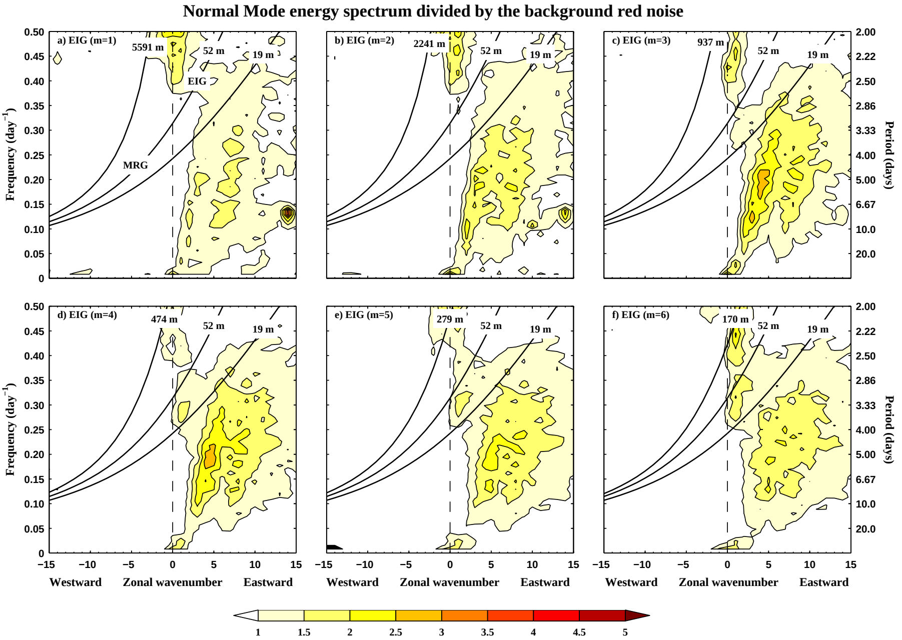
Normal Mode energy spectrum divided by the background red noise

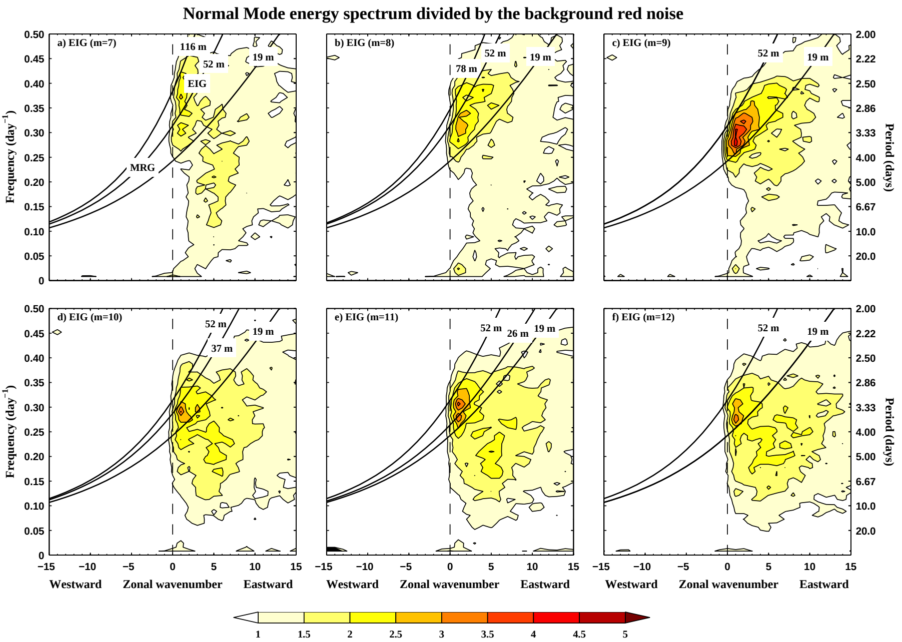




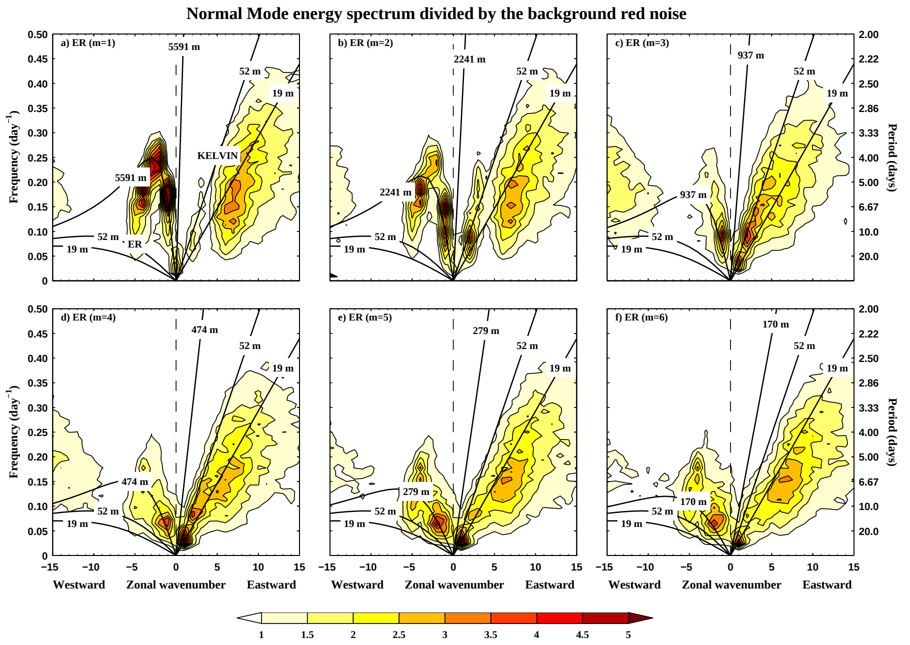
Normal Mode energy spectrum divided by the background red noise
a) MRG (m=1)
5591 m
52 m
19 m
EIG
MRG
b) MRG (m=2)
2241 m
c) MRG (m=3)
937 m
d) MRG (m=4)
474 m
e) MRG (m=5)
279 m
f) MRG (m=6)
170 m
Frequency (day^{-1})
Period (days)
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
2.00
2.22
2.50
2.86
3.33
4.00
5.00
6.67
10.0
20.0
−15
−10
−5
0
5
10
15
Westward
Zonal wavenumber
Eastward
1
1.5
2
2.5
3
3.5
4
4.5
5


Normal Mode energy spectrum divided by the background red noise
a) MRG (m=7)
b) MRG (m=8)
c) MRG (m=9)
d) MRG (m=10)
e) MRG (m=11)
f) MRG (m=12)
116 m
52 m
EIG
19 m
MRG
78 m
37 m
26 m
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
1
1.5
2
2.5
3
3.5
4
4.5
5

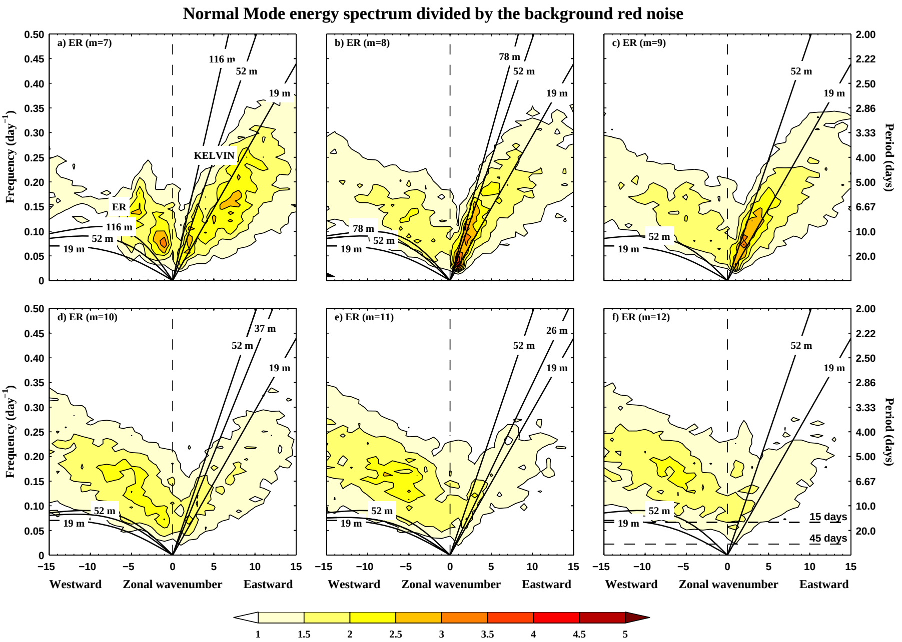

Normal Mode energy spectrum divided by the background red noise
a) EIG (m=1)
5591 m
52 m
19 m
EIG
MRG
b) EIG (m=2)
2241 m
c) EIG (m=3)
937 m
d) EIG (m=4)
474 m
e) EIG (m=5)
279 m
f) EIG (m=6)
170 m
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
1
1.5
2
2.5
3
3.5
4
4.5
5

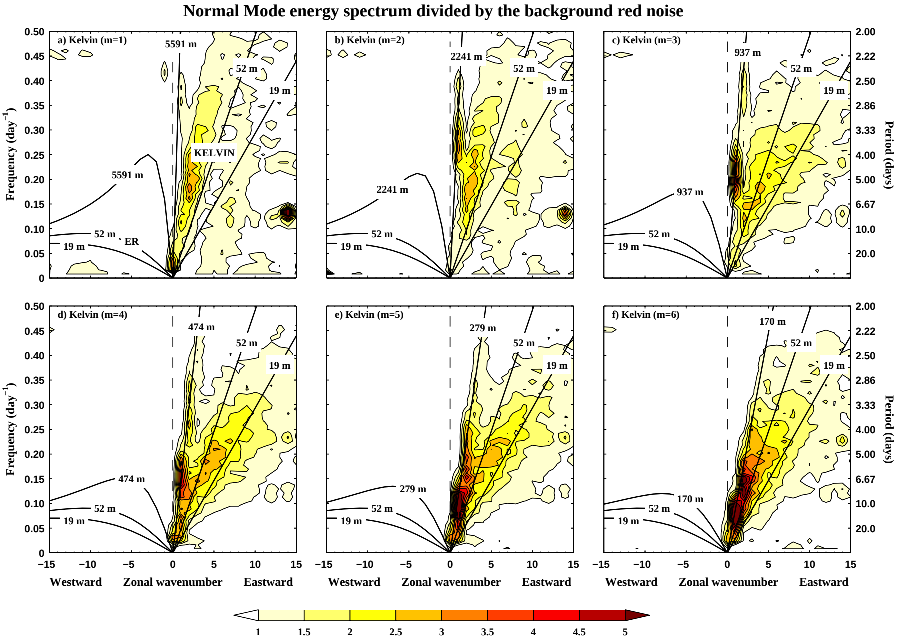


Normal Mode energy spectrum divided by the background red noise
a) EIG (m=7)
116 m
52 m
19 m
EIG
MRG
b) EIG (m=8)
78 m
52 m
19 m
c) EIG (m=9)
52 m
19 m
d) EIG (m=10)
52 m
37 m
19 m
e) EIG (m=11)
52 m
26 m
19 m
f) EIG (m=12)
52 m
19 m
Frequency (day^{-1})
Period (days)
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
2.00
2.22
2.50
2.86
3.33
4.00
5.00
6.67
10.0
20.0
−15 −10 −5 0 5 10 15
Westward
Zonal wavenumber
Eastward
1 1.5 2 2.5 3 3.5 4 4.5 5

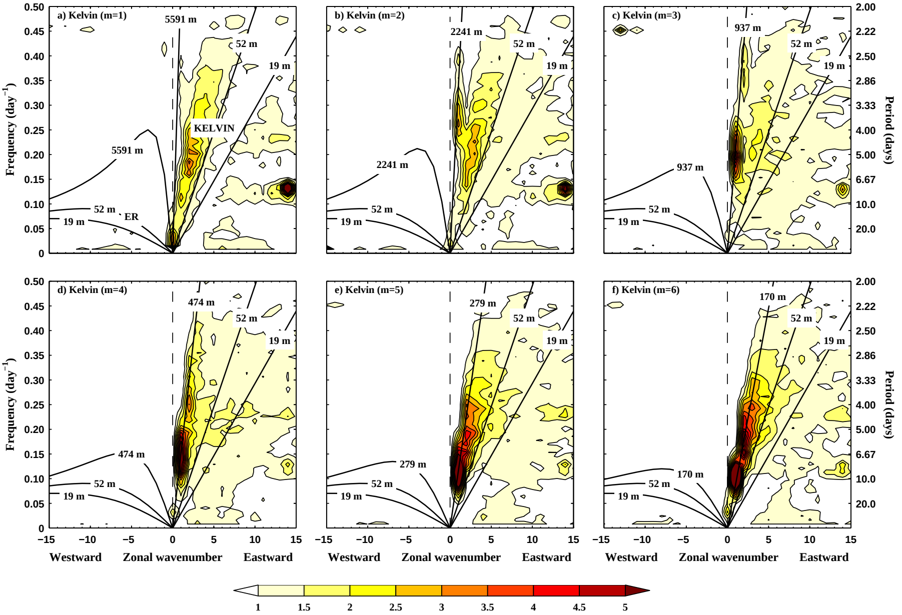


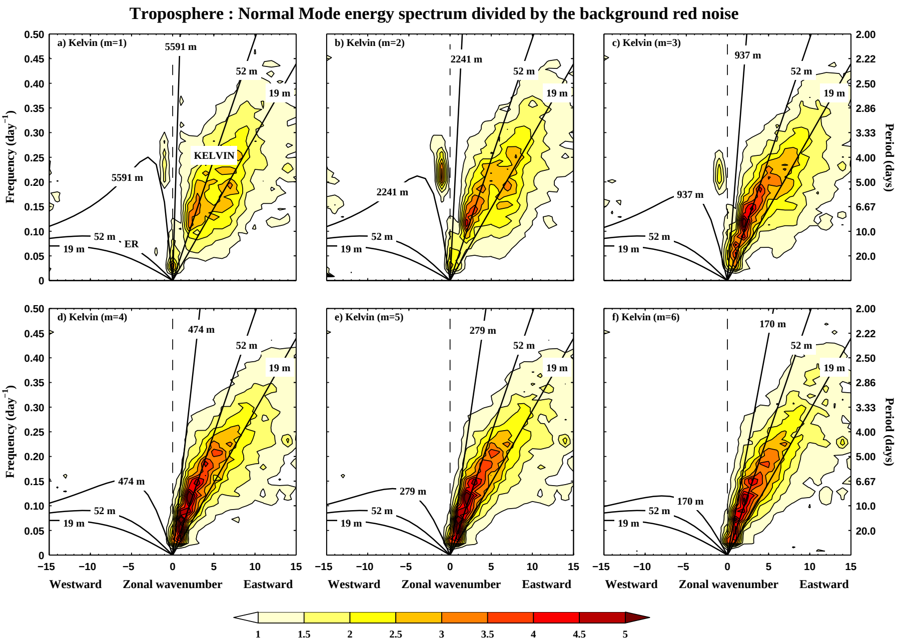
Normal Mode energy spectrum divided by the background red noise
a) ER (m=1)
b) ER (m=2)
c) ER (m=3)
d) ER (m=4)
e) ER (m=5)
f) ER (m=6)
5591 m
2241 m
937 m
474 m
279 m
170 m
52 m
19 m
KELVIN
ER
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward


Normal Mode energy spectrum divided by the background red noise
a) ER (m=7)
b) ER (m=8)
c) ER (m=9)
d) ER (m=10)
e) ER (m=11)
f) ER (m=12)
KELVIN
ER
116 m
78 m
52 m
37 m
26 m
19 m
15 days
45 days
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
1
1.5
2
2.5
3
3.5
4
4.5
5


Normal Mode energy spectrum divided by the background red noise
a) Kelvin (m=1)
b) Kelvin (m=2)
c) Kelvin (m=3)
d) Kelvin (m=4)
e) Kelvin (m=5)
f) Kelvin (m=6)
5591 m
2241 m
937 m
474 m
279 m
170 m
52 m
19 m
KELVIN
ER
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
1
1.5
2
2.5
3
3.5
4
4.5
5

Stratosphere : Normal Mode energy spectrum divided by the background red noise




Troposphere : Normal Mode energy spectrum divided by the background red noise
a) Kelvin (m=1)
b) Kelvin (m=2)
c) Kelvin (m=3)
d) Kelvin (m=4)
e) Kelvin (m=5)
f) Kelvin (m=6)
KELVIN
ER
5591 m
2241 m
937 m
474 m
279 m
170 m
52 m
19 m
Frequency (day⁻¹)
Period (days)
Westward
Zonal wavenumber
Eastward
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
2.00
2.22
2.50
2.86
3.33
4.00
5.00
6.67
10.0
20.0
−15
−10
−5
0
5
10
15
1
1.5
2
2.5
3
3.5
4
4.5
5

QBO effects

- The vertical propagation of equatorial waves was analysed for the two phases of the QBO. Considering the daily zonal mean 30-hPa zonal wind at the equator the QBO phases were defined as follows

$$\overline{u}(0\,\mathrm{N}, 30\,\mathrm{hPa}) > 5\,\mathrm{m\,s^{-1}} \quad \Rightarrow \quad \textit{Westerly phase}$$

$$\overline{u}(0\,\mathrm{N}, 30\,\mathrm{hPa}) < -5\,\mathrm{m\,s^{-1}} \quad \Rightarrow \quad \textit{Easterly phase}$$

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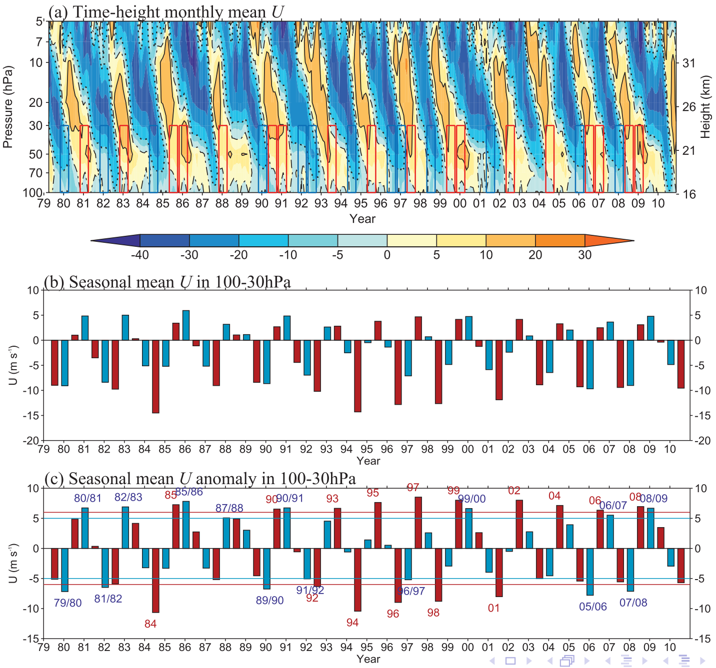
- From the linear theory and using the slow-variation WKBJ approximation (*Andrews et al.*, 1987), waves with zonal phase velocity c can propagate vertically only if the zonal winds satisfy

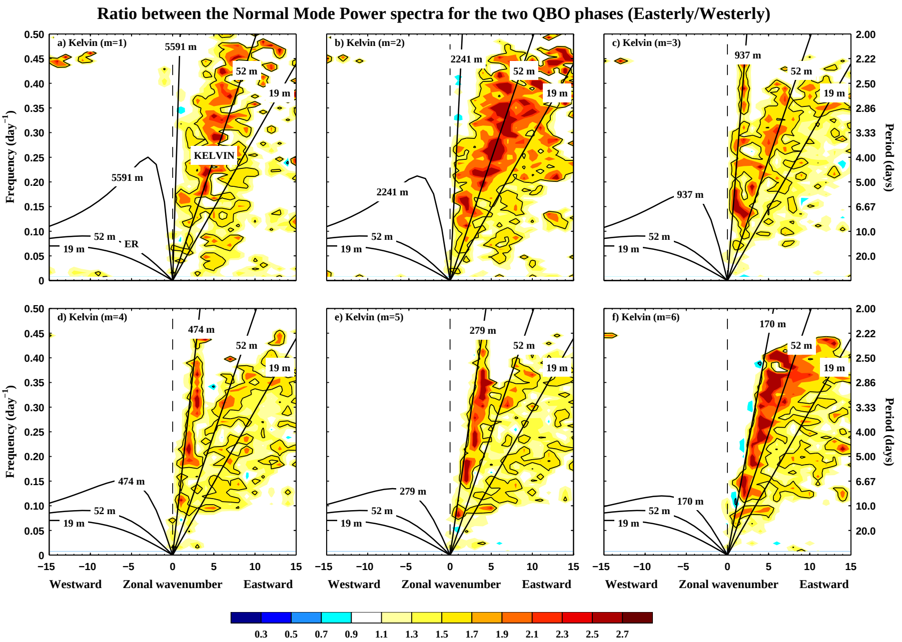
$$c - \overline{u}(z) > 0 \qquad \textit{for Kelvin waves}$$

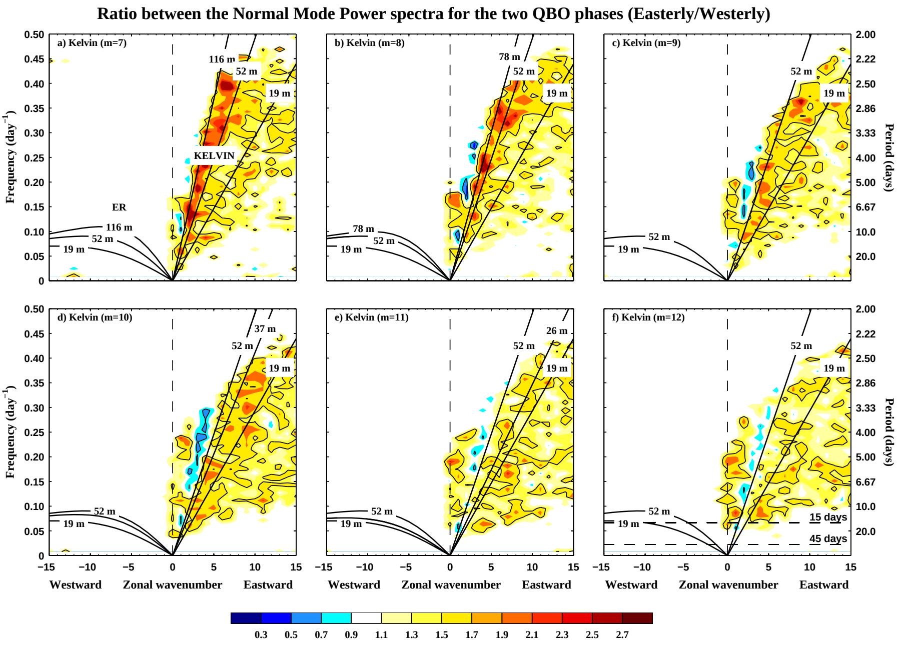
$$-\beta/k^2 < c - \overline{u}(z) < 0 \qquad \textit{for MRG waves}$$


(a) Time-height monthly mean U
(b) Seasonal mean U in 100-30hPa
(c) Seasonal mean U anomaly in 100-30hPa
Pressure (hPa)
Height (km)
U (m s⁻¹)
Year

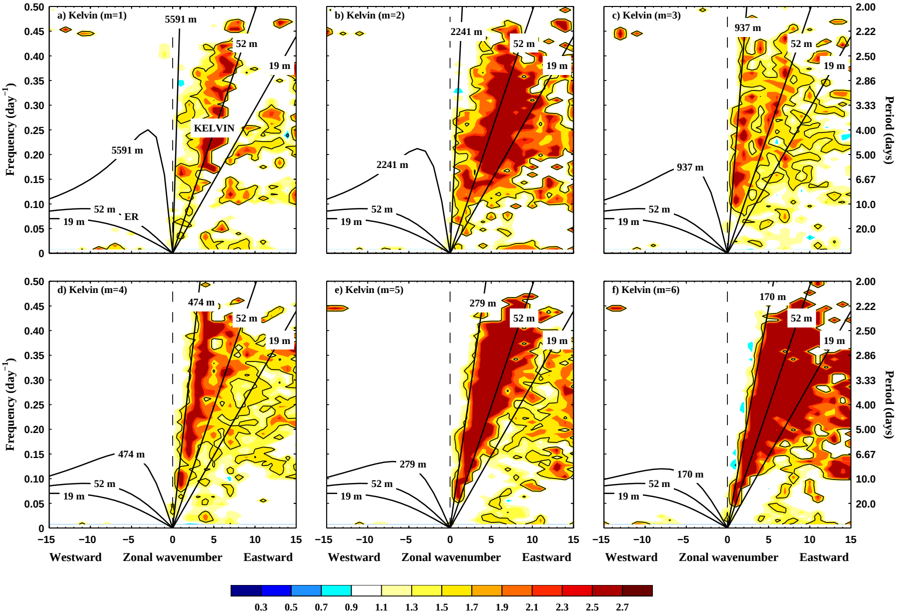
- The ratios between the power spectra calculated for each QBO phase are represented in the next Figures

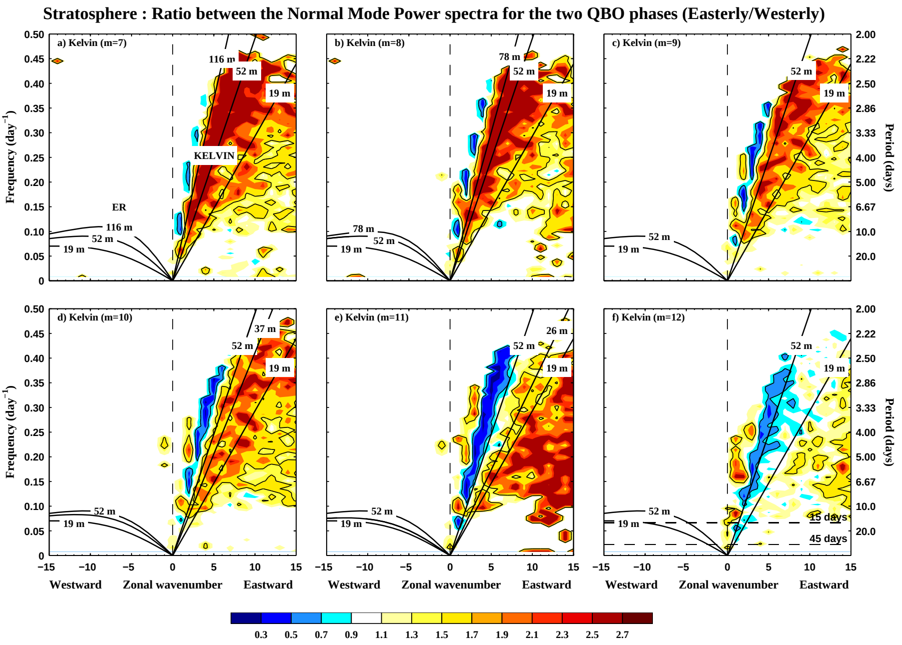
$$F = \frac{P^{E-QBO}}{P^{W-QBO}}$$


Ratio between the Normal Mode Power spectra for the two QBO phases (Easterly/Westerly)
a) Kelvin (m=1)
b) Kelvin (m=2)
c) Kelvin (m=3)
d) Kelvin (m=4)
e) Kelvin (m=5)
f) Kelvin (m=6)
5591 m
2241 m
937 m
474 m
279 m
170 m
52 m
19 m
KELVIN
ER
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
0.3 0.5 0.7 0.9 1.1 1.3 1.5 1.7 1.9 2.1 2.3 2.5 2.7


Ratio between the Normal Mode Power spectra for the two QBO phases (Easterly/Westerly)
a) Kelvin (m=7)
b) Kelvin (m=8)
c) Kelvin (m=9)
d) Kelvin (m=10)
e) Kelvin (m=11)
f) Kelvin (m=12)
KELVIN
ER
116 m
78 m
52 m
37 m
26 m
19 m
15 days
45 days
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
0.3 0.5 0.7 0.9 1.1 1.3 1.5 1.7 1.9 2.1 2.3 2.5 2.7

Stratosphere : Ratio between the Normal Mode Power spectra for the two QBO phases (Easterly/Westerly)





Stratosphere : Ratio between the Normal Mode Power spectra for the two QBO phases (Easterly/Westerly)
a) Kelvin (m=7)
b) Kelvin (m=8)
c) Kelvin (m=9)
d) Kelvin (m=10)
e) Kelvin (m=11)
f) Kelvin (m=12)
KELVIN
ER
116 m
78 m
52 m
37 m
26 m
19 m
15 days
45 days
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
2.5
2.7

- The vertical wavenumber of Kelvin waves is given by

$$m(z) = \mp \frac{N}{c - \overline{u}}$$

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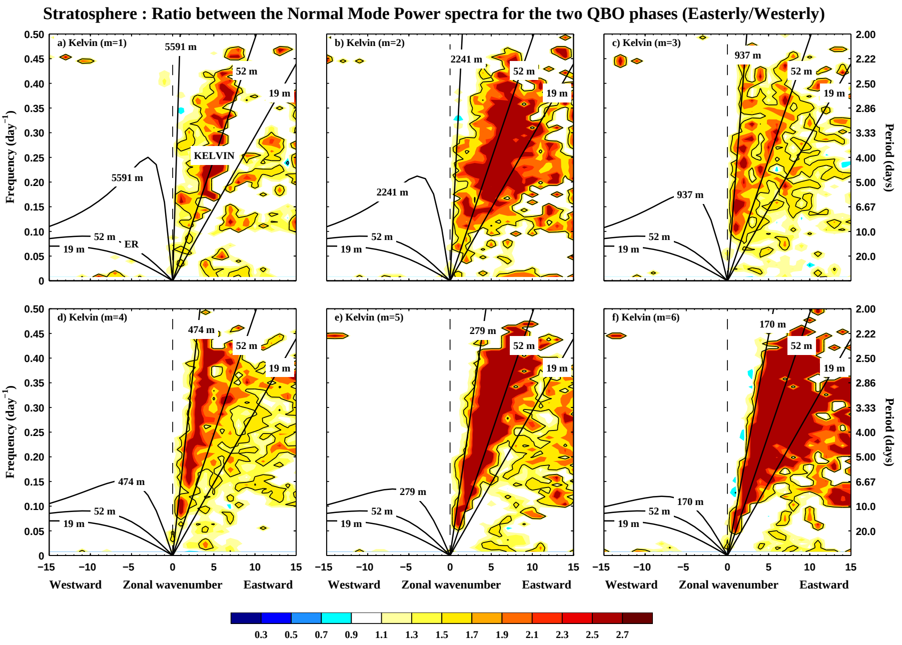
- Therefore the westerly QBO phase is associated with smaller vertical wavelengths.


Stratosphere : Ratio between the Normal Mode Power spectra for the two QBO phases (Easterly/Westerly)
a) Kelvin (m=1)
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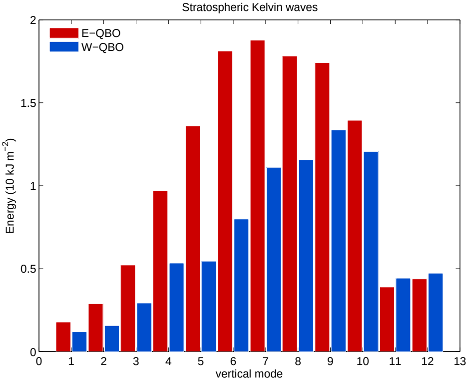
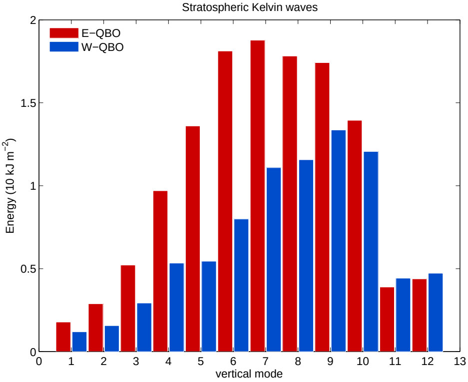
e) Kelvin (m=5)
f) Kelvin (m=6)
5591 m
2241 m
937 m
474 m
279 m
170 m
52 m
19 m
KELVIN
ER
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
2.5
2.7

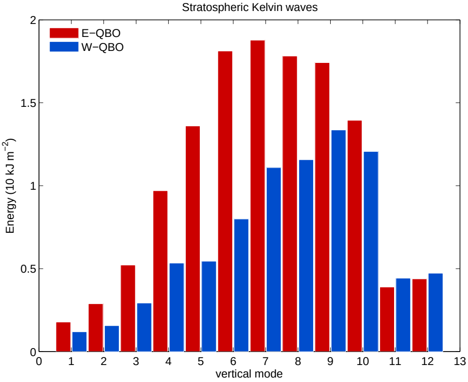


Stratospheric Kelvin waves
E−QBO
W−QBO
Energy (10 kJ m^{-2})
vertical mode
0
0.5
1
1.5
2
0 1 2 3 4 5 6 7 8 9 10 11 12 13



- Using typical values for the variables in the theoretical relationship

$$h_e = \frac{N^2}{g\left(m^2 + \frac{1}{4H^2}\right)}$$



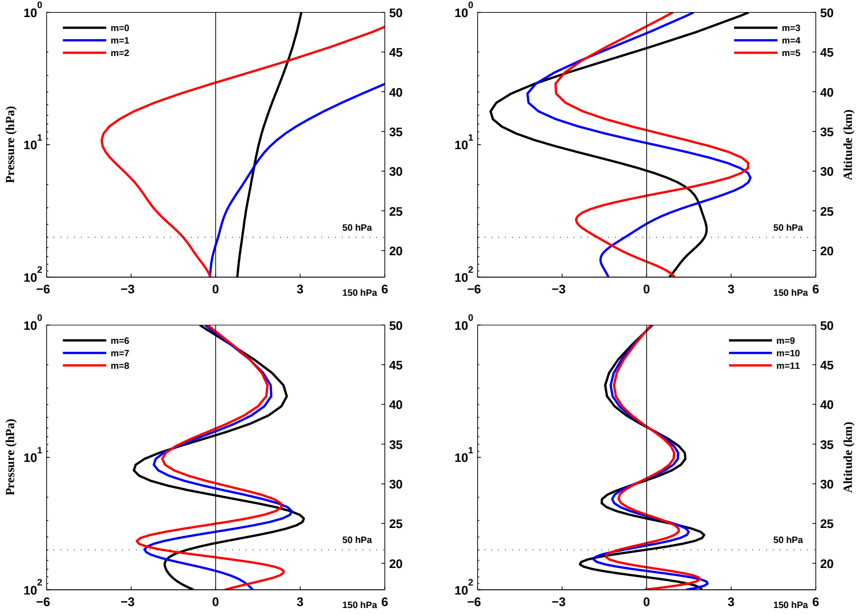
- Using typical values for the variables in the theoretical relationship

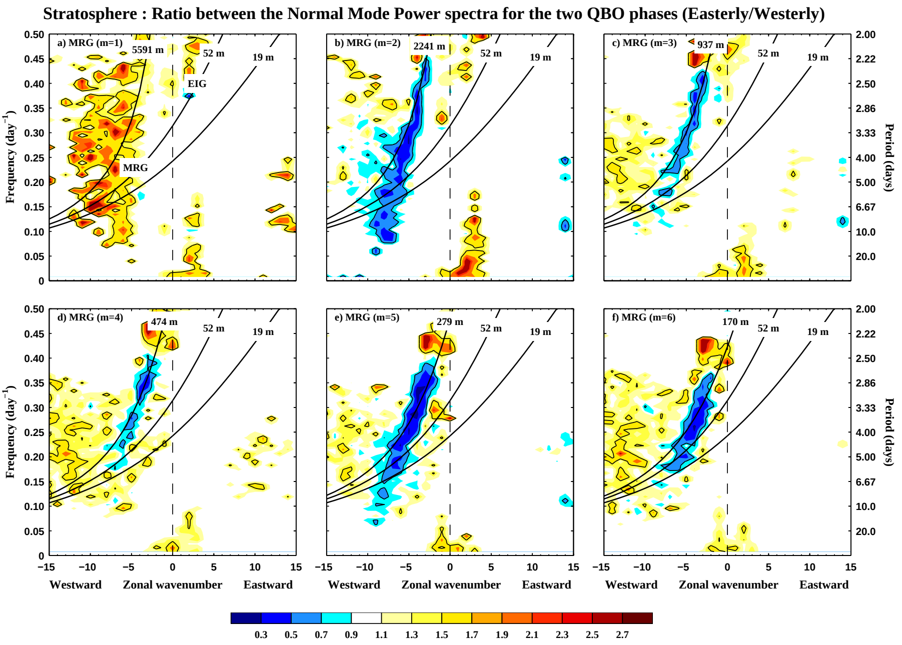
$$h_e = \frac{N^2}{g\left(m^2 + \frac{1}{4H^2}\right)}$$

- one obtains

$$h_7 = 116\,\mathrm{m} \quad \Rightarrow \quad L_z = 9.7\,\mathrm{km}$$
$$h_9 = 52\,\mathrm{m} \quad \Rightarrow \quad L_z = 6.5\,\mathrm{km}$$

Vertical Structure Functions





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19 m
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MRG
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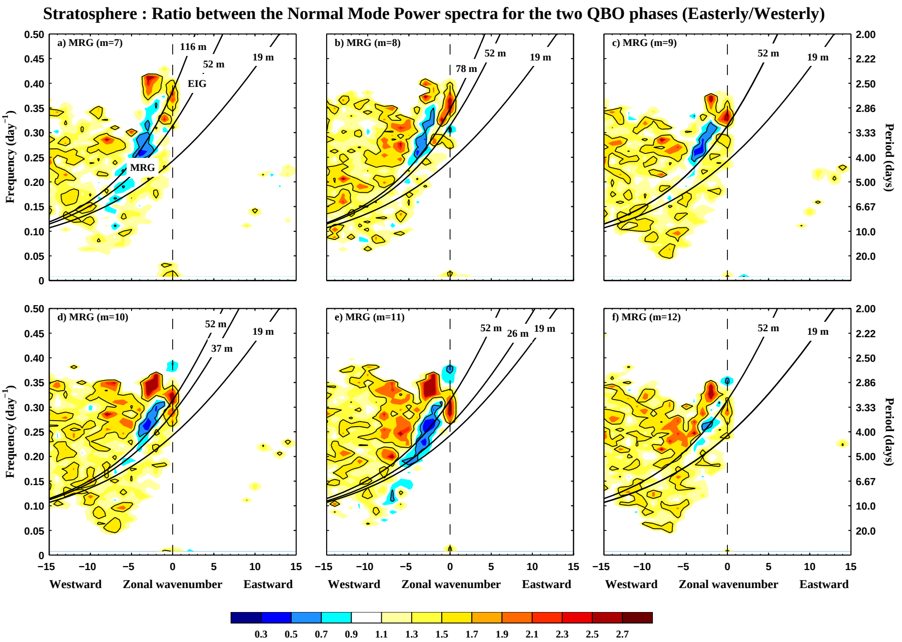
279 m
f) MRG (m=6)
170 m
Frequency (day^{-1})
Period (days)
Westward
Zonal wavenumber
Eastward
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
2.5
2.7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Period (days)
0.50
0.45
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0.30
0.25
0.20
0.15
0.10
0.05
0
2.00
2.22
2.50
2.86
3.33
4.00
5.00
6.67
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20.0
−15
−10
−5
0
5
10
15
Westward
Zonal wavenumber
Eastward
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
2.5
2.7

Conclusions

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Thank you!