The Tangent Linear Normal Mode Constraint in GSI:

Applications in the NCEP GFS/GDAS Hybrid EnVar system and Future Developments

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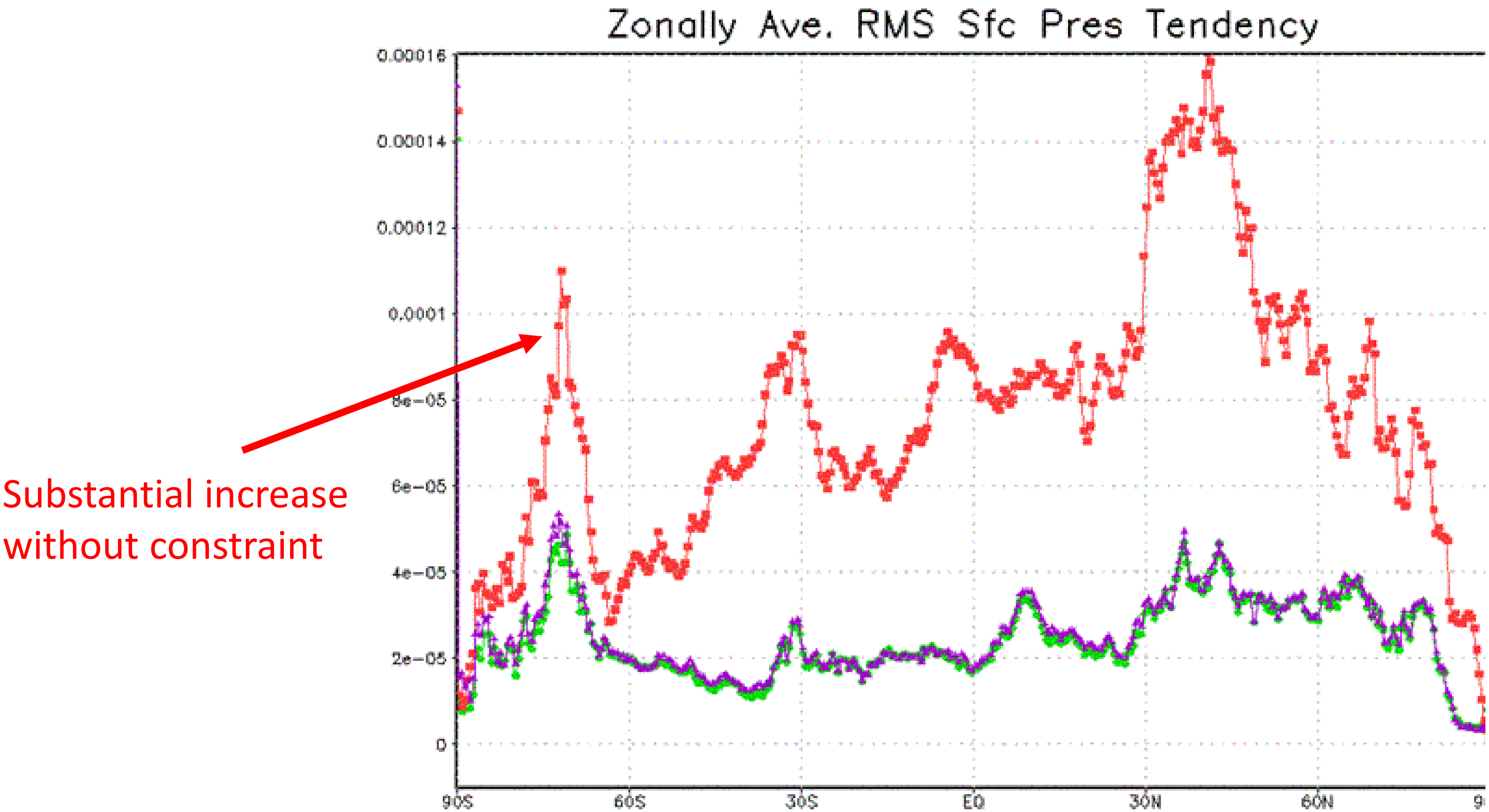
Some History

- GSI was effort started in 2000s
  - Merge regional (Eta) and global (SSI) DA systems, collaborate with JCSDA (GMAO)
  - Grid-space, modified B, no balancing
- However, initial results with GFS were discouraging
  - In order to make GSI operational for GFS/GDAS, needed to pursue improved B and/or balance operators
- Initial attempts were weak constraint formulations
  - First, based on incremental tendencies
  - Then, normal modes, eventually yielding the TLNMC….
- GSI finally implemented in 2007

Increase in Ps tendency found in GSI/3DVAR analyses



Zonal-average surface pressure tendency for background (green), unconstrained GSI analysis (red), and GSI analysis with TLNMC (purple).

Potential Corrections for Noise / Imbalance

- Noise in the background (first guess/model forecast)
  - ***Digital filters***
  - Initialization (Nonlinear Normal Mode Initialization)
    - Analysis draws to data, initialization pushes away from observations
- Noise in the analysis increment
  - Improved multivariate variable definition
  - Dynamic weak constraint
    - This was our first attempt but:
      - Poor convergence / ill-conditioned
      - Scale selectivity was an issue
      - Significant degradation in analysis fits to the data, similar to full field initialization
  - ***Incremental normal mode initialization***

Tangent Linear Normal Mode Constraint
Kleist et al. (2009)

$$J(\mathbf{x}_c') = \frac{1}{2}(\mathbf{x}_c')^T \mathbf{C}^{-T}\mathbf{B}^{-1}\mathbf{C}^{-1}(\mathbf{x}_c') + \frac{1}{2}(\mathbf{y}_o' - \mathbf{H}\mathbf{x}_c')^T \mathbf{R}^{-1}(\mathbf{y}_o' - \mathbf{H}\mathbf{x}_c') + J_c$$

$$\mathbf{x}_c' = \mathbf{C}\mathbf{x}'$$

- analysis state vector after incremental NMI
  - **C** = Correction from incremental normal mode initialization (NMI)
    - represents correction to analysis increment that filters out the unwanted projection onto fast modes
- No change necessary for **B** in this formulation
- Based on:
  - Temperton, C., 1989: “Implicit Normal Mode Initialization for Spectral Models”, MWR, vol 117, 436-451.

* Similar idea developed and pursued independently by Fillion et al. (2007)

"Strong Constraint" Procedure

$$\mathbf{C}=[\mathbf{I}-\mathbf{DFT}]\mathbf{x}'$$

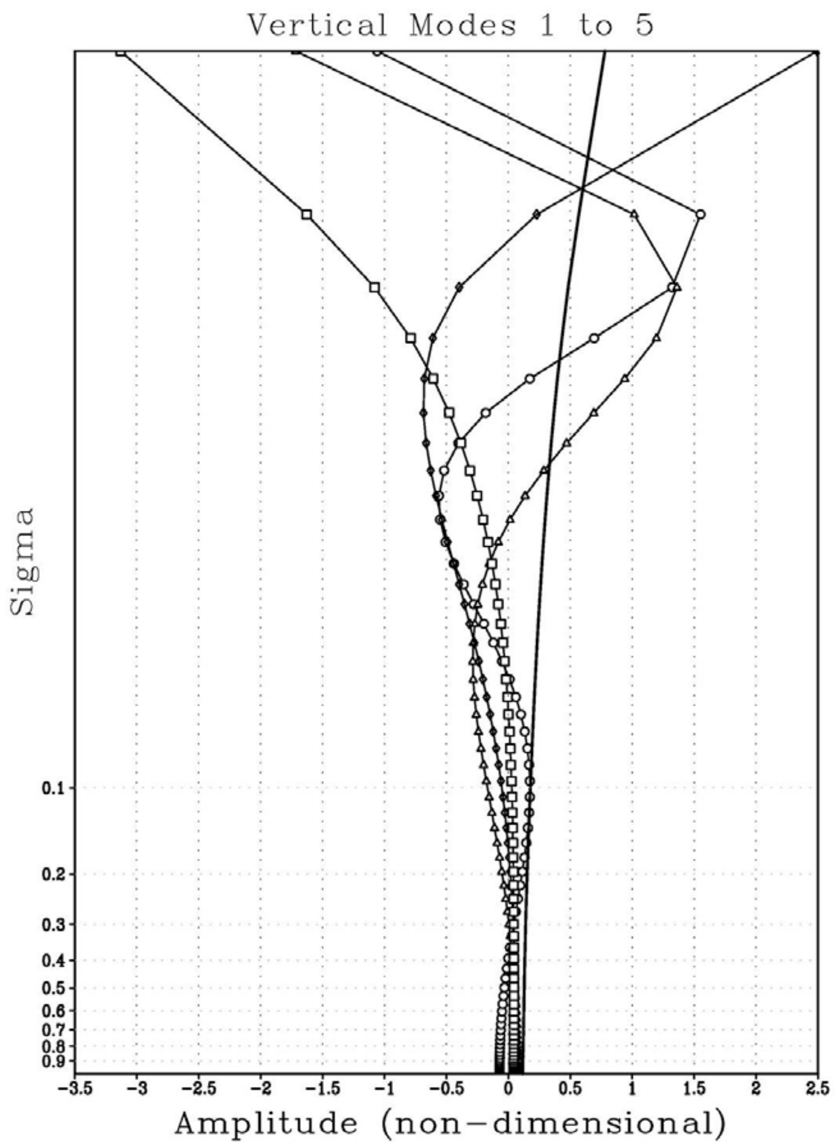
T →	F →	D
n x n	m x n	n x m
$\frac{d\mathbf{x}'}{dt}$	$\frac{d\mathbf{x}'_g}{dt}$	$-\Delta\mathbf{x}'$
Dry, adiabatic tendency model	**Projection onto m modes** **m-2d shallow water problems**	**Correction matrix to reduce gravity mode tendencies** **Spherical harmonics used for period cutoff**

- Practical Considerations:
  - **C** is operating on **x'** only, and is the tangent linear of NNMI operator
  - Only need one iteration in practice for good results
  - Adjoint of each procedure needed as part of minimization/variational procedure

Tangent Linear Normal Mode Constraint

- Performs correction to *increment* to reduce gravity mode tendencies
- Applied during minimization to *increment*, not as post-processing of analysis fields
- Little impact on speed of minimization algorithm
- $\mathbf{CBC}^T$ becomes effective background error covariances for balanced increment
  - Not necessary to change variable definition/**B** (unless desired)
  - Adds implicit flow dependence
- Requires time tendencies of increment
  - **Implemented dry, adiabatic, generalized coordinate tendency model (TL and AD)**

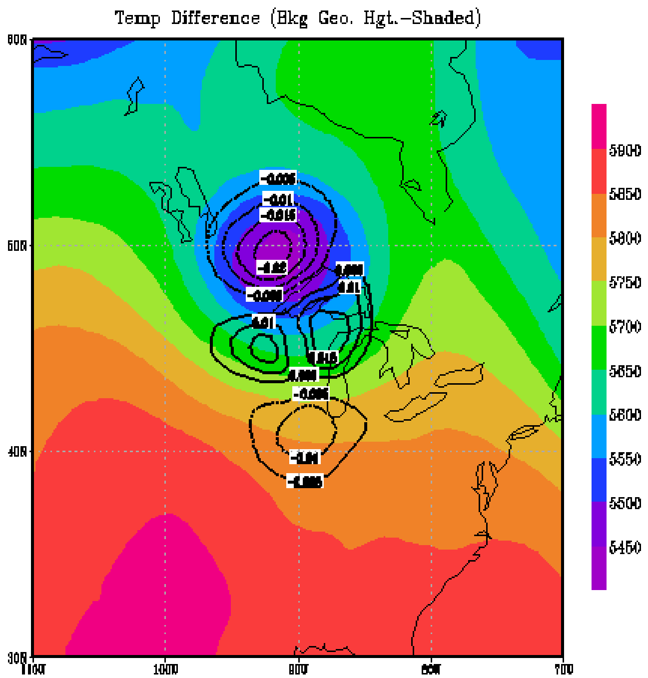
Vertical Modes

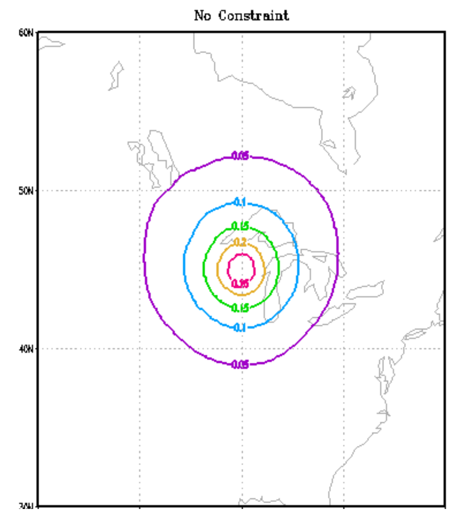


- Global mean temperature and pressure for each level used as reference
- ***First 8 vertical modes are used*** in deriving incremental correction in global implementation

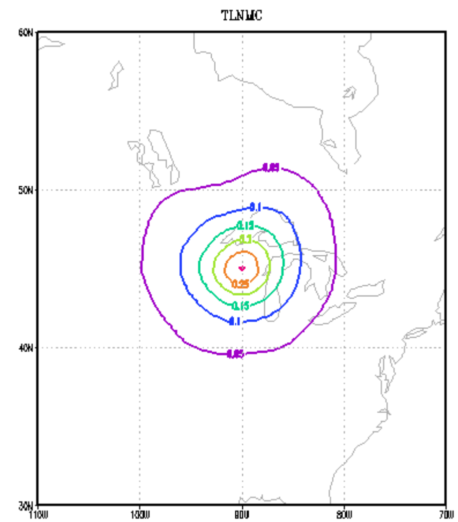
Single observation test (T observation)

- Magnitude of TLNMC correction is small
- TLNMC adds flow dependence even when using same isotropic **B**





Isotropic response



Flow dependence added

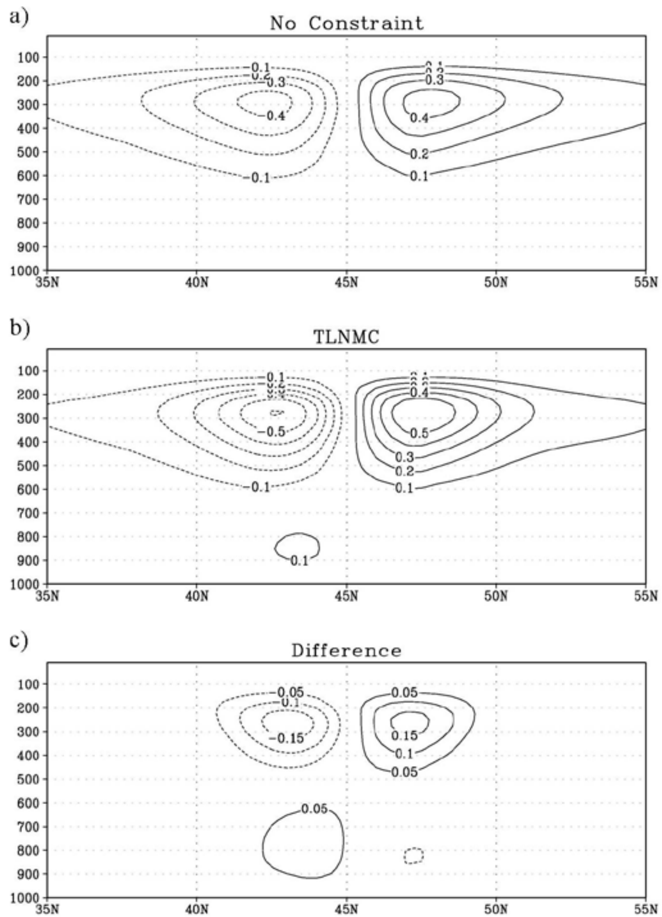
500 hPa temperature increment (right) and analysis difference (left, along with background geopotential height) valid at 12Z 09 October 2007 for a single 500 hPa temperature observation (1K O-F and observation error)

Single observation test (T observation)

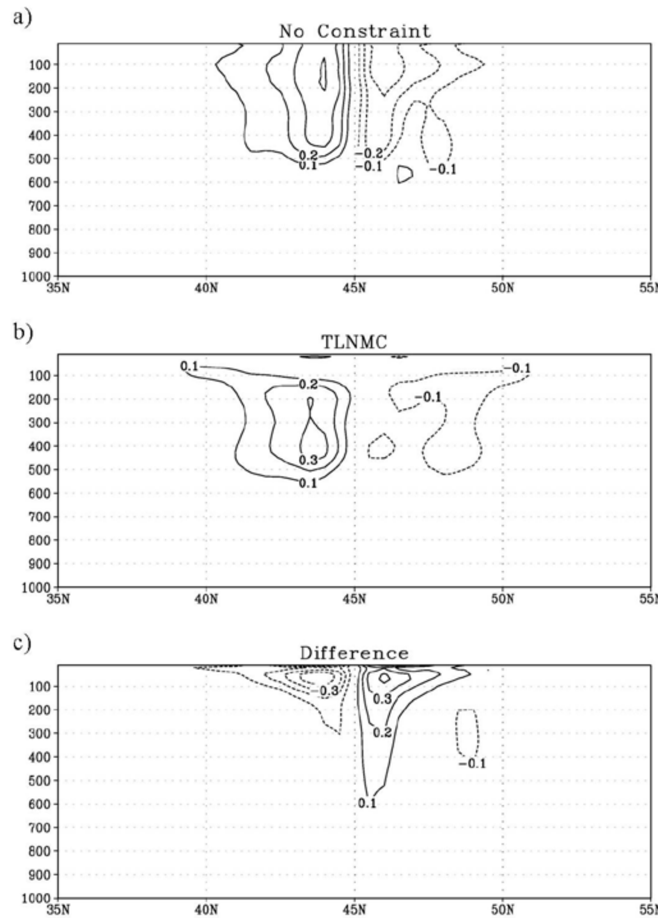
U wind

From multivariate B

TLNMC corrects



Ageostrophic U wind

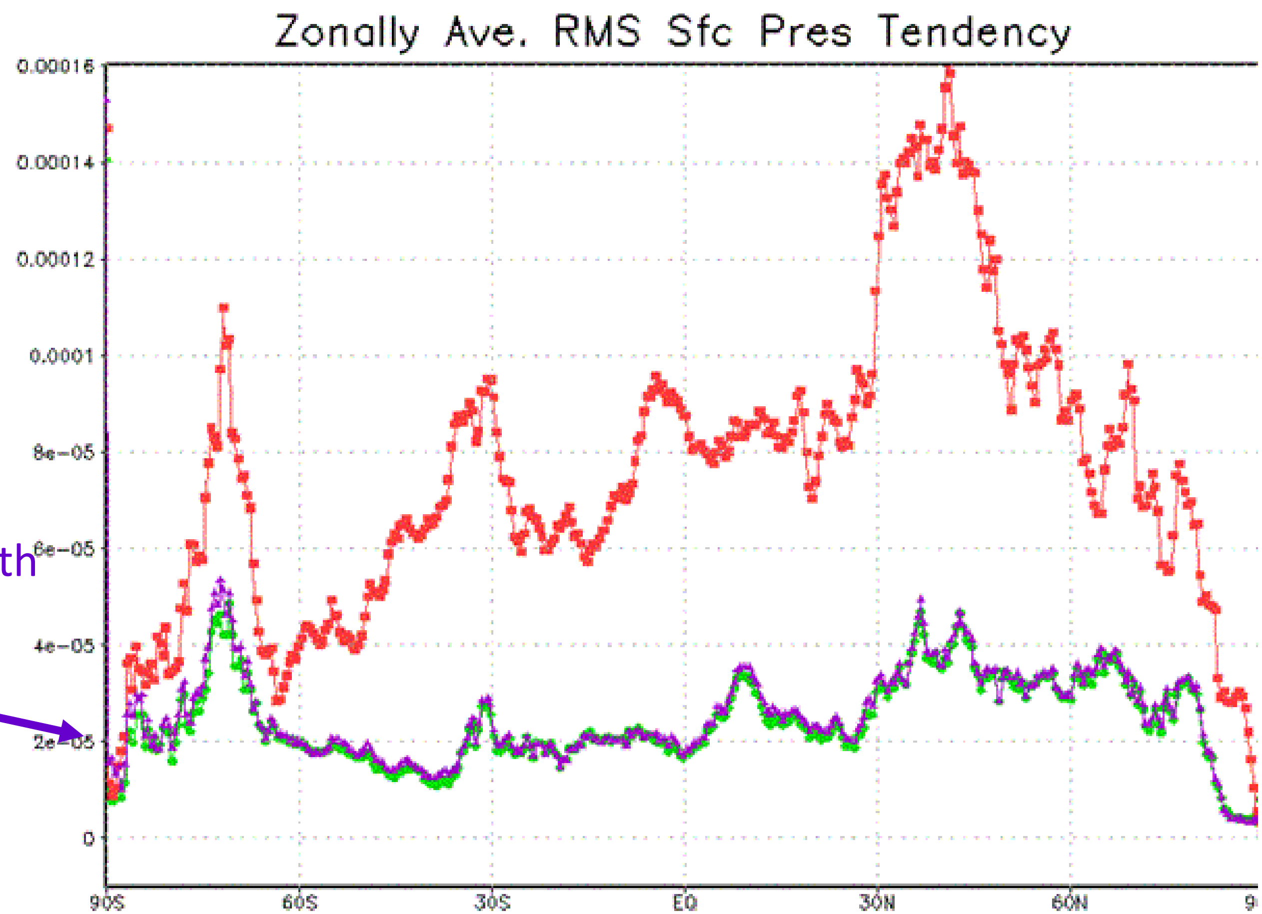


Smaller ageostrophic component

Cross section of zonal wind increment (and analysis difference) valid at 12Z 09 October 2007 for a single 500 hPa ***temperature*** observation (1K O-F and observation error)

Surface Pressure Tendency Revisited

Minimal increase with TLNMC



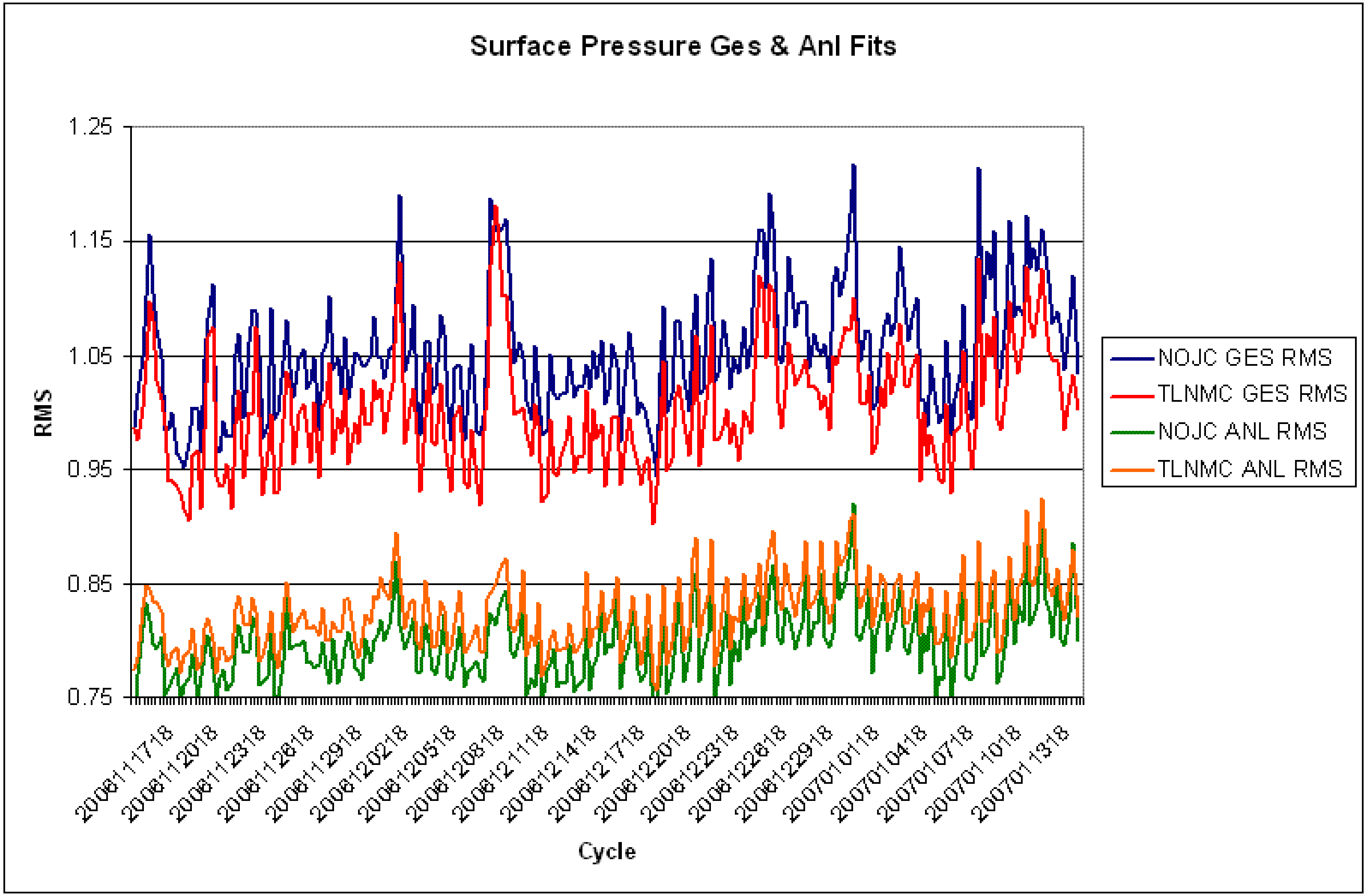
Zonal-average surface pressure tendency for background (green), unconstrained GSI analysis (red), and GSI analysis with TLNMC (purple)

"Balance"/Noise Diagnostic

- Compute RMS sum of incremental tendencies in spectral space (for vertical modes kept in TLNMC) for final analysis increment
  - Unfiltered: S_{uf} (all) and S_{uf_g} (projected onto gravity modes)
  - Filtered: S_f (all) and S_{f_g} (projected onto gravity modes)
  - Normalized Ratio:
    - $R_f = S_{f\_g} / (S_f - S_{f\_g})$
    - $R_{uf} = S_{uf\_g} / (S_{uf} - S_{uf\_g})$

	S_{uf}	S_{uf_g}	R_{uf}	S_f	S_{f_g}	R_f
NoJC	**1.45×10^{-7}**	**1.34×10^{-7}**	**12.03**	1.41×10^{-7}	1.31×10^{-7}	**12.96**
TLNMC	**2.04×10^{-8}**	**6.02×10^{-9}**	**0.419**	1.70×10^{-8}	3.85×10^{-9}	**0.291**

Fits of Surface Pressure Data in Parallel Tests GSI 3DVAR with TLNMC implement 1 May 2007



Hybrid EnVar

Lorenc (2003), Buehner (2005), Wang et al.(2007)

$$J_{Hyb}(\mathbf{x}_c,\alpha) = \beta_c \frac{1}{2}(\mathbf{x}_f^{'})^T \mathbf{B}^f (\mathbf{x}_f^{'}) + \beta_e \frac{1}{2}(\alpha)^T \mathbf{L}^{-1}(\alpha) + \frac{1}{2}(\mathbf{y}_o^{'} - \mathbf{H}\mathbf{x}_t^{'})^T \mathbf{R}^{-1}(\mathbf{y}_o^{'} - \mathbf{H}\mathbf{x}_t^{'})$$

$$\mathbf{x}'_t = \mathbf{x}'_f + \sum_{m=1}^{M}(\alpha^m \circ \mathbf{x}_e^m)$$

β_f & β_e: weighting coefficients for clim. (var) and ensemble covariance respectively

$\mathbf{x}_t'$: (total increment) sum of increment from fixed/static **B** ($\mathbf{x}_f'$) and ensemble **B**

$\boldsymbol{a}_k$: extended control variable; $\mathbf{x}_e^m$:ensemble perturbations

- analogous to the weights in the LETKF formulation

$$(\mathbf{w}_{k,m} = (\mathbf{Y}^b_{k,m})^T[\mathbf{Y}^b_{k,m}(\mathbf{Y}^b_{k,m})^T + \mathbf{R}]^{-1}\mathbf{d}_k)$$

L: correlation matrix [effectively the localization of ensemble perturbations]

TLNMC in Hybrid Context

- Apply to static contribution only
  - Non-filtering of ensemble contribution

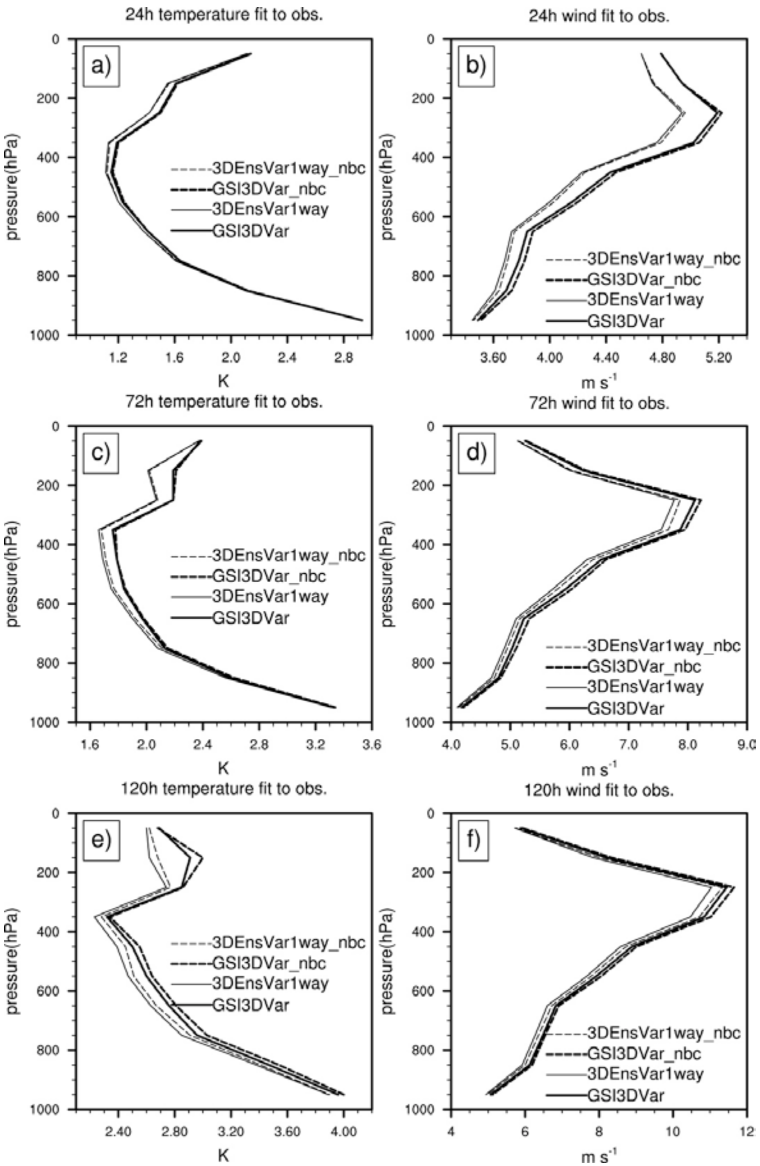
$$\mathbf{x}'_{\mathrm{t}} = \mathbf{C}\mathbf{x}'_{\mathrm{f}} + \sum_{m=1}^{M} (\alpha^{m} \circ \mathbf{x}_{\mathrm{e}}^{m})$$

- **Apply to total increment * (method of choice)**
  - Helps mitigate imbalance/noise associated with static B (as before) and ensemble contribution (localization, etc.)

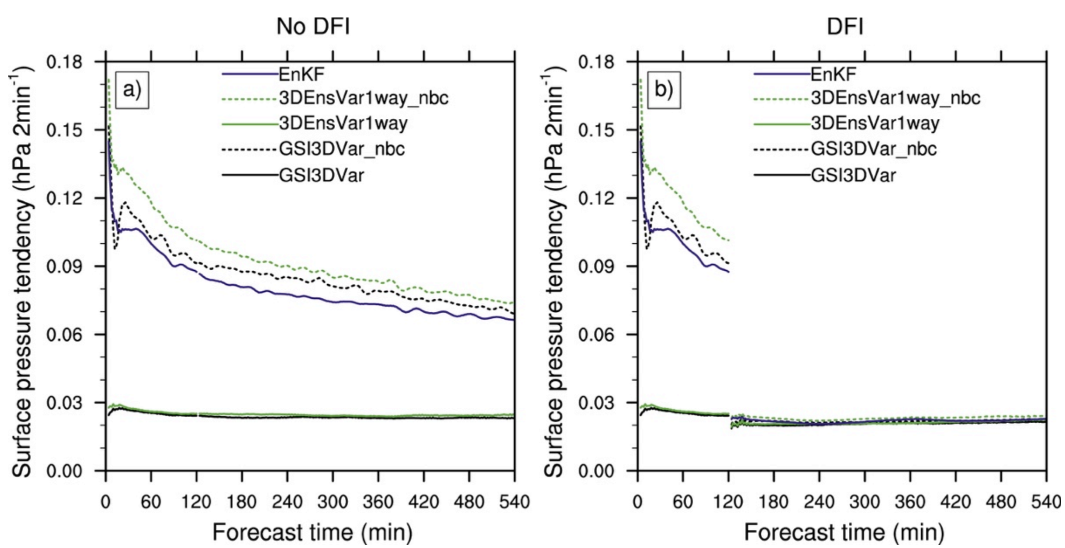
$$\mathbf{x}'_{\mathrm{t}} = \mathbf{C}[\mathbf{x}'_{\mathrm{f}} + \sum_{m=1}^{M} (\alpha^{m} \circ \mathbf{x}_{\mathrm{e}}^{m})]$$

Impact of TLNMC in 3D Hybrid

Wang et al. (2013)



Observation Fits (dashed are without TLNMC)

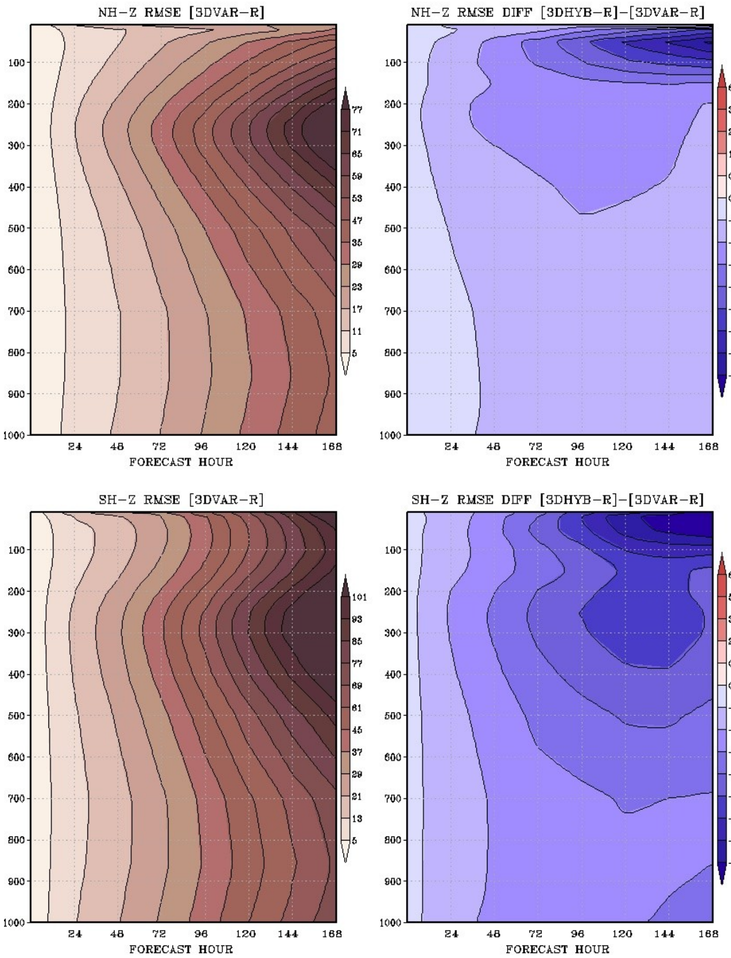


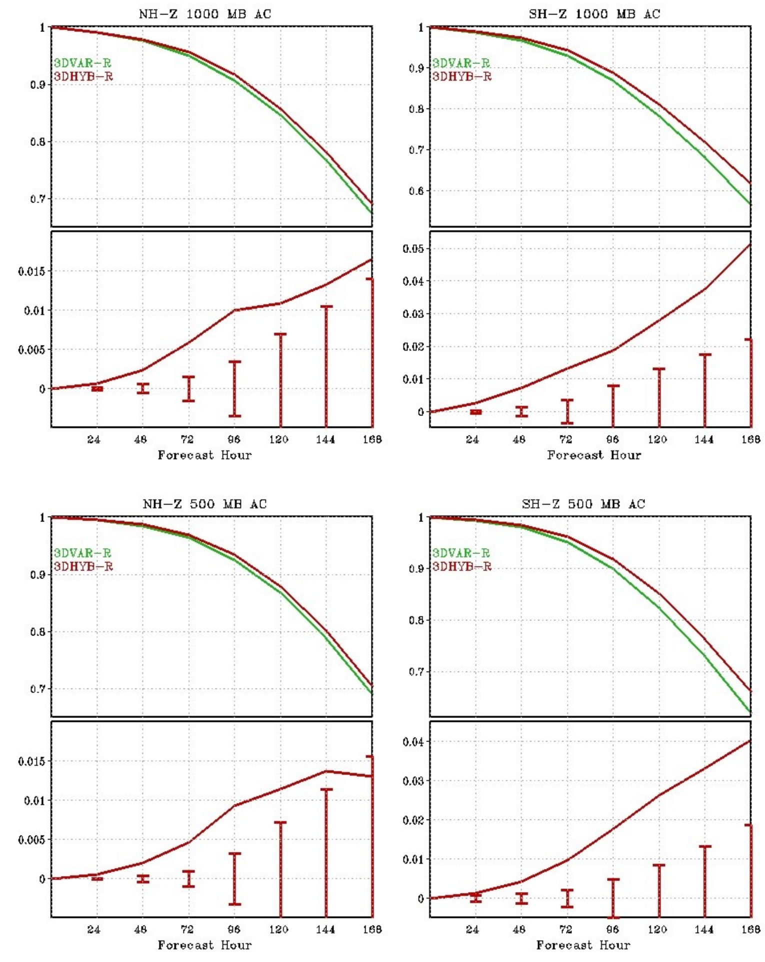
Surface Pressure Tendency

Hybrid 3D EnVar ***with TLNMC*** Implemented in May 2012

NH

SH





1000 mb

500 mb

4D Ensemble Var (Liu et al, 2008)
GSI - Hybrid 4D-EnVar

Wang and Lei (2014); Kleist and Ide (2015)

The Hybrid EnVar cost function can be easily extended to 4D and include a static contribution (ignore preconditioning)

$$J(\mathbf{x}'_{\mathrm{f}},\alpha)=\beta_{\mathrm{f}}\frac{1}{2}(\mathbf{x}')^{\mathrm{T}}\mathbf{B}_{\mathrm{f}}^{-1}(\mathbf{x}')+\beta_{\mathrm{e}}\frac{1}{2}\alpha^{\mathrm{T}}\mathbf{L}^{-1}\alpha+\frac{1}{2}\sum_{k=1}^{K}(\mathbf{y}'_k-\mathbf{H}_k\mathbf{x}'_k)^{\mathrm{T}}\mathbf{R}_k^{-1}(\mathbf{y}'_k-\mathbf{H}_k\mathbf{x}'_k)$$

Jo term divided into observation "bins" as in 4DVAR

Where the 4D increment is prescribed through linear combinations of the 4D ensemble perturbations plus static contribution, i.e. it is not itself a model trajectory

$$\mathbf{x}'_k=\mathbf{x}'_{\mathrm{c}}+\sum_{m=1}^{M}(\alpha^m\circ(\mathbf{x}_{\mathrm{e}})_k^m)$$

Here, static contribution is time invariant. No TL/AD in Jo term (**M** and $\mathbf{M}^{\mathrm{T}}$)

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Constraint Options in 4D EnVar

- Tangent Linear Normal Mode Constraint
  - Based on past experience and tests with 3D hybrid, default configuration includes TLNMC ***over all time levels*** (quite expensive)

$$\mathbf{x}'_k = \mathbf{C}_k \left[\mathbf{x}'_\mathrm{f} + \sum_{n=1}^{N} \left(\alpha^n \circ \left(\mathbf{x}_\mathrm{e} \right)_k^n \right) \right]$$

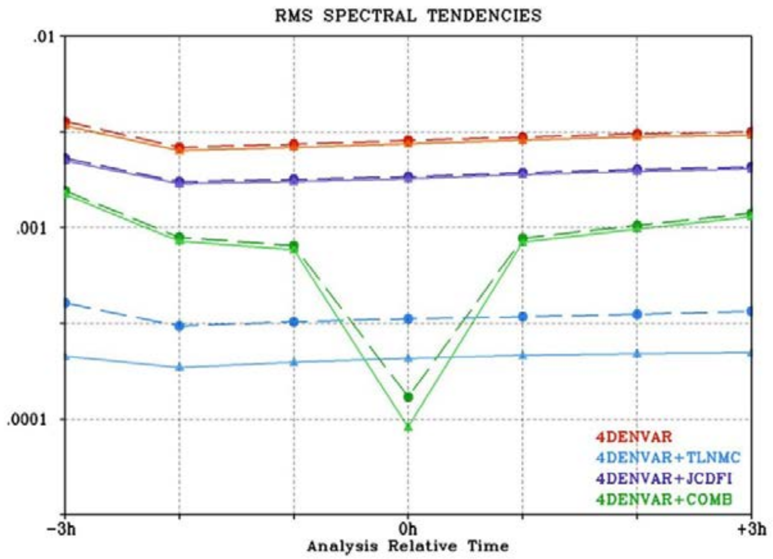
- Weak Constraint "Digital Filter"
  - Construct filtered/initialized state as weighted sum of 4D states

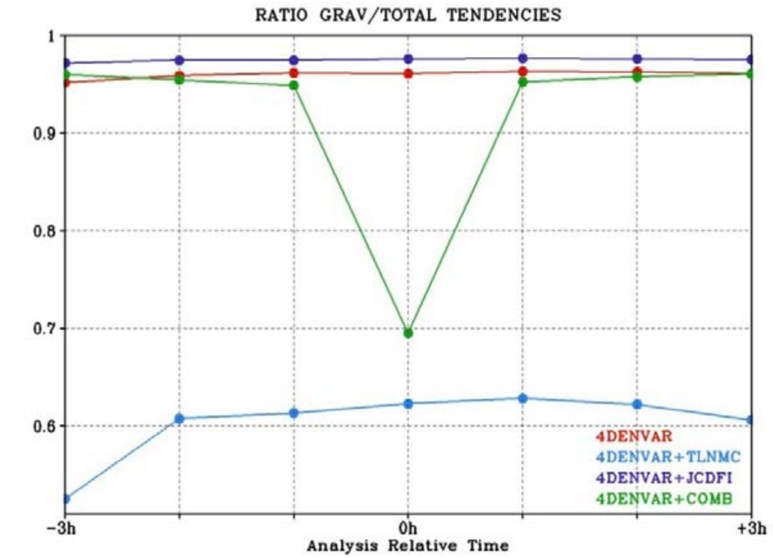
$$J_{dfi} = \chi \left\langle \mathbf{x}_m - \mathbf{x}_m^\mathrm{i}, \mathbf{x}_m - \mathbf{x}_m^\mathrm{i} \right\rangle$$

$$\mathbf{x}_m^\mathrm{i} = \sum_{k=1}^{K} \mathrm{h}_{k-m} \mathbf{x}_k^\mathrm{u}$$

- Combination of the two
  - Apply TLNMC to center of assimilation window only in combination with JcDFI (Cost effective alternative?)

Constraint impact (single case)

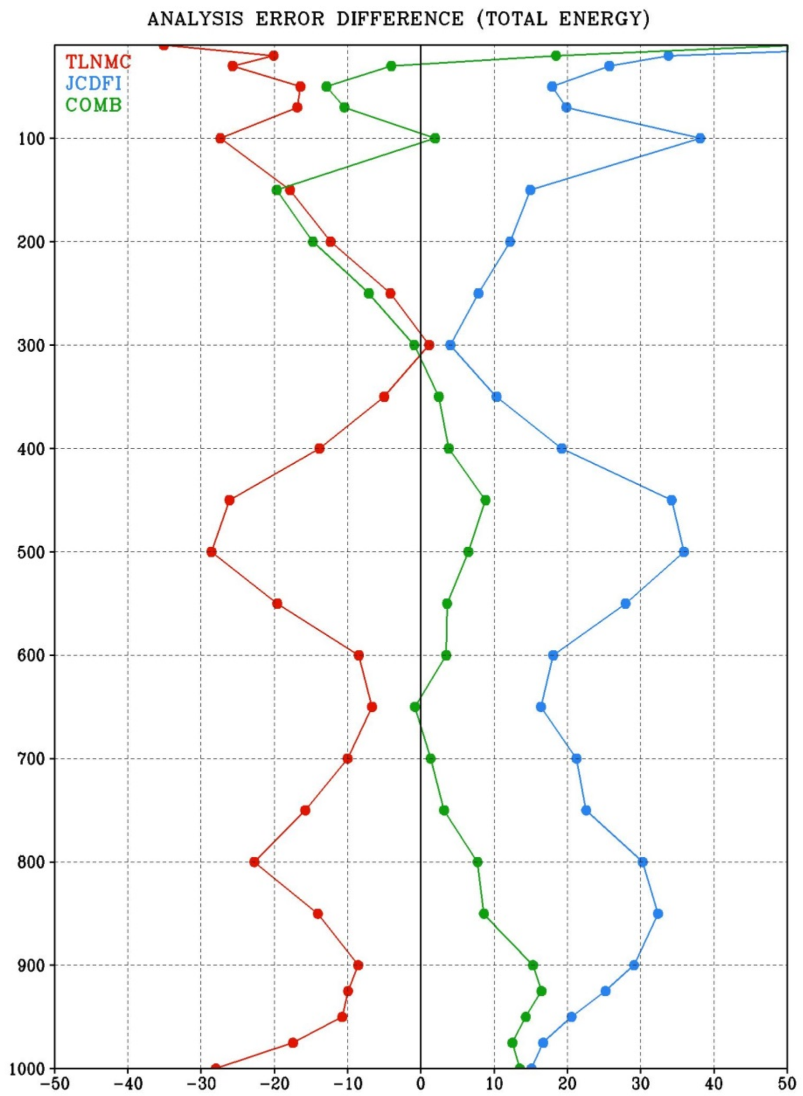




- **Impact on tendencies**
  - Dashed: Total tendencies
  - Solid: Gravity mode tendencies
  - All constraints reduce incremental tendencies

- **Impact on ratio of gravity mode/total tendencies**
  - JcDFI increases ratio of gravity mode to total tendencies
  - TLNMC most effective (but most expensive)
  - Combined constraint potential (cost effective alternative)

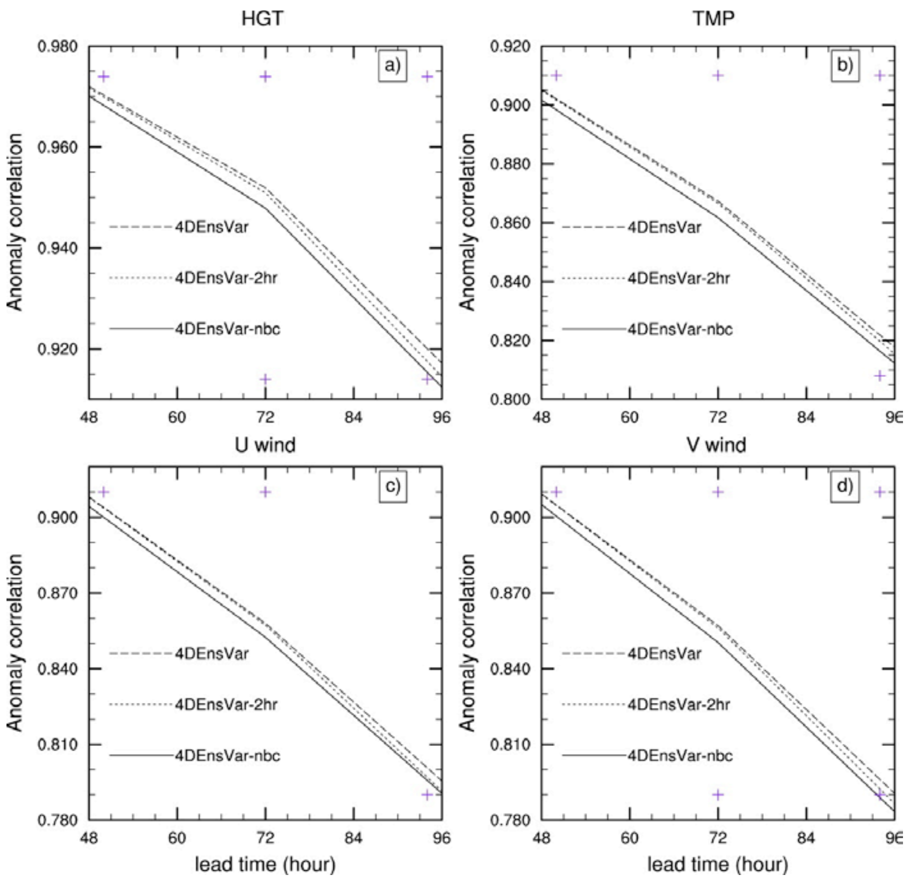
Analysis Error (cycled OSSE)



- Time mean (August) change in analysis error (total energy) ***relative*** to 4D hybrid EnVar experiment that utilized no constraints at all
  - **TLNMC universally better**
  - Combined constraint mixed
  - JcDFI increases analysis error

TLNMC impact in 4D EnVar (real obs)

Wang and Lei (2014)



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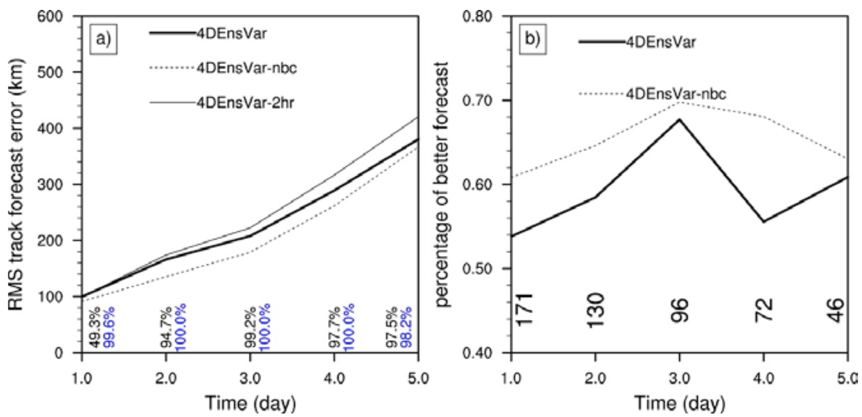
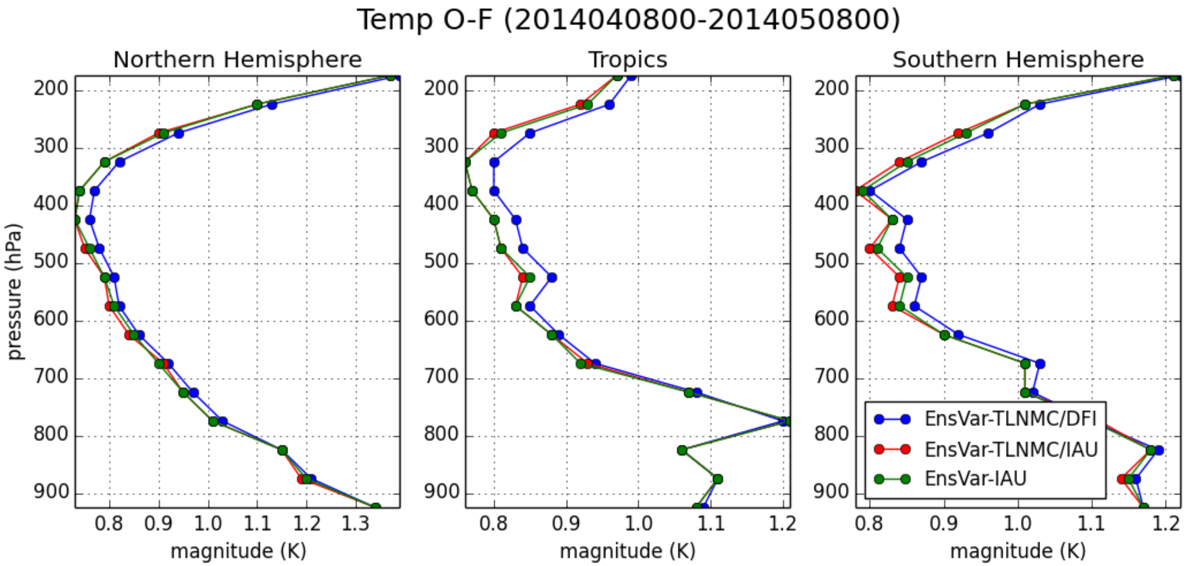
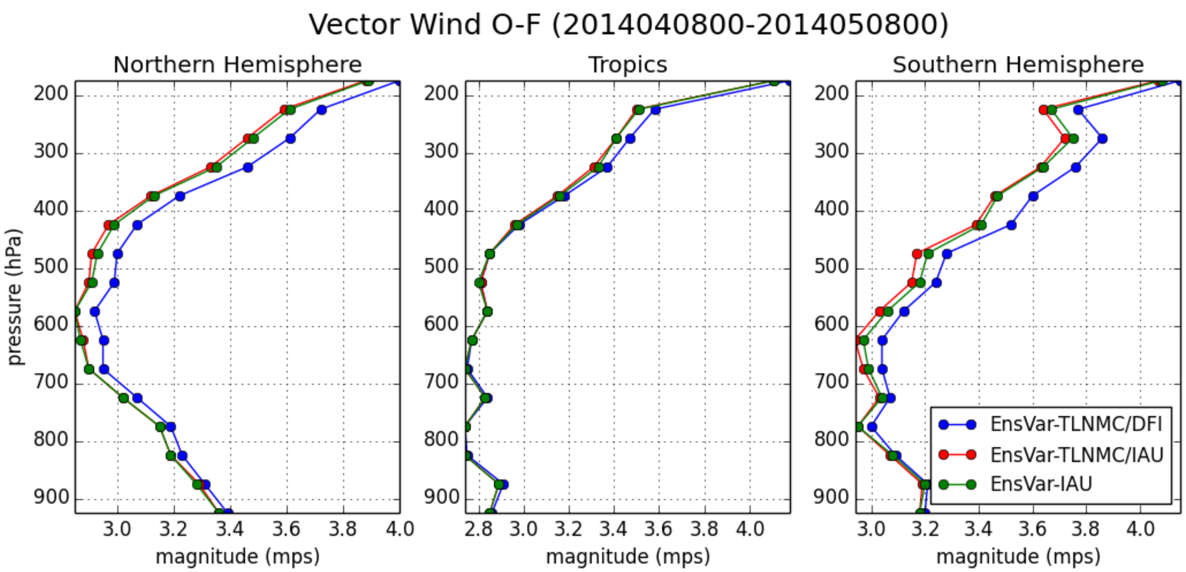


TC Track Errors

Impact in 4D hybrid with IAU

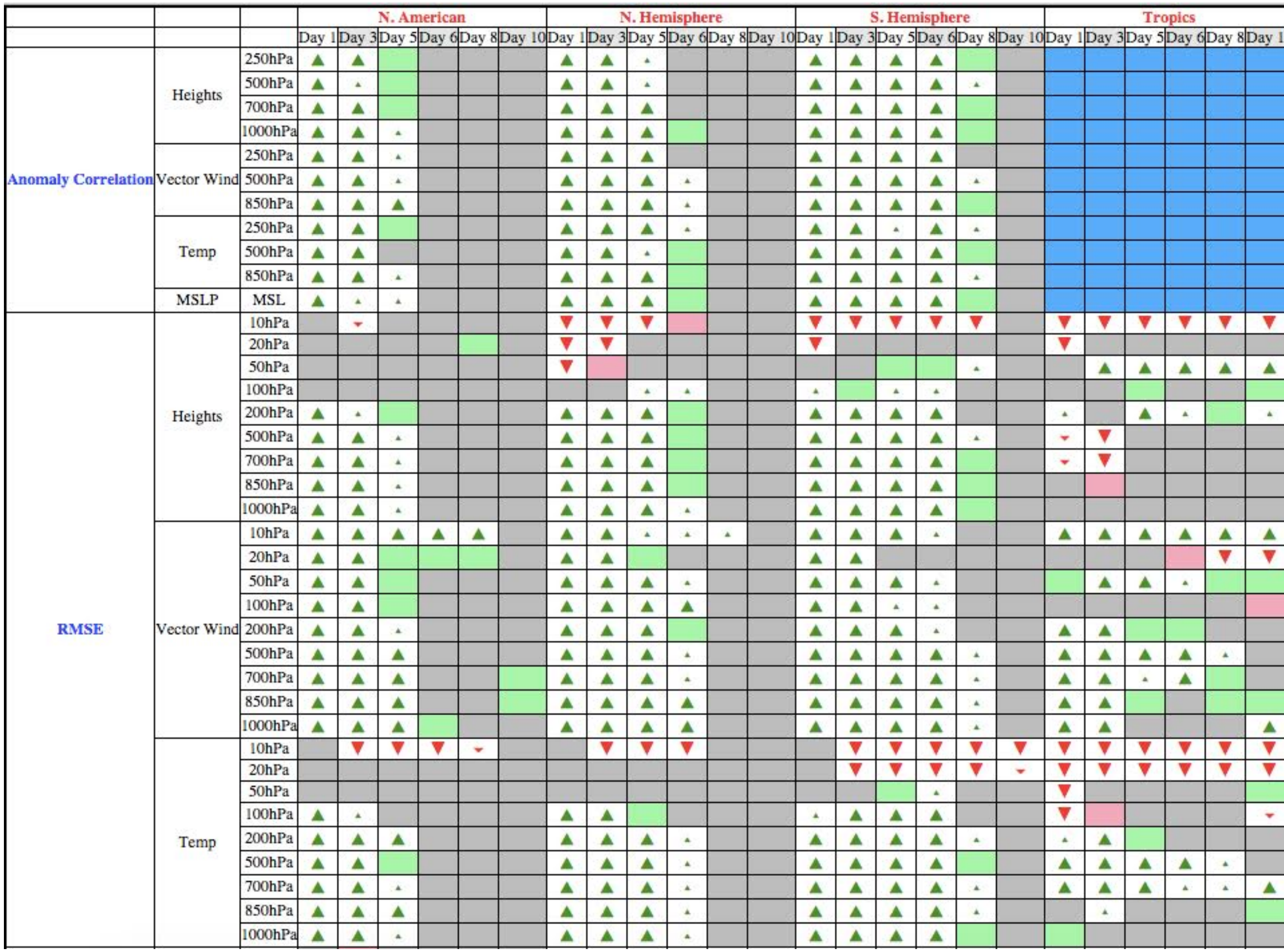
Courtesy: Lili Lei





Hybrid 4D EnVar ***with TLNMC***

To be implemented in early 2016



Full Resolution (T1534/T574) Trials:
Summary Scorecard
(02-01 through 04-29 2015)

TLNMC Summary

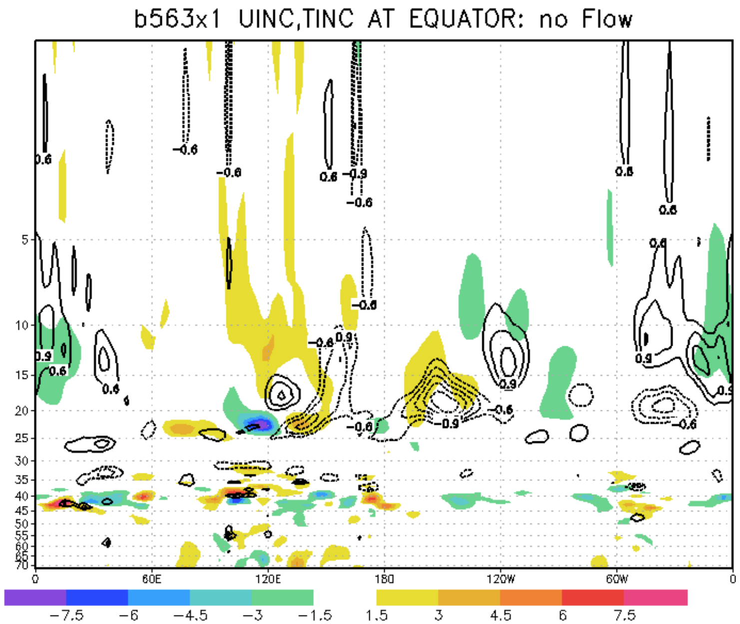
- A scale-selective dynamic constraint has been developed based upon the ideas of NNMI
  - Successful implementation of TLNMC into global version of GSI at NCEP and GMAO
  - Incremental: does not force analysis (much) away from the observations compared to an unconstrained analysis
  - Improved analyses and subsequent forecast skill, particularly in extratropical mass fields
  - ***Key contribution in 3DVAR and hybrid 3D/4D EnVar***

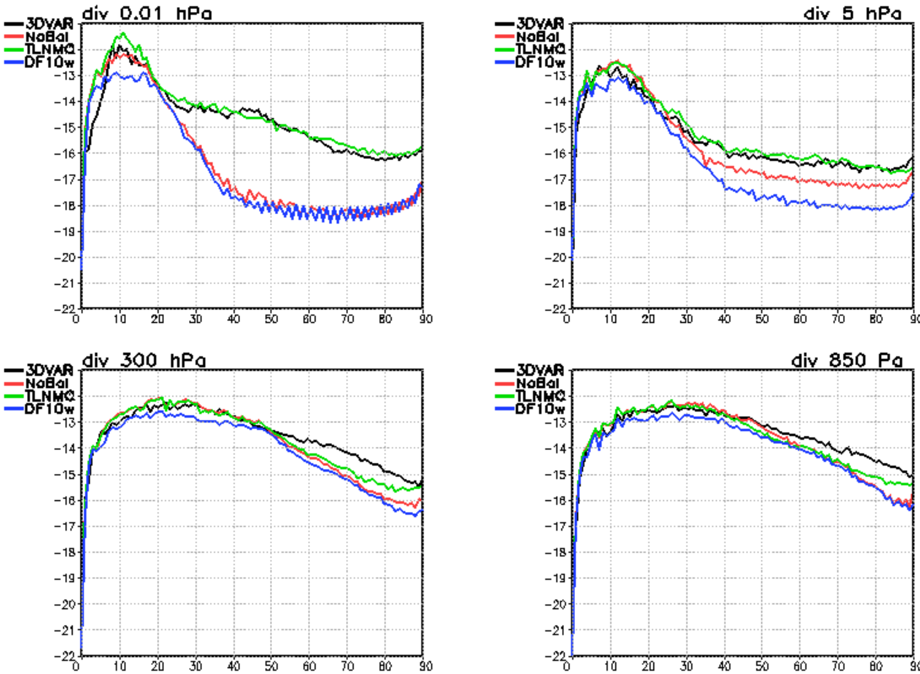
- Work is on-going to apply TLNMC to regional applications & domains (Dave Parrish – NCEP, Part I)

TLNMC Summary

- Some of the negatives
  - Slightly detrimental in tropics
    - Dry, adiabatic tendency model
    - Formulation modifications/additions
  - Computational cost in context of 4D EnVar
    - Application over *k* time levels
    - Currently using single basic state, need to expand to time dimension
  - Large corrections at very high levels
    - Potentially problematic if interested in upper atmosphere

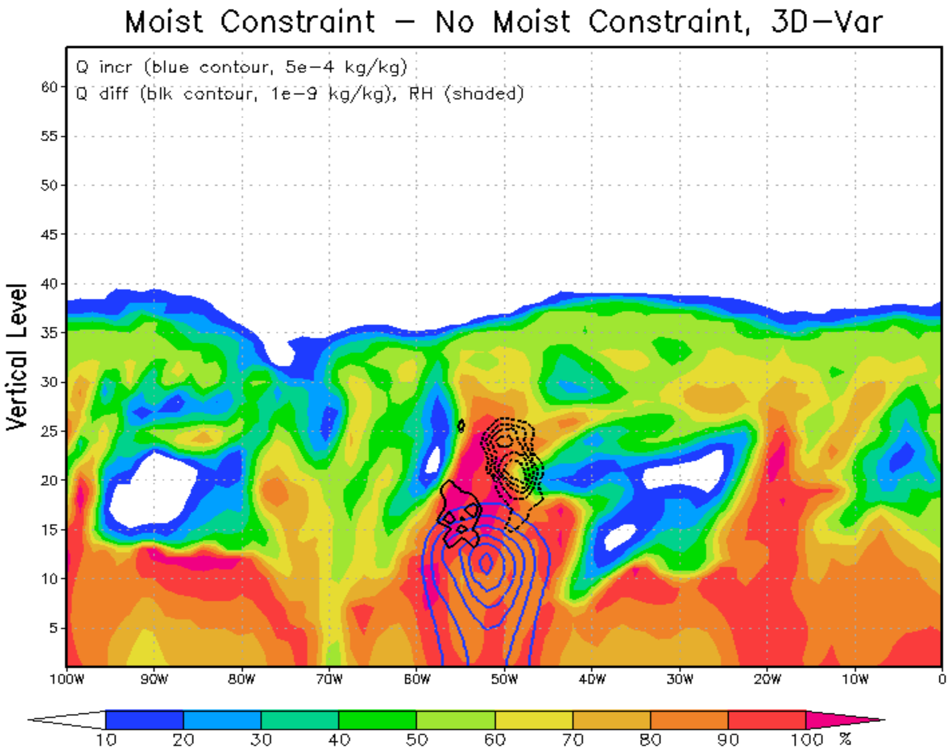
Upper Atmosphere Increment From GMAO





Large corrections (increments) at upper levels away from observations due to projections from below

Linearized "Moist Physics" in the TLNMC (Cathy Thomas)

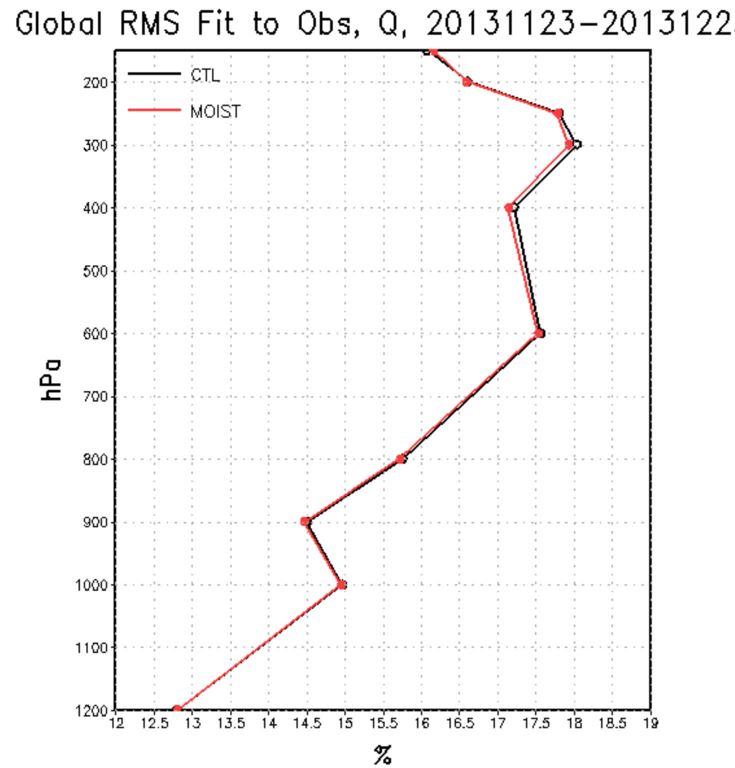


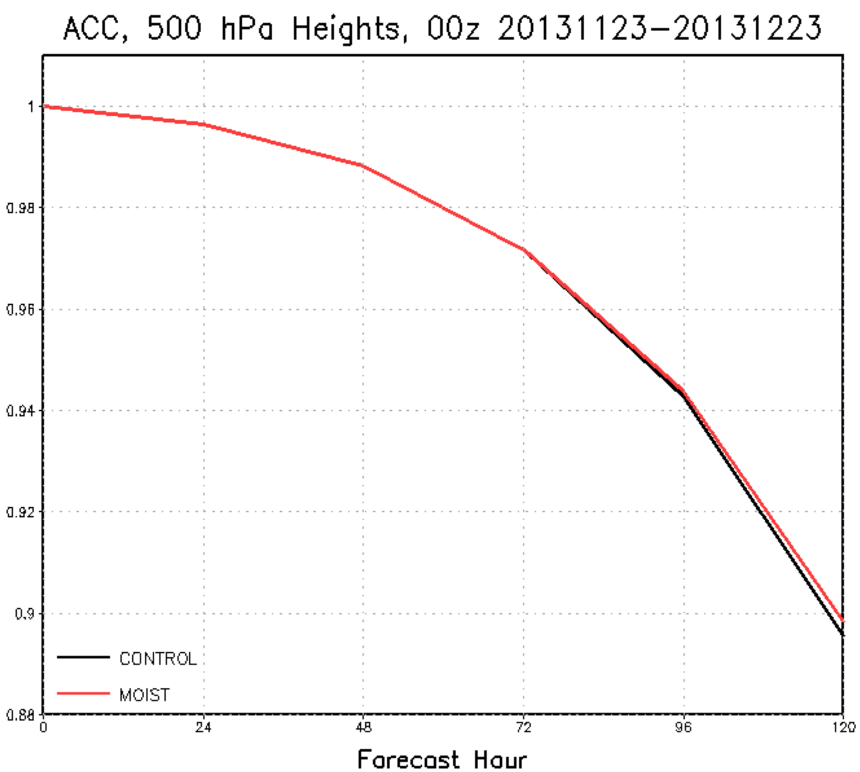
- Linearized processes added:
  - Grid scale condensation
  - Large scale precipitation
- No direct correction to moisture within the TLNMC
- Added physics modifies the temperature tendencies only

Shaded – Background Humidity
Blue Contours – Humidity Analysis Increment
Black Contours – Difference with and without Moist Processes

Results

- T254 Eulerian GFS
- Hybrid 3DEnVar
- 22 Nov - 23 Dec 2013





- Moist experiment performs slightly better globally, but results are not statistically significant and are dependent on region and variable.
- Northern Hemisphere generally improved, with tropics slightly degraded, neither significant.

Summary

- Initialization still matters
- Work ongoing to optimize TLNMC for operational use and next-generation DA
  - Higher resolution, clouds, tropical modes, etc.
  - Software optimization
  - More linear physics, computational optimization
- Testing (potential) alternatives such as IAU, though it seems best (overall) results come from system with TLNMC