Generalizing the Concepts of Normal Mode Theory for Diagnosis of Climate Variability.

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• Some of my past publications at NCAR relevant to talk:

- ***A. Modes for initialization***: Dickinson, R.E. and D.L. Williamson, 1972: Free oscillations of a discrete stratified fluid with Application to Numerical Weather Prediction, *J. of the Atmospheric Science*, **29**, 623-640
- Williamson, D.L. and R.E. Dickinson, 1972: A note on periodic updating of meteorological variables, *J. Atmos. Sci*., **29**, 190-193

- ***B Modeling Hadley Cells Analytically balancing radiative and frictional damping:*** Dickinson, R.E., 1971: Analytical model for zonal winds in the tropics. I. Details of the model and simulation of gross features of the zonal mean troposphere, *Mon. Wea. Rev*., **99**, 501–510
- Dickinson, R.E., 1971: Analytical model for zonal winds in the tropics. II. Variation of the tropospheric mean structure with season and differences between hemispheres, *Mon. Wea. Rev*., **99**, 511-523

- **C. Advective controls can dominate** : Rossby waves leave tropics with strong westerly winds. Dickinson, R.E., 1971: Cross-equatorial eddy momentum fluxes as evidence of tropical planetary wave sources, *Quart. J. Roy. Meteor. Soc*., **97**, 554-558

- ***D. Analytic Solution for thermosphere and application to Venus***
- Dickinson, R.E., 1969: The steady circulation of a nonrotating, viscous, heat conducting atmosphere, *J. Atmos. Sci*., **26**, 1199-1215
- Dickinson, R.E., 1971: Circulation and thermal structure of the Venusian thermosphere, *J. Atmos. Sci*., **28**, 889-894

- Many later papers from **D** with Ridley and Roble leading to the TGCM
- Same general idea of balancing dissipation and radiation damping as **B** in later papers of Gill 1980 QJRMS and many subsequent papers for tropical circulation but focussed on longitudinal structure

Quick summary of classical MODES theory

(6 basic equations, simplify with f plane, linearize, D = horizontal divergence, ζ = vorticity, vert. coord.: $z = -\log(p/p_0)$, ϕ = geopot., buoyancy $b = g\rho'/\rho_0$, $N^2 = R(\Gamma+\partial T/\partial z)/H^2$, N= buoyancy frequency, H = scale height.

- $b = g\, T'/T$ (equation of state)
- $RT' = \partial\phi/\partial z$ (hydrostatic equation)
- $\partial w/\partial z - w + D = 0.$ (continuity equation)
- $\partial\zeta/\partial t = f\, \partial w/\partial z$ (vorticity equation)
- $\partial D/\partial t + f\, \zeta = \Delta\phi$ (divergence equation)
- $C_p\, \partial T'/\partial t + H^2 N^2 w = 0$ (thermodyn. equation)

Classical MODES

- .Use 19th-20th century math. Can boil down to a single PDE (should be on sphere but f plane easier to show):
- $$\partial/\partial t\ [(f^2+\partial^2/\partial t^2)(\partial^2/\partial z^2-\partial/\partial z) + N^2\Delta]w = 0$$
- On relatively long time scales the double time derivative drops out and you have a conserved potential vorticity (Rossby mode).
- MODES follow from assuming oscillatory solutions : $\partial/\partial t \;\Rightarrow\; i\nu$

- Separate into horizontal and vertical structure equations using equivalent ocean depth D.
- Horizontal becomes shallow water equation.
- Boundary conditions needed to proceed.
- Get discrete eigenfunctions for horizontal structure depending on geometry.. Hough modes on sphere.
- Can find solutions only for certain values of v depending on D , i.e. the spectrum of the equation.
- For vertical structure unbounded atmosphere, a discrete external mode mode in the vertical and a discrete and continuous spectum of internal modes defining what D is. For numerical model all discrete modes depending on top boundary condition. Can become complex with dissipation or radiation top.

- Idea of MODES filter is to project initial conditions on the modes and remove those believed to be largely noise.

Time scales ≥ 10 days, need to include frictional and radiative damping terms

- For simplicity to look at climate modes can assume both damping terms are approximated by a linear plus diffusive term, i.e., $[a - K\,\Delta]$; away from boundary layer, friction more diffusive and radiation more linear so simplest inclusion is to add to $\partial T'/\partial t$ the term $a\,T'$, “Newtonian cooling”, and to $\partial \zeta/\partial t$ the term $-K\Delta\zeta$;
- For time scales >> 10 days (climate scales) only the damping terms remain.

Key point:

atmospheric dynamical systems on climate time scales may be largely controlled by balances between frictional and radiative damping terms

- Additional factors to consider:
  - Eddy fluxes
  - Coupling to boundary layer.
  - Strong advective terms: $U/L \approx a \approx 1/10$ day

Historic example

- Simplest thermosphere model. Assume Nonrotating (more directly led to more advanced models of Venus thermosphere)
- Expand horizontal structure in spherical harmonics
- Time derivatives replaced by : $-k\,(\partial^2/\partial^2 z - \partial/\partial z)$
- Reduces to a 6^{th} order hypergeometric equation: i.e.,

$$[\exp(z)(\partial^2/\partial^2 z)\exp(z)\,(\partial^4/\partial^4 z) - r]\,h = f$$

- Like Bessel functions: solve by power series and tie together with contour integral representations of solutions and asymptotic expansions.
- Also can solve numerically. (suggested and done by Cicely Ridley)

Boundary layer circulations.

- Pressure forcing prescribed or calculated from temperature horizontal variability.
- Vorticity tendency balanced by vortex stretching, frictional loss of vorticity, and northward advection of planetary vorticity (βv)
- Nigam – Lindzen used for tropical atmospheric structure over ocean as alternative to Gill
- We use to explain nightime peak in precipitation in Eastern Great Plains and interannual variability over Gulf states (Pu and Dickinson, 2014; Pu et al, 2016)

Forcing of Winds by BLB Gradients

Top of Boundary Layer

- Low pressure

High pressure

Warm air

Cold air

Summer Continent

Summer Ocean

Much can be learned by examining a boundary layer slab model, i.e.,

$$\frac{\partial u}{\partial t} = fv - g\frac{\partial h}{\partial x} - \varepsilon u - \phi_x$$

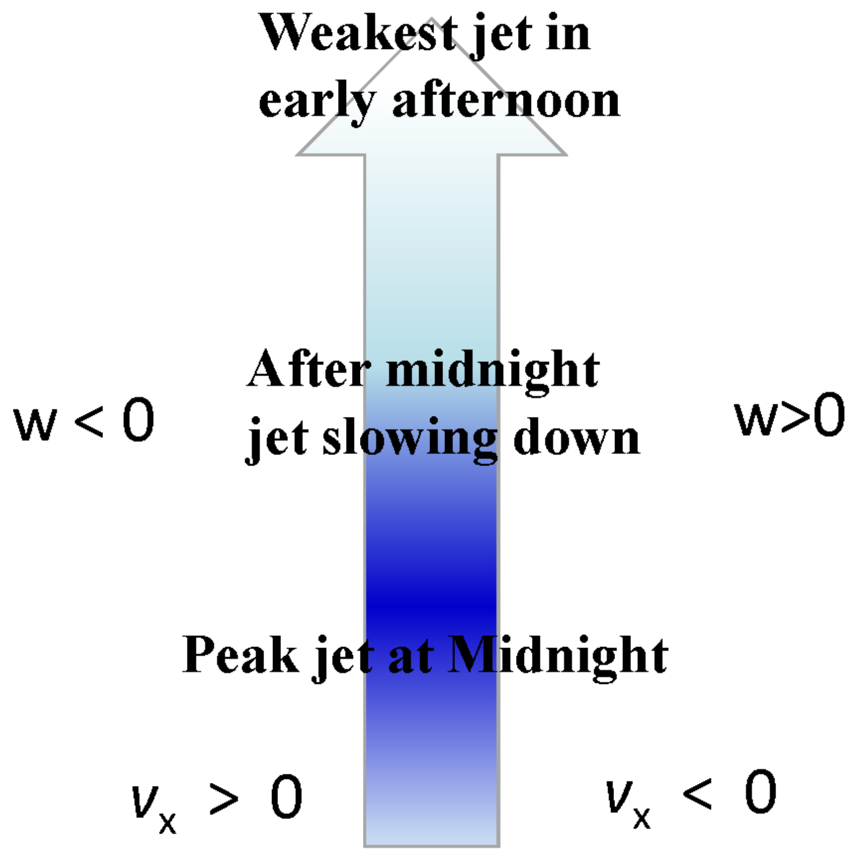
$$\frac{\partial v}{\partial t} = -fu - g\frac{\partial h}{\partial y} - \varepsilon v - \phi_y$$

$$h = -\tau H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

Lindzen and Nigam (1987)

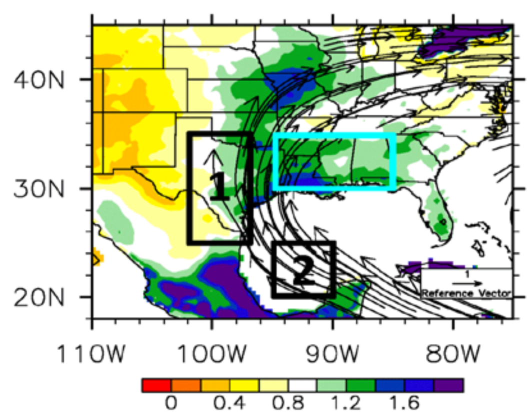
Great Plains Low Level Jet

(from Pu and Dickinson 2014)



Gulf states have the strongest summer interannual variation of precipitation in the US. Why?

- Simplest explanation is that it is from interannual variation of winds (e.g. from NASH). What this wind variability does can be interpreted in terms vorticity equation (Pu and Dickinson, in review.).
- Relevant wind can be represented by averages on northward wind over two boxes.



Toy model for dynamics of summer Great Plains drought

- Assume negligible mean advection and that transient disturbances act as an eddy friction.
- Circulation is forced by the negative diabatic heating occuring with drought
  - negative anomaly in latent heating
  - positive anomaly in longwave radiative cooling (lack of clouds, reduced humidity)
  - role of anomaly in surface solar heating?

- The balance between dissipation and generation of vorticity can be reduced to:

$$K\,\varphi/R^4 = f^2\,\partial w/\partial z, \qquad (1)$$

where K is a horizontal diffusion coeficient, and a double Laplacian has been reduced to the $1/R^4$ by assuming horizontal struction an eigenfunction of Laplacian.

For forcing by cooling Eq. (1) represents dissipation of anticyclonic vorticity by mixing balanced by a downward reduction of sinking motion (vortex compression).

- Thermodynamic equation, consists of Newtonian cooling of the temperature anomaly and adiabatic cooling from vertical motion balancing diabatic cooling anomaly. Temperature replaced by geopotential from hydrostatics:

$$a\, \partial \varphi / \partial z + (HN)^2\, w = \kappa\, Q(z)\ . \qquad (2)$$

where $Q(z)$ is the diabatic heating per unit mass

What happens?

- Get single equation in w. Solve for Greens function. Negative heating forces downward motion whose adiabatic warming reduces the cold temperature anomaly from what it would be with only balancing from Newtonian cooling.
- The cold temperature anomaly creates high in geopotential (pressure) balanced geostrophically by anticyclonic circulation whose loss of negative vorticity by friction is balanced by generation from vortex compression (downward vertical motion decreasing as approach surface)

What else?

- Can add a boundary layer and surface heating
- Minor effects except surface heating reduces high pressure near surface or in extreme conditions, changes to a heat low (e.g. Sahara)
- Various radiative feedbacks implied, working through how to quantify.