MODES software and its application to the ERA Interim reanalyses

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NCAR

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Based on
Žagar, N., A. Kasahara, K. Terasaki, J. Tribbia and H. Tanaka: Normal-mode function representation of global 3-D data sets: open-access software for the atmospheric research community. Geoscientific Model Development, 8, 1169-1195.

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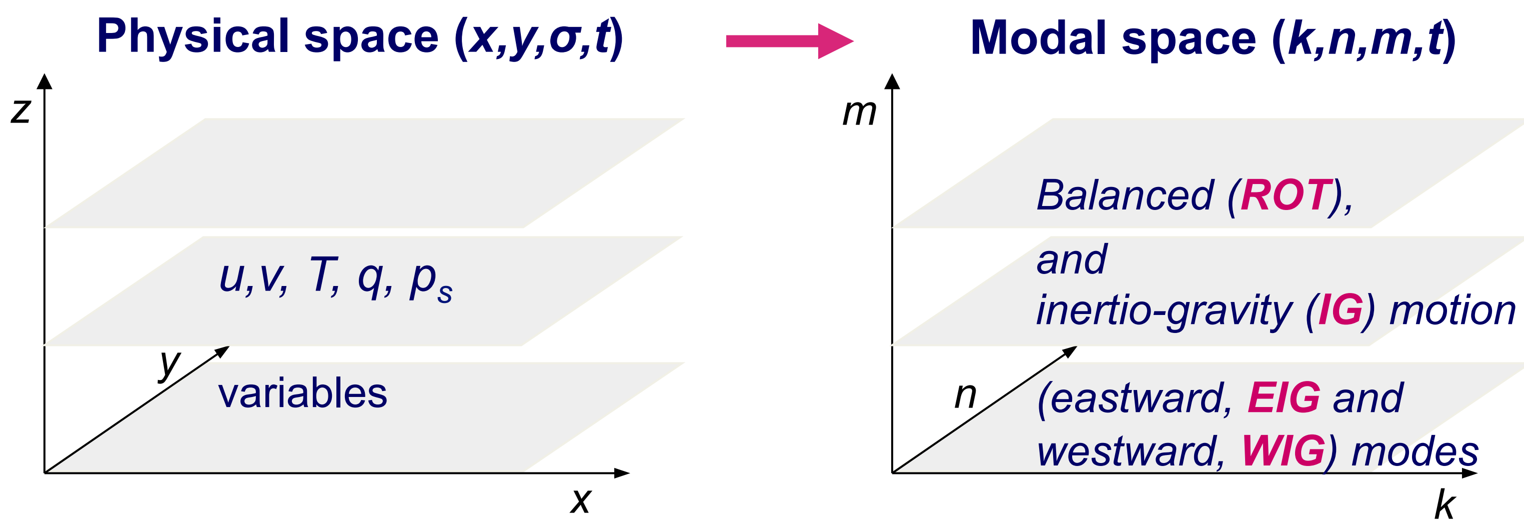
Outline

- Introduction
- Development of the normal-mode function (NMF) representation in the terrain-following coordinate
- Solution of NMF problem: vertical structure functions and horizontal structure functions
- Construction of the MODES software
- Some outputs of the MODES applied to the ERA Interim dataset
- Conclusions and Outlook

Normal mode function (NMF) representation



Balance: part of the circulation that is associated with the Rossby type (quasi-geostrophic, ROT) of solutions to the linearized primitive equations.

The unbalanced part projects onto the inertio-gravity solutions that propagate eastward (EIG) or westward (WIG).

Derivation of NMF following Kasahara&Puri, 1981

Vertical coordinate $\sigma = p/p_s$

A new geopotential variable $P = \Phi + RT_0 \ln(p_s) \quad h' = P/g$

System of linearized equations describing osclillations (u',v',h') superimposed on a basic state of rest with $T_0(\sigma)$:

$$\frac{\partial u'}{\partial t} - 2\Omega v' \sin(\varphi) = -\frac{g}{a\cos(\varphi)} \frac{\partial h'}{\partial \lambda},$$

$$\frac{\partial v'}{\partial t} + 2\Omega u' \sin(\varphi) = -\frac{g}{a} \frac{\partial h'}{\partial \varphi},$$

$$\frac{\partial}{\partial t}\left[\frac{\partial}{\partial \sigma}\left(\frac{g\sigma}{R\Gamma_0}\frac{\partial h'}{\partial \sigma}\right)\right] - \nabla \cdot \mathbf{V}' = 0.$$

The static stability parameter: $\Gamma_0 = \frac{\kappa T_0}{\sigma} - \frac{\mathrm{d}T_0}{\mathrm{d}\sigma}$

Derivation of NMF following Kasahara&Puri, 1981

Solutions in terms of horizontal and vertical dependencies:

$$[u',v',h']^{\mathrm{T}}(\lambda,\varphi,\sigma,t)=[u,v,h]^{\mathrm{T}}(\lambda,\varphi,t)\times G(\sigma)$$

An equation for the vertical structure and a set of equations for 2D horizontal motions identical in form to the global shallow water equations. 3D and 2D motions are connected by particular values of a separation parameter – “**equivalent depth” D**.

The vertical structure equation:

$$\frac{d}{d\sigma}\left(\frac{\sigma}{S}\frac{dG}{d\sigma}\right)+\frac{H_*}{D}G=0$$

and its boundary conditions

$$\frac{dG}{d\sigma}+rG=0 \text{ where } r=\frac{\Gamma_o}{T_o} \text{ at the bottom } \sigma=1$$

$$\sigma\frac{dG}{d\sigma}=0 \text{ at the model top } \sigma=\sigma_T$$

provide solutions for G(σ) which are orthogonal $\int_{\sigma_T}^{1} G_i(\sigma)G_j(\sigma)d\sigma=\delta_{ij}\,,$

Vertical expansion of discrete data onto NMF

An input data vector **X** is defined on the horizontal regular Gaussian grid and vertical sigma levels at time *t*:

$$\mathbf{X}(\lambda,\varphi,\sigma)=\left(u,v,h\right)^T$$

Projection of a single data point on j-th sigma level is performed on the precomputed vertical structure functions **G**, the horizontal Hough vector functions in the meridional direction and waves in the longitudinal direction:

$$\mathbf{X}(\lambda,\varphi,\sigma_j)=\sum_{m=1}^{M}\mathbf{S}_m\mathbf{X}_m(\lambda,\varphi)\cdot G_m(j) \quad (1)$$

$$\mathbf{X}_m(\lambda,\varphi)=\left(\tilde{u}_m,\tilde{v}_m,\tilde{h}_m\right)^T=\left(\frac{u_m}{\sqrt{gD_m}},\frac{v_m}{\sqrt{gD_m}},\frac{h_m}{D_m}\right)^T$$

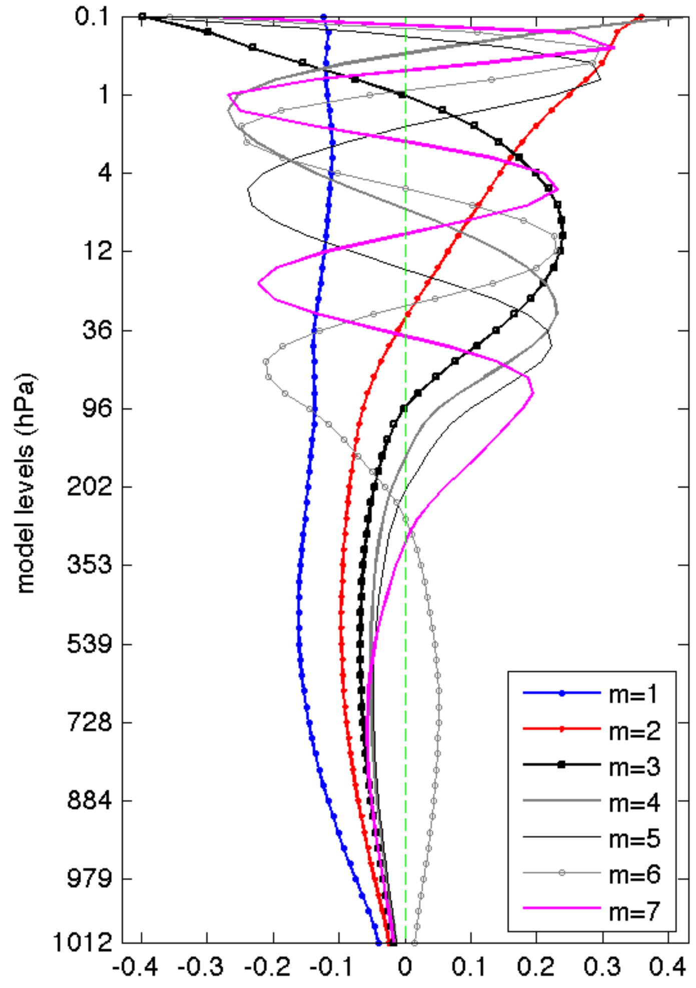
Normalization factors define diagonal elements of 3x3 matrix normalization matrix **S**

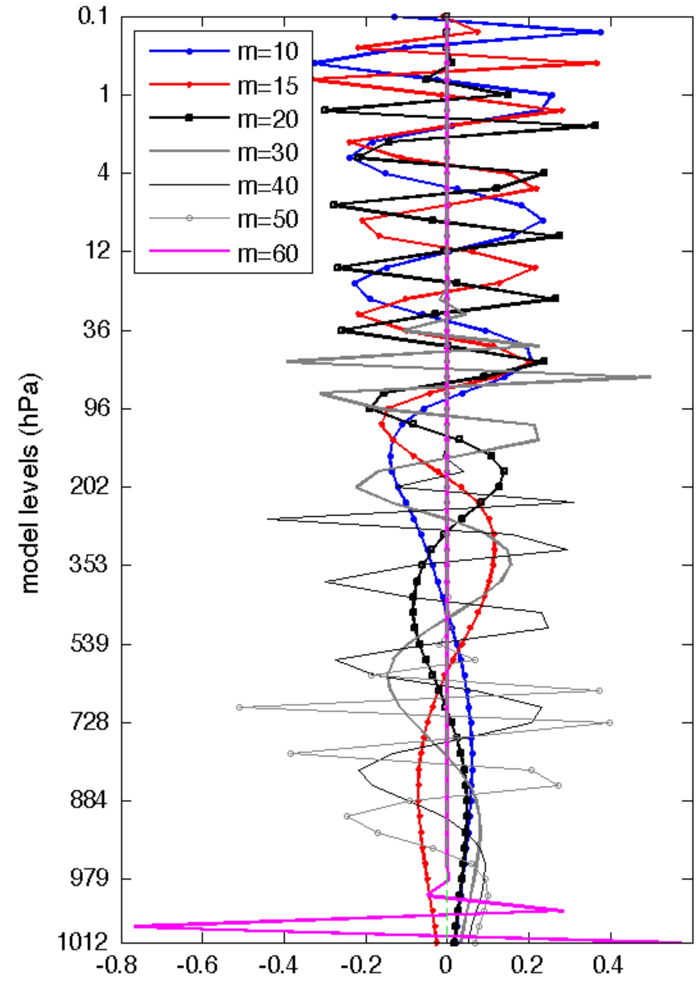
The vector $\mathbf{X}_m$ is obtained by the reverse transform of (1):

$$\mathbf{X}_m(\lambda,\varphi)=\mathbf{S}_m^{-1}\sum_{j=1}^{J}\left(u,v,h\right)_j^T G_m(j) \quad (2)$$

Solutions of the vertical structure function

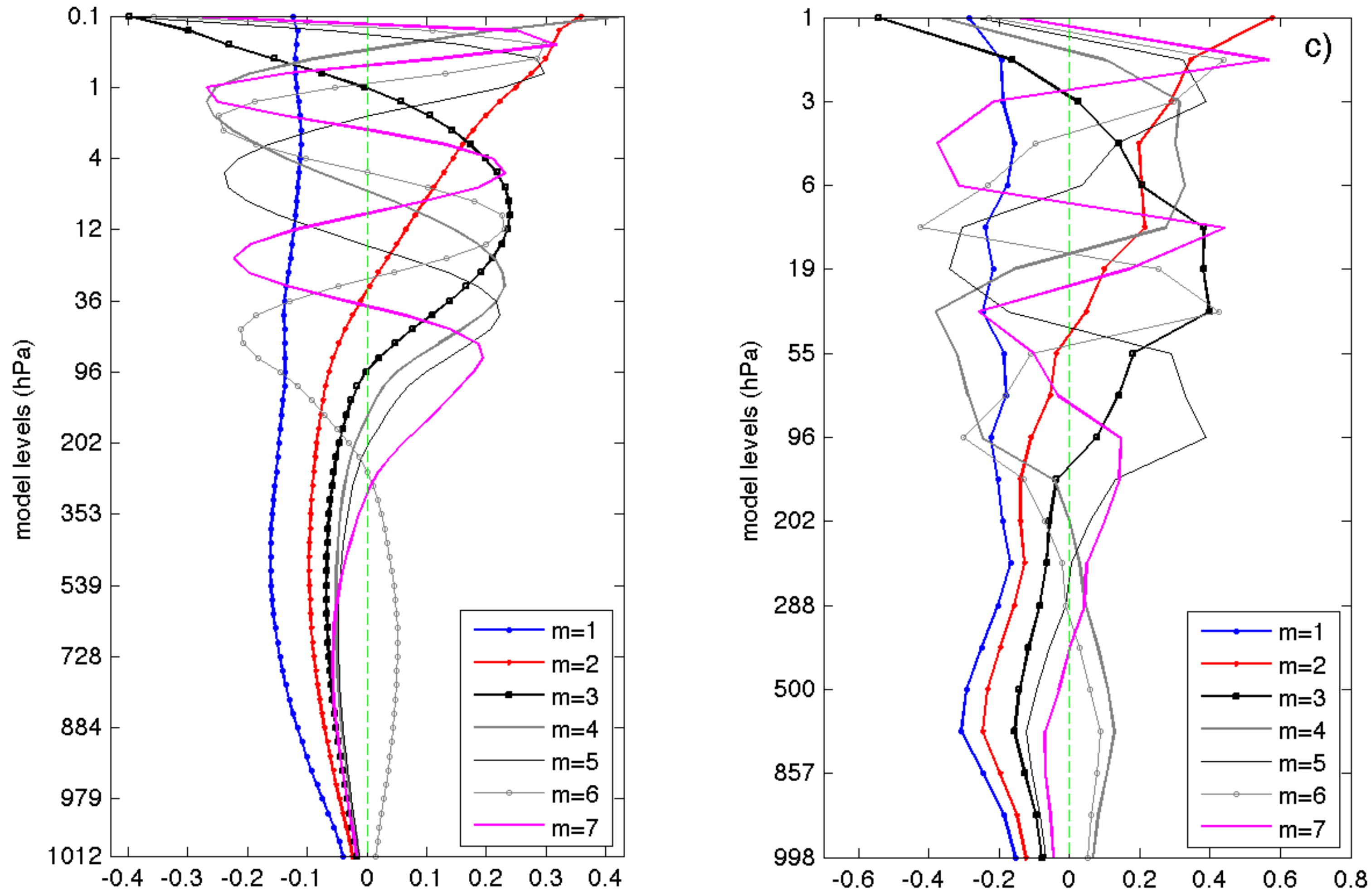
Example of ERA Interim dataset: L60





Solutions of the vertical structure function

Example of ERA Interim dataset: L60 versus L21 (standard p levels)



Derivation of NMF following Kasahara&Puri, 1981

The horizontal structure equation

$$\frac{\partial}{\partial t}\mathbf{X}_m + \mathbf{L}\mathbf{X}_\mathbf{m} = 0$$

describe 2D motions for vector $\mathbf{X}_m(\lambda,\varphi)$ every equivalent depth D_m:

$$\mathbf{X}_m(\lambda,\varphi)=\left(\tilde{u}_m,\tilde{v}_m,\tilde{h}_m\right)^T = \left(\frac{u_m}{\sqrt{gD_m}},\frac{v_m}{\sqrt{gD_m}},\frac{h_m}{D_m}\right)^T$$

L is the linear differential matrix operator

$$\mathbf{L} = \begin{vmatrix} 0 & -\sin(\varphi) & \frac{\gamma}{\cos(\varphi)}\frac{\partial}{\partial\lambda} \\ \sin(\varphi) & 0 & \gamma\frac{\partial}{\partial\varphi} \\ \frac{\gamma}{\cos(\varphi)}\frac{\partial}{\partial\lambda} & \frac{\gamma}{\cos(\varphi)}\frac{\partial}{\partial\varphi}[\cos(\varphi)()] & 0 \end{vmatrix}$$

Solution $\mathbf{X}_m$ is harmonic in time:

$$\mathbf{X}_m(\lambda,\varphi,\tilde{t})=\mathbf{H}_n^k(\lambda,\varphi)e^{-i\nu_n^k\tilde{t}}$$

With global inner product defined as $\langle\mathbf{X}_\mathbf{l},\mathbf{X}_\mathbf{m}\rangle = \frac{1}{2\pi}\int_0^{2\pi}\int_{-1}^{1}\left(\tilde{u}_l\tilde{u}_m^* + \tilde{v}_l\tilde{v}_m^* + \tilde{h}_l\tilde{h}_m^*\right)d\mu d\lambda$

functions **H** are expressed as $\mathbf{H}_n^k(\lambda,\varphi) = \Theta_n^k e^{ik\lambda}$ Hough harmonics

Horizontal expansion of discrete data onto NMF

The horizontal coefficient vector $\mathbf{X}_m$ for a given vertical mode is projected onto the Hough harmonics $\mathbf{H}^n{}_k(\lambda,\varphi,m)$ as

$$\mathbf{X}_m(\lambda,\varphi) = \sum_{n=1}^{R} \sum_{k=-K}^{K} \chi_n^k(m) \mathbf{H}_n^k(\lambda,\varphi,m) \qquad (3)$$

The subscript *n* indicates all meridional modes including rotational (ROT), and eastward and westward propagating inertio-gravity (EIG and WIG, respectively) modes

The scalar complex coefficients χ are obtained as

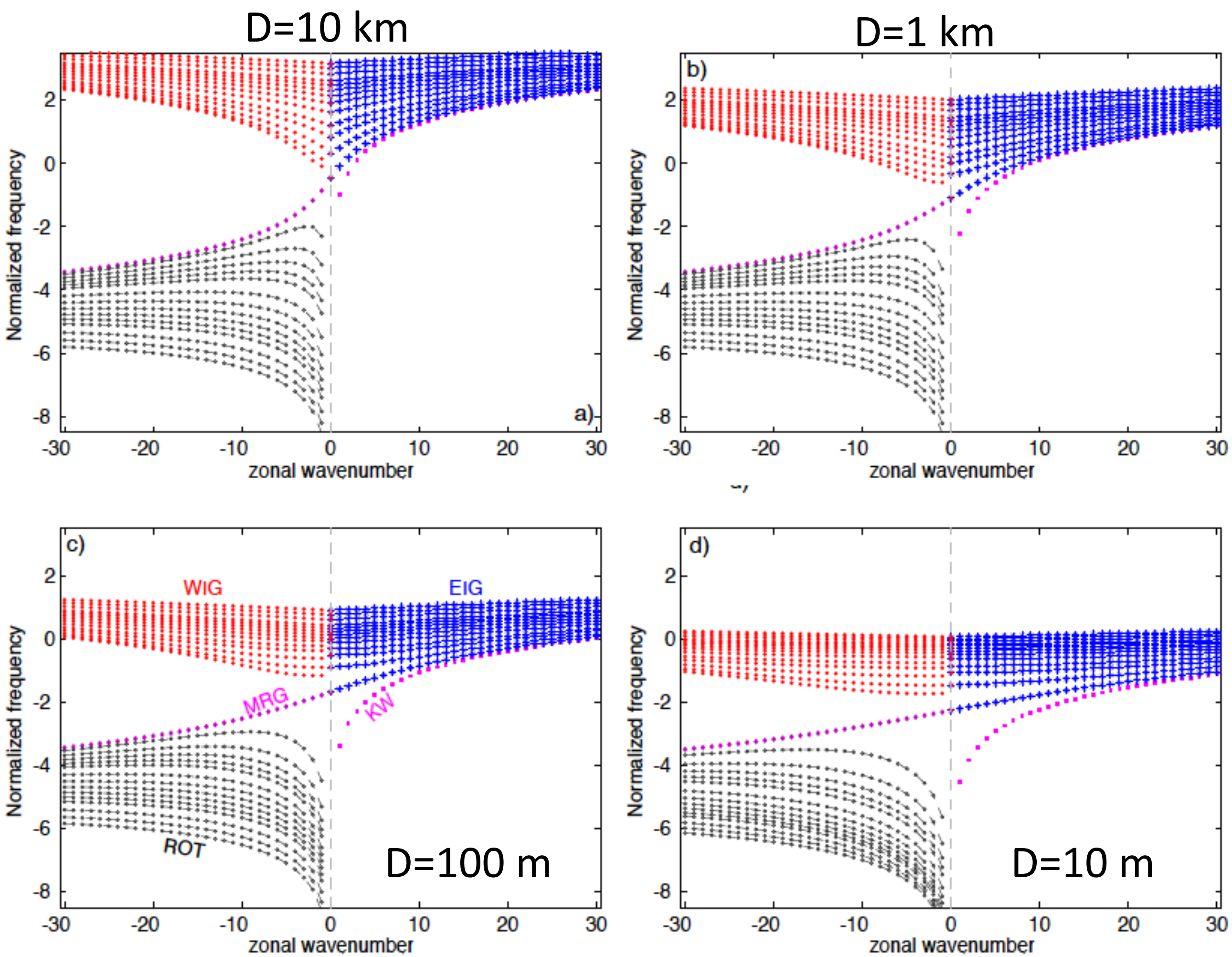
$$\chi_n^k(m) = \frac{1}{2\pi} \int_0^{2\pi} \int_{-1}^{1} \left(\tilde{u}_m, \tilde{v}_m, \tilde{h}_m\right)^T \left[\mathbf{H}_{n'}^{k'}\right]^* d\mu d\lambda \qquad (4)$$

Here, $\mu = \sin(\varphi)$.

Equations (3-4) are the horizontal transform pair.

Equations (1-2) represent the vertical transform pair.

Two kinds of Hough functions



Frequencies of spherical normal modes for different equivalent depths

NMF expansion: horizontal expansion functions

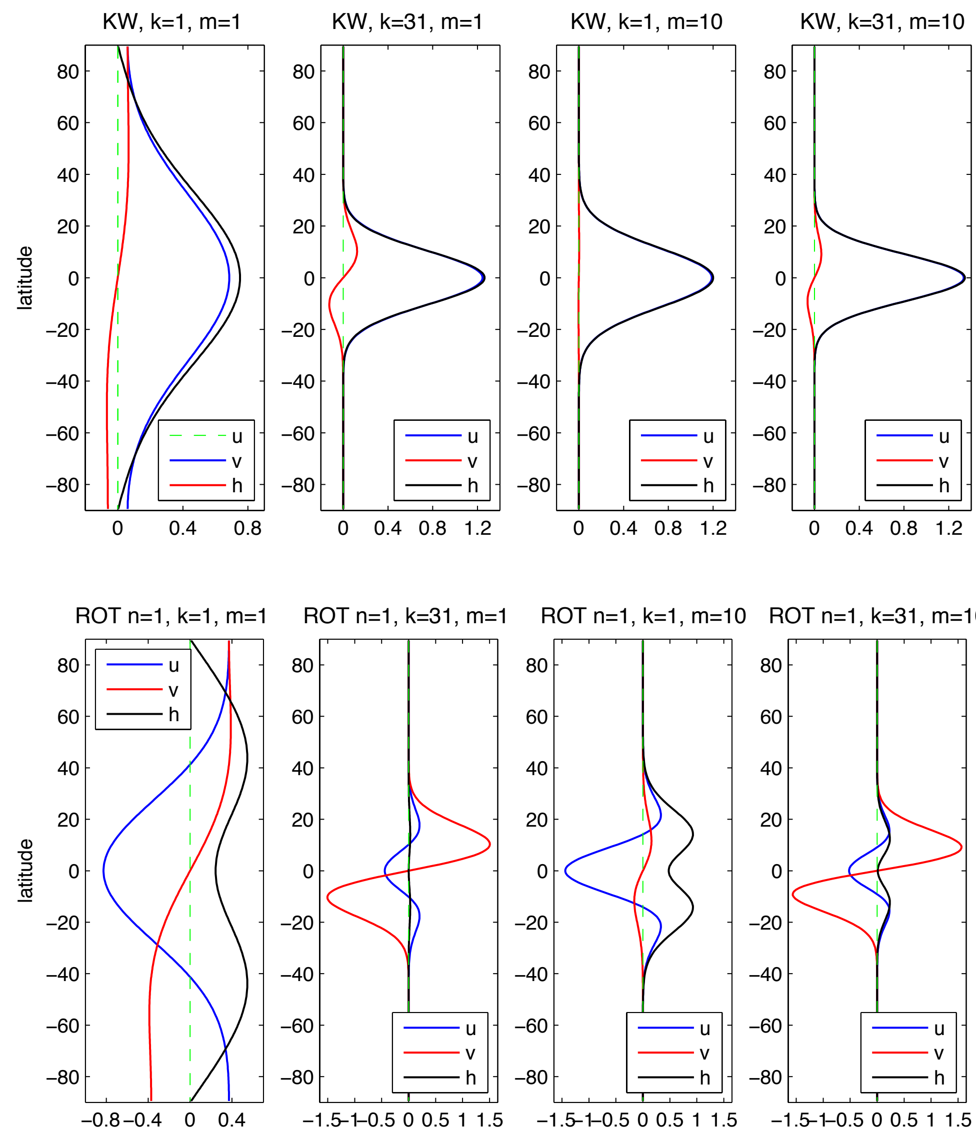
HSFs are pre-computed for a given number of vertical modes, M

For every m=1,…,M, i.e. for every D_m

Meridional structure for Hough functions is computed for a range of the zonal wavenumbers K, k=-K,..,0,...,K

and a range of meridional modes for the balanced, N_{ROT}, a range of EIG, N_{EIG}, and a range of WIG, N_{WIG}, modes.

$R = N_{ROT} + N_{EIG} + N_{WIG}$



Expansion of 3D data onto NMF: MODES software

```
# --------------------------------------------------------------
TOPPATH=/Users/nedjeljka/NMF/Sigma/NMF_MODES
# --------------------------------------------------------------
INSTALLDIR=$(TOPPATH)/bin
SHAREDIR=$(TOPPATH)/share
HOUGHDIR=$(TOPPATH)/hough
NORMALDIR=$(TOPPATH)/normal
VSFDIR=$(TOPPATH)/VSF
MAINDIR=$(TOPPATH)/main
LIBDIR=/usr/local/lib
# ----------------------- FOR LINUX ---------------------------
SYSTEM = LINUX
LAPACK_LIB=/usr/local/lib
ALFPACK_LIB=/usr/local/lib
GRIB_API_LIB=/usr/local/grib_api-1.9.18/lib
GRIB_API_INCLUDE=/usr/local/grib_api-1.9.18/include
NETCDF_LIB=/opt/local/lib
NETCDF_INCLUDE=/opt/local/include
LIBGRIB_F90=/usr/local/grib_api-1.9.18/lib/libgrib_api_f90.a
LIBGRIB=/usr/local/grib_api-1.9.18/lib/libgrib_api.a

FC = gfortran
FFLAGS= -O3 -I$(SHAREDIR) -I$(HOUGHDIR) -I$(NORMALDIR) -I$
(VSFDIR) -L$(LIBDIR) -fopenmp -I$(GRIB_API_INCLUDE)
LD = $(FC)
LFLAGS = -O3 -fopenmp
```

Makefile and namelist controlled

Required libraries for the input data in grib and netcdf format

Input data on the Gaussian grid and model levels (sigma or hybrid)

Uses LAPACK or equivalent

Preparation steps requires computation of the stability profile

Five executables which are run in subsequent steps:

- Preparation of the horizontal grid
- Computation of the vertical structure functions
- Computation of the horiozntal structure functions
- Projection
- Filtering of selected modes to physical space

Expansion of 3D data onto NMF: MODES software

```
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ALFPACK_LIB=/usr/local/lib
GRIB_API_LIB=/usr/local/grib_api-1.9.18/lib
GRIB_API_INCLUDE=/usr/local/grib_api-1.9.18/include
NETCDF_LIB=/opt/local/lib
NETCDF_INCLUDE=/opt/local/include
LIBGRIB_F90=/usr/local/grib_api-1.9.18/lib/libgrib_api_f90.a
LIBGRIB=/usr/local/grib_api-1.9.18/lib/libgrib_api.a

FC = gfortran
FFLAGS= -O3 -I$(SHAREDIR) -I$(HOUGHDIR) -I$(NORMALDIR) -I$
(VSFDIR) -L$(LIBDIR) -fopenmp -I$(GRIB_API_INCLUDE)
LD = $(FC)
LFLAGS = -O3 -fopenmp
```

```
& vsfcalc_cnf
     stab_fname    =  'stability_L60.data',
     vgrid_fname    =   'sigma_levels_L60.data',
     vsf_fname    =   'vsf_L60.data',
     equiheight_fname  =   'equivalent_height_L60.data',
     num_vmode  = 60,
     mp    = 60,
     hstd    = 8000.0d0,
     suft    = 288.0d0,
     given_stability      = .true.,
    ocheck     = .true.,
/
```

```
&normal_cnf_inverse
  nx         = 512,
  ny         = 256,
  nz         = 56,
  coef3DNMF_fname = 'Houghcoeff_',
  inverse_fname   = ' Inv_',
  inv2hybrid      = .false.,
  ps_fname     = ' Ps_',
  meant_fname     = 'Tmean_',
  saveasci        = .true.,
  afname          = 'Inverse_ascii_',
  aformat        = '(512E16.4,1x)',
/
```

```
& filter_cnf
     eig_n_s        = 1,
     eig_n_e        = 70,
     wig_n_s        = 1,
     wig_n_e        = 70,
     rot_n_s        = 2,
     rot_n_e        = 70,
     kmode_s        = 100,
     kmode_e        = 85,
     vmode_s        = 410,
     vmode_e        = 70,
/
```

NMF software structure: grid and vsf namelists

```
& gaussian
    N = 256,
    gauss_fname = 'gauss256.data’,
/
```

```
& vsfcalc_cnf
    stab_fname    =   'stability_L60.data’,
    vgrid_fname     =    'sigma_levels_L60.data’,
    vsf_fname     =    'vsf_L60.data’,
    equiheight_fname  =    'equivalent_height_L60.data',
    num_vmode  = 60,
    mp    = 60,
    hstd   = 8000.0d0,
    suft    = 288.0d0,
    given_stability      = .true.,
   ocheck    = .true.,
/
```

NMF software structure: hsf namelists

```
&houghcalc_cnf
      szw       =  0,
      ezw       =  200,
      maxl      =  70,
      my        =  256,
      freq_fname     = 'freq.data',
      ocheck         = .true.,
/
```

```
& output
      output_gmt    = .false.,
      ofname_gmt   = 'hough_gmt',
      ofname_bin    = 'hough',
      bin_combine    = 'zonal',
/
```

```
& meridional_grid
   ygrid_fname = 'gauss256.data',
/
```

```
& vsf_cnf
    equiheight_fname  = 'equivalent_height.data',
    num_vmode   = 43,
/
```

NMF software structure: projection namelists

```
& normal_cnf
   nx          = 512,
   ny          = 256,
    nz         =  56,
   nstep       =   1,
   coef3DNMF_fname = 'Hough_coeff_',
   output_3DNMF    = .true.,
   saveps          = .true.,
   savemeant       = .true.,
   ps_fname        = 'Ps_',
   meant_fname     = 'Tmean_',
   saveasci        = .true.,
   afname          = 'Indata_',
   aformat         = '(512E20.6,1x)',
/
```

```
& time
      datetype       = 'yyyymmddhh',
      syear          = 2007,
      smon           =   07,
      sday           =   01,
      shour          =   12,
      smins          =   00,
      ssec           =   00,
      slen           =   00,
      eyear          = 2007,
      emon           =   07,
      eday           =   01,
      ehour          =   12,
      emins   =   00,
      esec    =   00,
      elen    =  00,
     dt      = 86400,
/
& input_data
   dataformat_input = 'grib',
   orig        = 'ECMWF',
   zgrid_type  = 'hybrid'
  ifile_grib_head(1)   =  'erai_N128_'
/
```

Energy product

Starting from the linearized system (u',v',h') and its solutions by using the variable separation, the partition of total energy into the kinetic and available potential energy for every vertical mode is written as :

$$\frac{\partial}{\partial t}\int_0^{2\pi}\int_{-1}^{1}\sum_m \frac{1}{2}\left(u_m^2 + v_m^2 + \frac{g}{D_m}h_m^2\right)a^2 d\mu d\lambda = 0$$

Global energy product of the m-th vertical mode defined as

$$I_m = \frac{1}{2} g D_m \sum_{n=1}^{R}\sum_{k=-K}^{K} \chi_n^k(m)\left[\chi_n^k(m)\right]^*$$

is equivalent to

$$I_m = \frac{1}{2} g D_m \int_0^{2\pi}\int_{-1}^{1}\left(\tilde{u}_m^2 + \tilde{v}_m^2 + \tilde{h}_m^2\right)\mathrm{d}\lambda\,\mathrm{d}\mu = \int_0^{2\pi}\int_{-1}^{1}\left(K_m + P_m\right)\mathrm{d}\lambda\,\mathrm{d}\mu$$

NMF representation of ERA Interim dataset

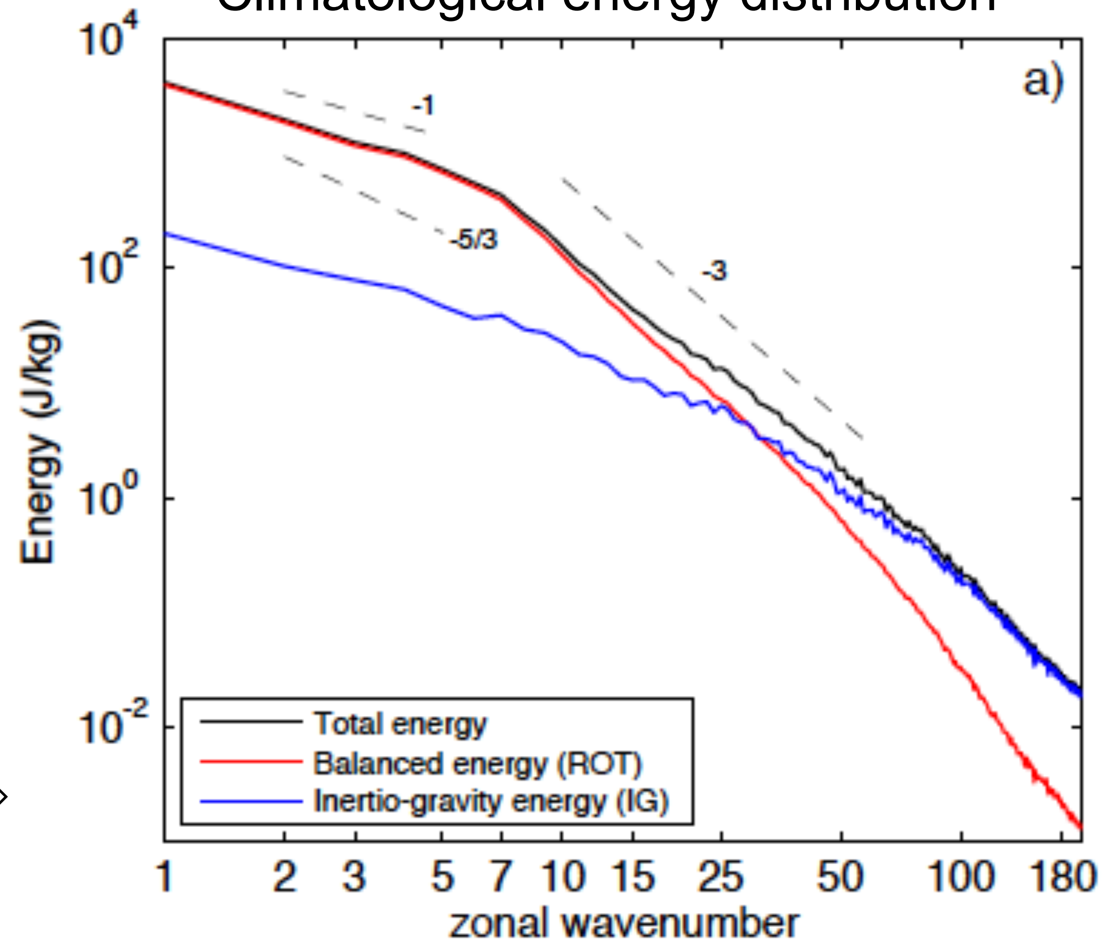
30-year period between 1980 and 2009

12 UTC time

K=200;
N=70, R=210
M=45

About 10% of wave energy in IG modes ⇔ 1/3 of circulation

Climatological energy distribution



NMF representation of ERA Interim dataset

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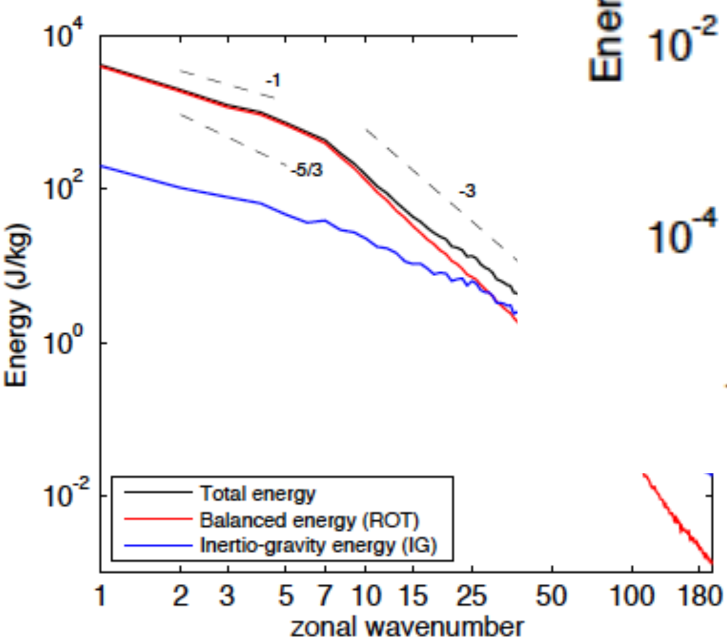
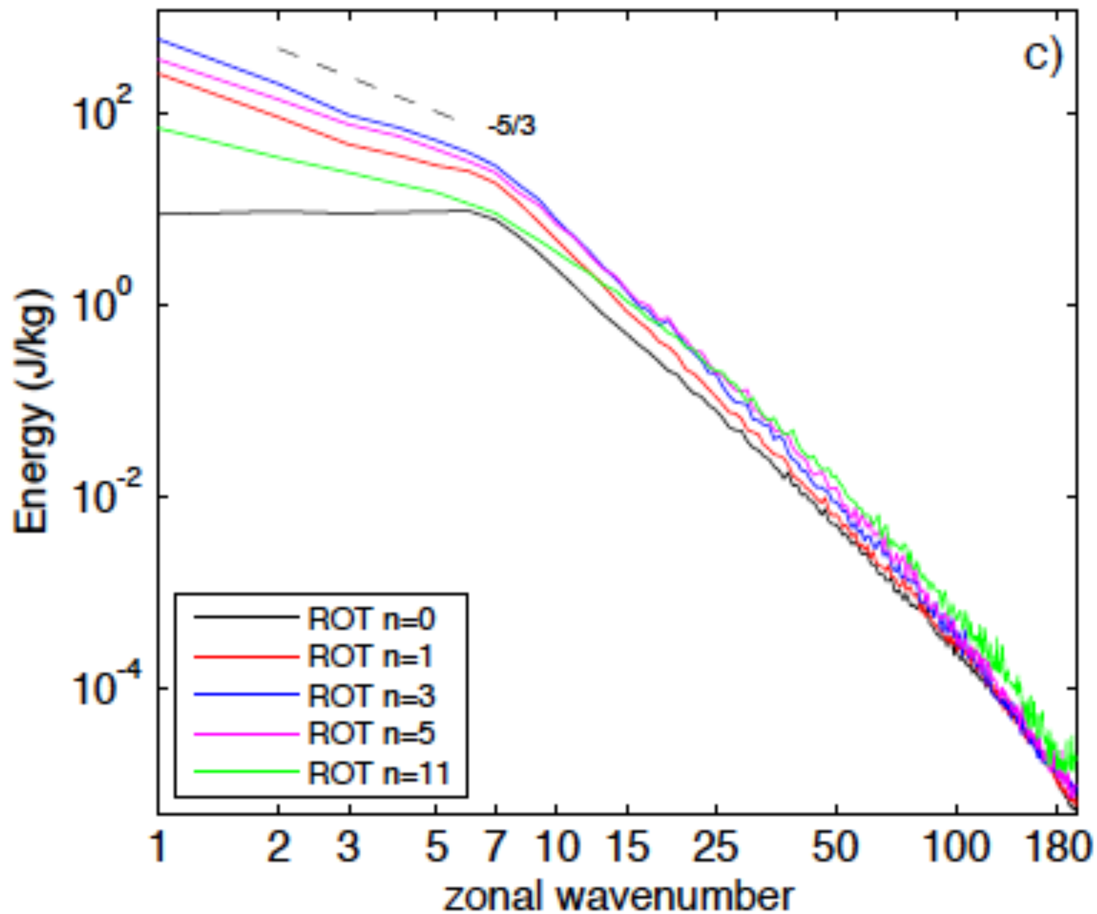


Spectra for various balanced modes



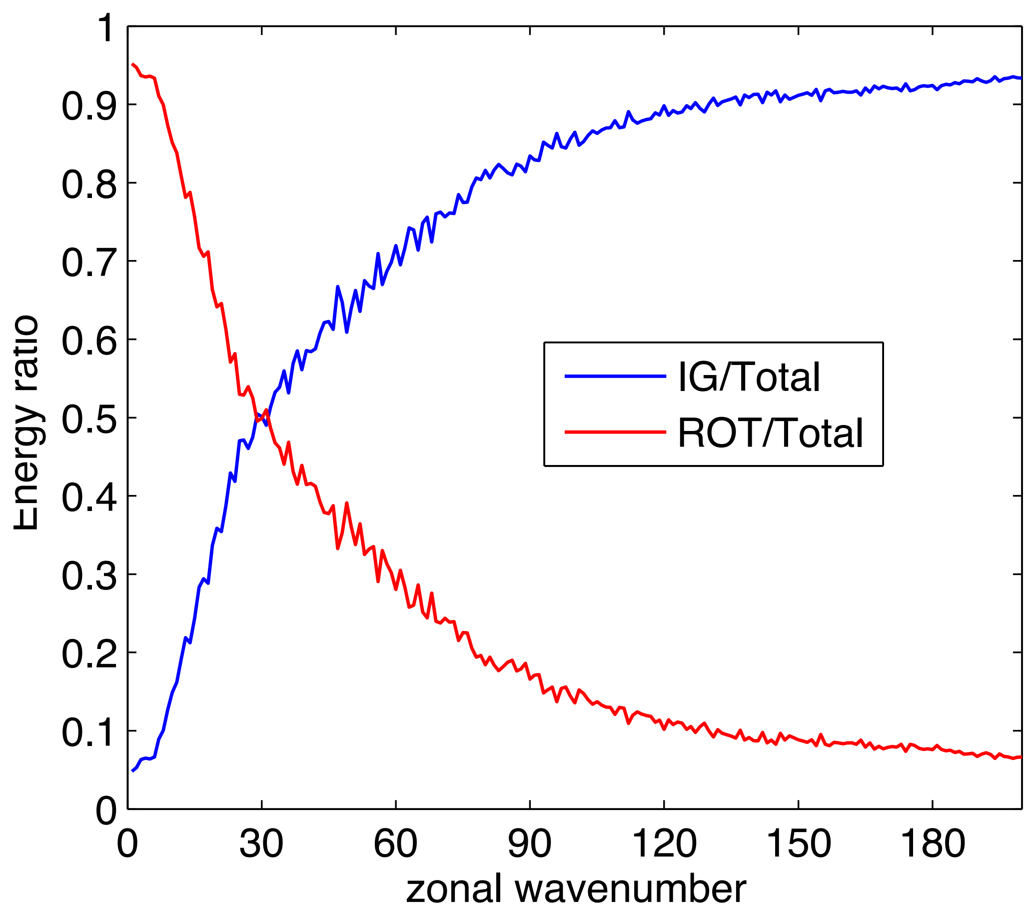
Scale-dependent climatological distribution of atmospheric total energy

Based on 30-year period between 1980 and 2009

12 UTC time

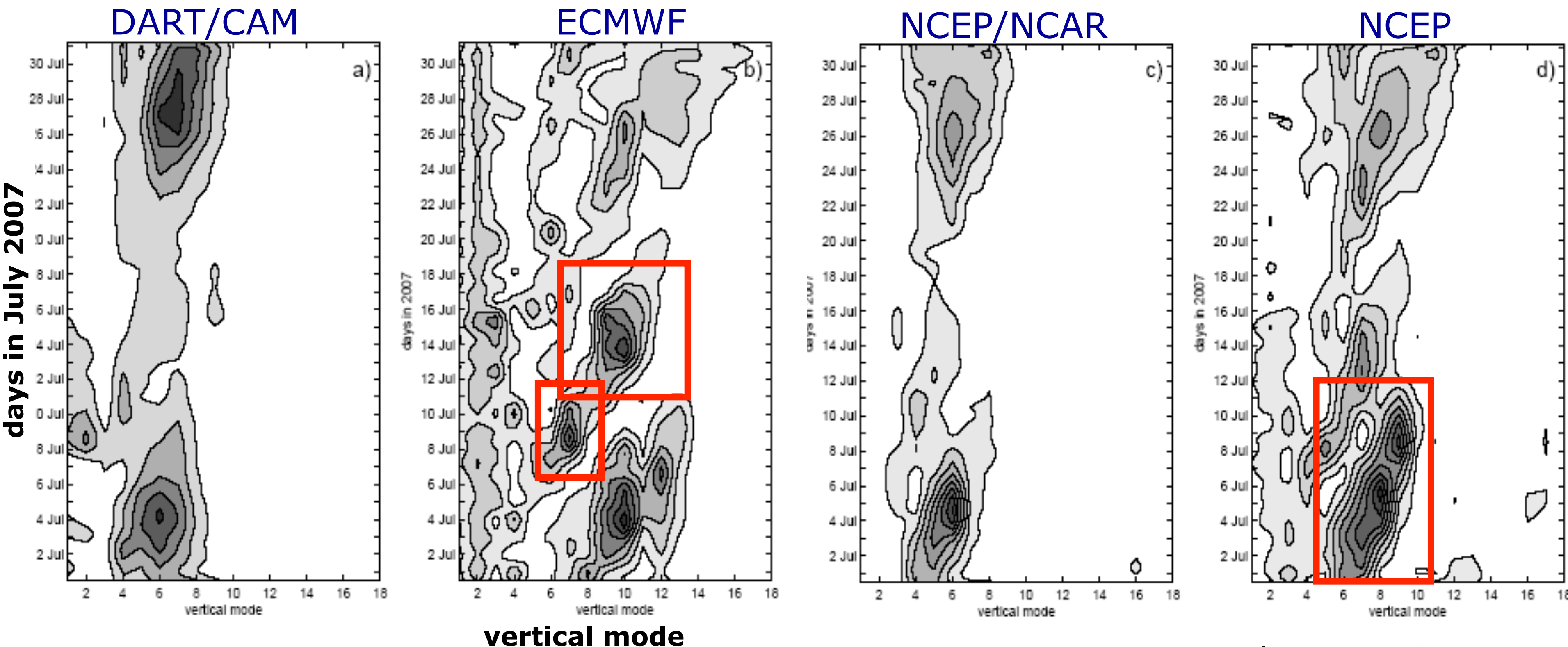
Up to 10% of wave energy in IG modes ⇔ 1/3 of circulation

Percentage of ROT/IG energy in each scale



Application of the normal mode representation

Modal-space diagnosis of the vertical energy propagation by the Kelvin waves



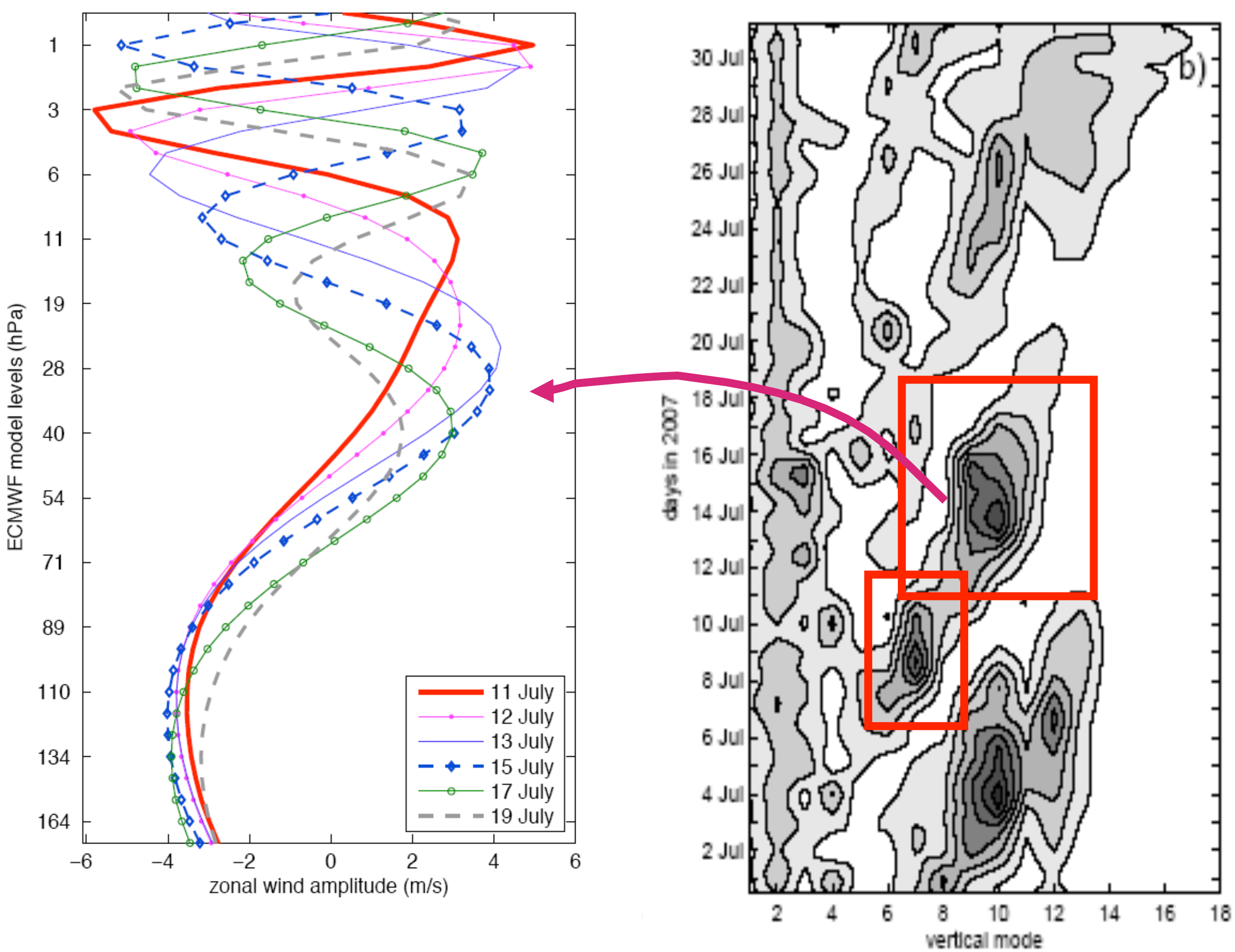
Zagar et al., MWR, 2009

Evolution of the zonal wavenumber k=1 Kelvin wave in July 2007, filtered for periods shorter than 36 hours. The best agreement between the datasets exists for the Kelvin wave.

Application of the normal mode representation

Modal-space diagnosis of the vertical energy propagation by the Kelvin waves

ECMWF



The zonal wind component in the Kelvin wave at subsequent days (left) shows the downward phase propagation (upward energy propagation).

The difference in the depth of the atmosphere in DART/CAM and NCEP/NCAR on one hand and ECMWF and NCEP on the other appears to be one reason for different propagation properties.

(Zagar et al., 2009)

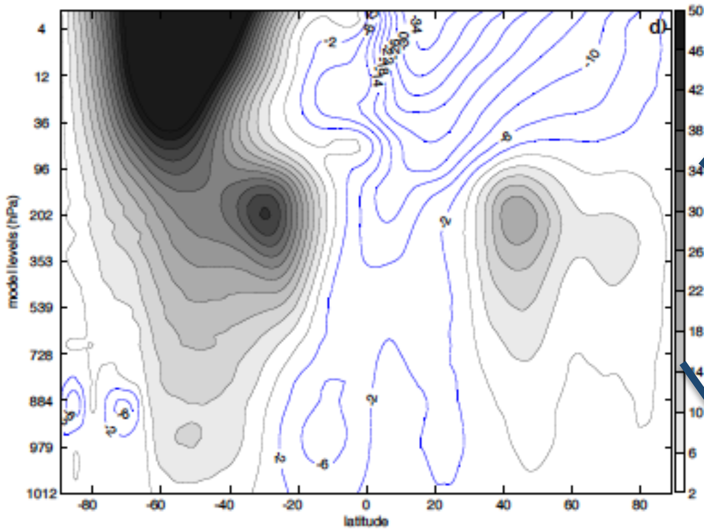
NMF software structure: filtering namelists

```
&normal_cnf_inverse
   nx          = 512,
   ny          = 256,
   nz          = 56,
   coef3DNMF_fname = 'Houghcoeff_',
   inverse_fname   = ' Inv_',
   inv2hybrid      = .false.,
   ps_fname    = ' Ps_',
   meant_fname     = 'Tmean_',
   saveasci        = .true.,
   afname          = 'Inverse_ascii_',
   aformat         = '(512E16.4,1x)',
/
```

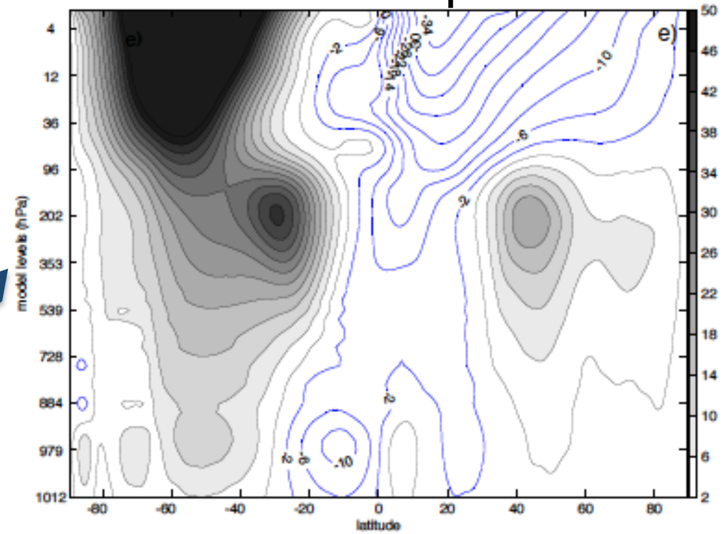
```
& filter_cnf
      eig_n_s        = 1,
      eig_n_e        = 70,
      wig_n_s        = 1,
      wig_n_e        = 70,
      rot_n_s        = 2,
      rot_n_e        = 70,
      kmode_s        = 100,
      kmode_e        = 85,
      vmode_s        = 410,
      vmode_e        = 70,
/
```

NMF representation of ERA Interim dataset

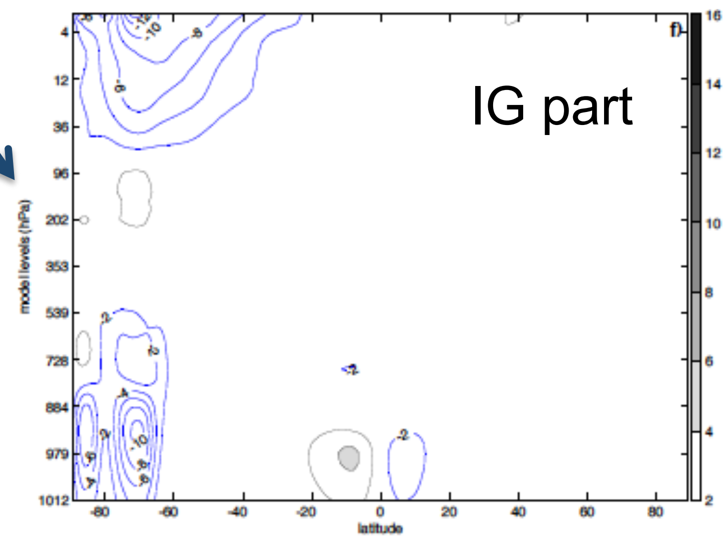
Zonally-averaged zonal winds, **July**



Balanced part

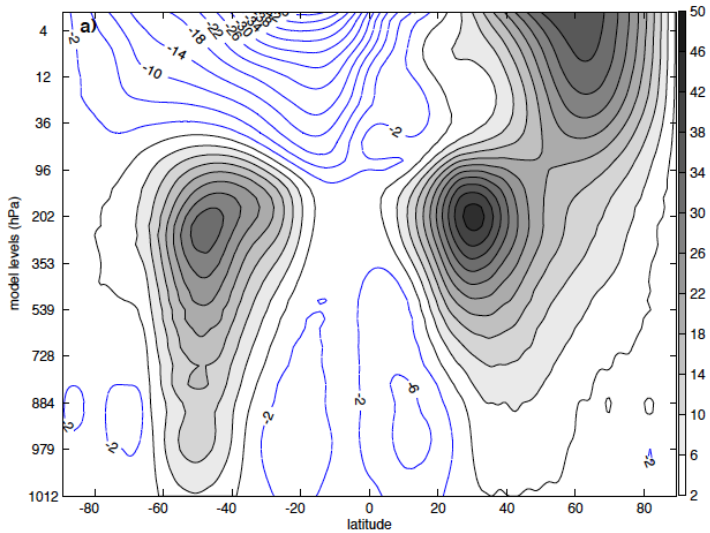


IG part

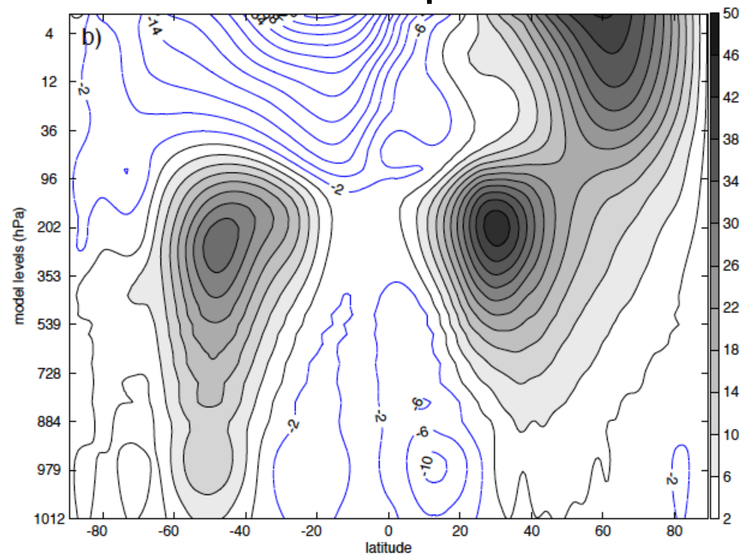


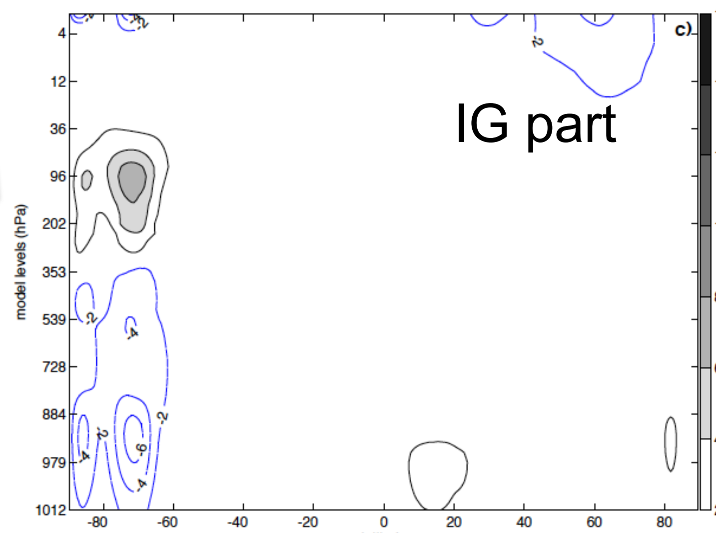
NMF representation of ERA Interim dataset

Zonally-averaged zonal winds, **January**

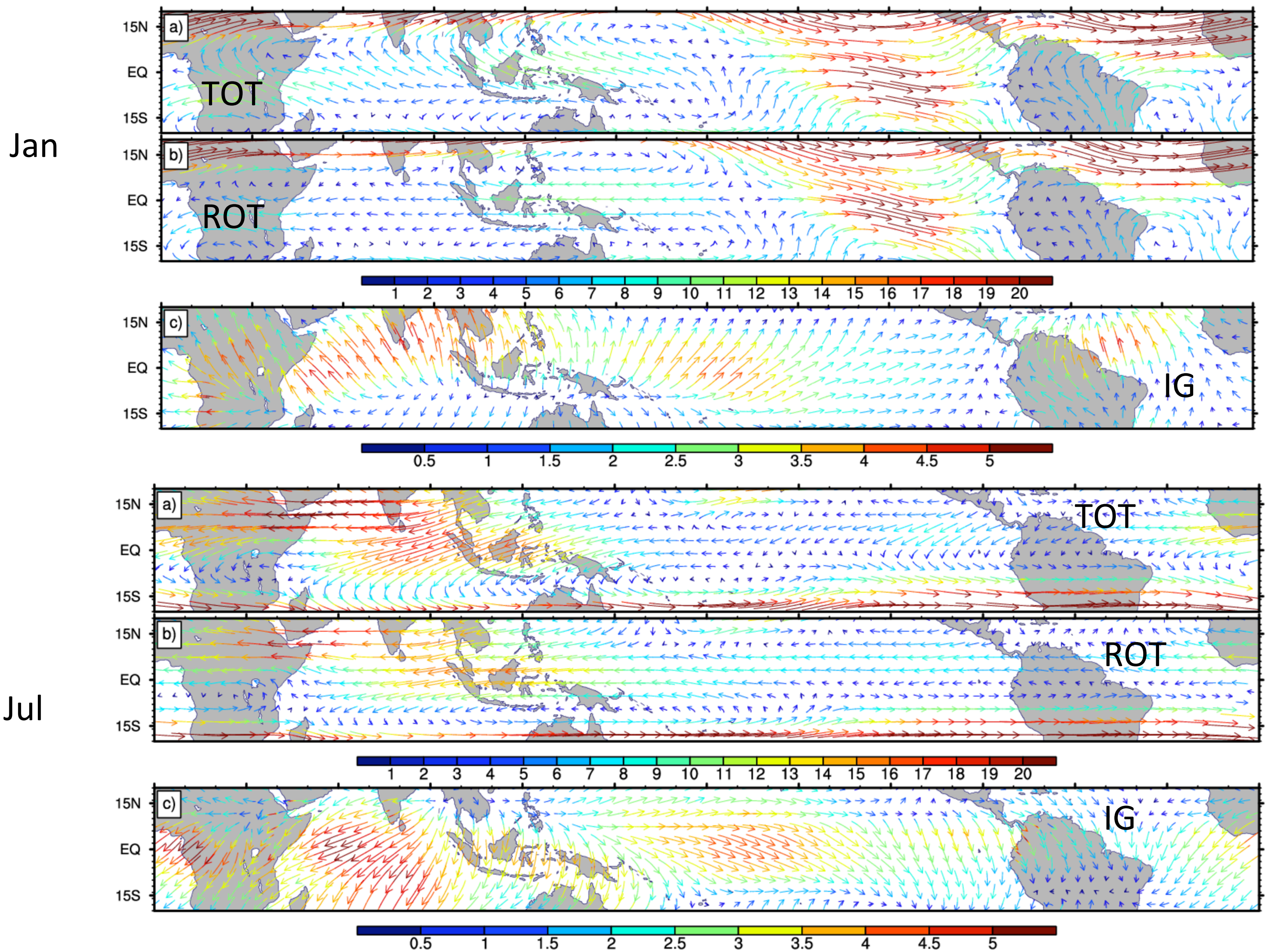


Balanced part



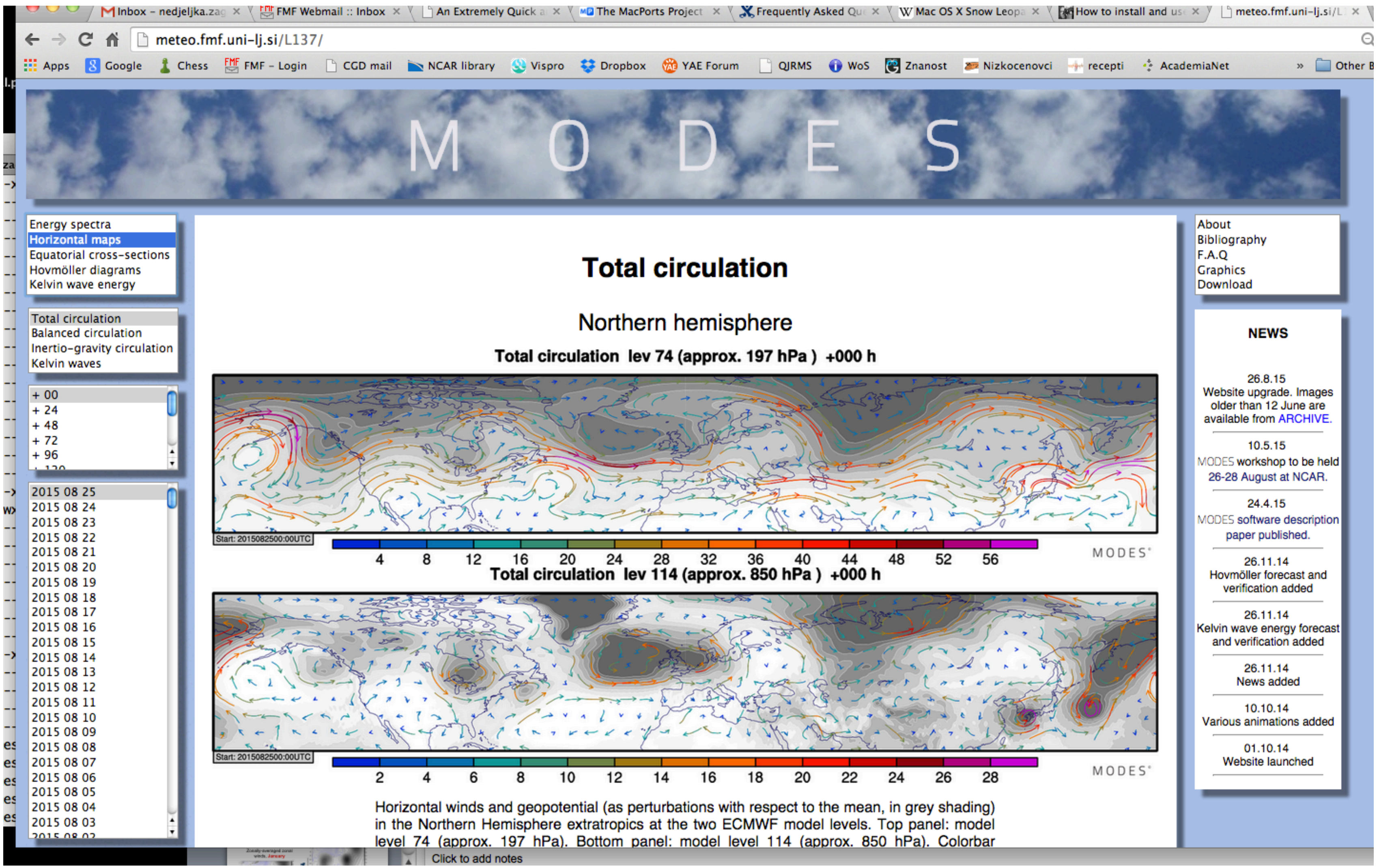


Climatological circulation in the tropics



Real-time modal view of ECMWF forecast

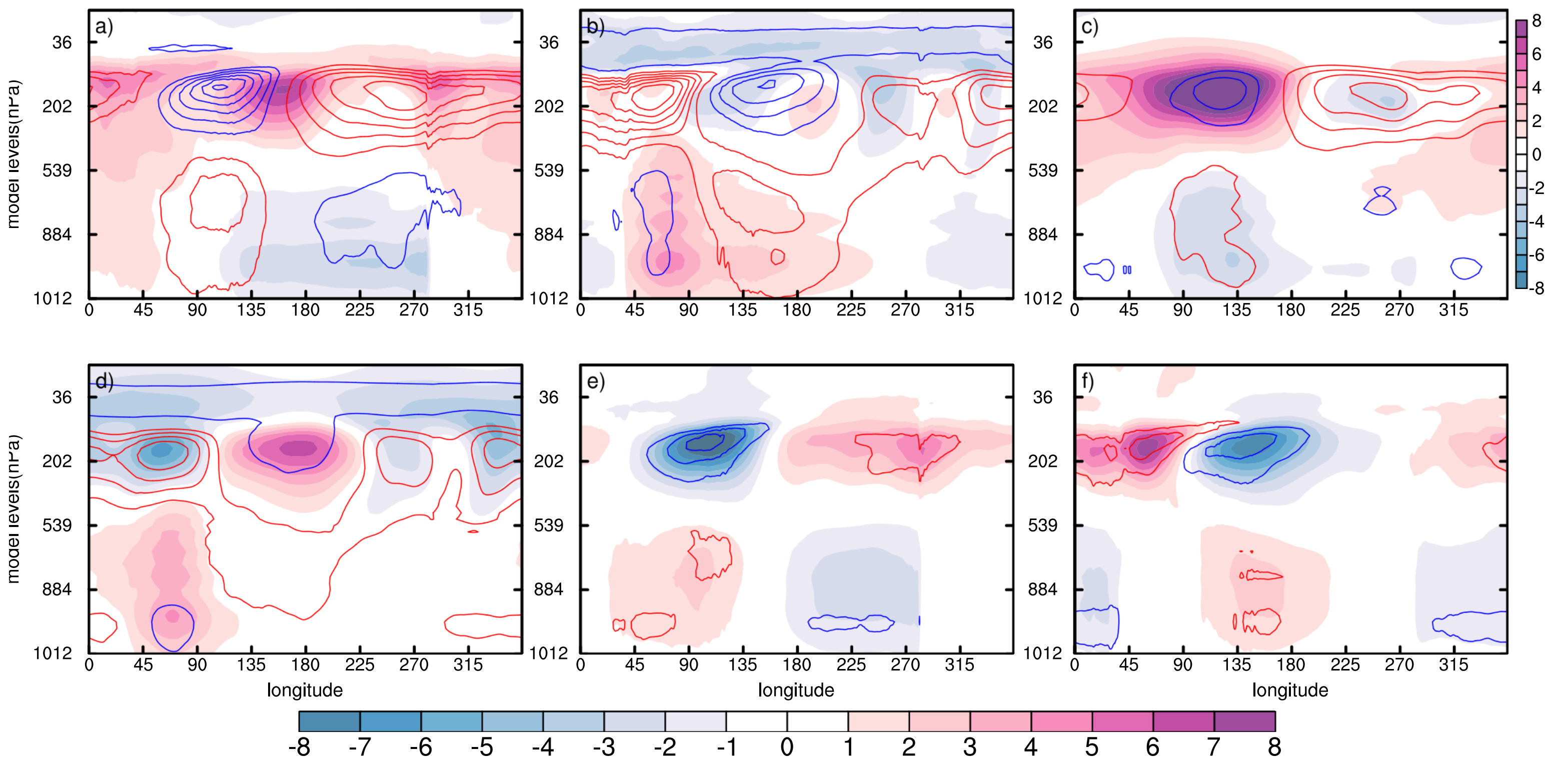
http://meteo.fmf.uni-lj.si/MODES



Modal representation of the MJO

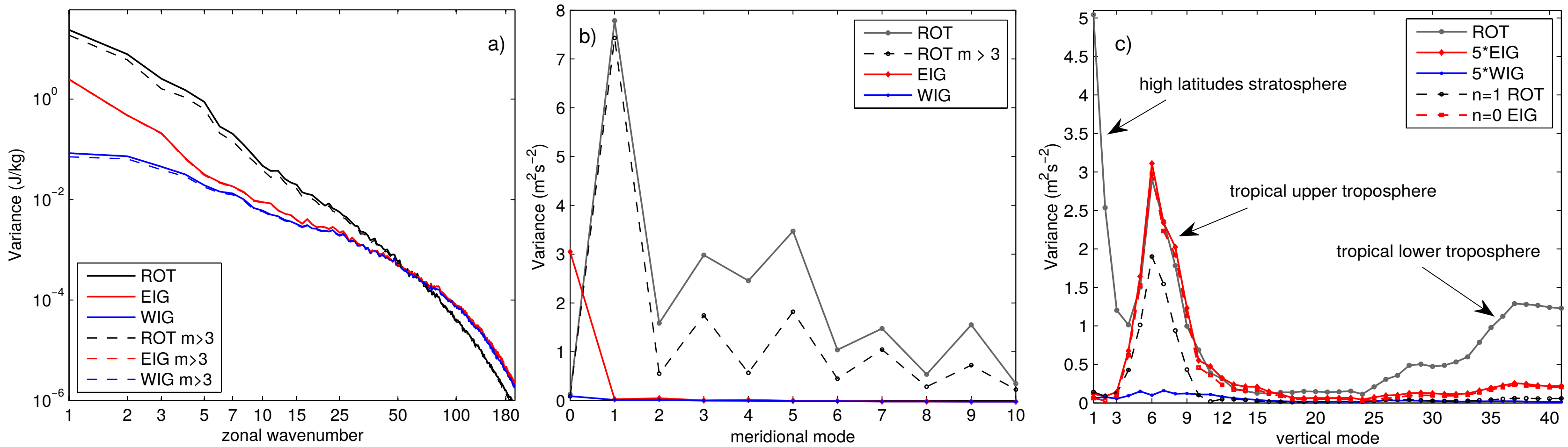
Regression between the MJO index and ERA Interim

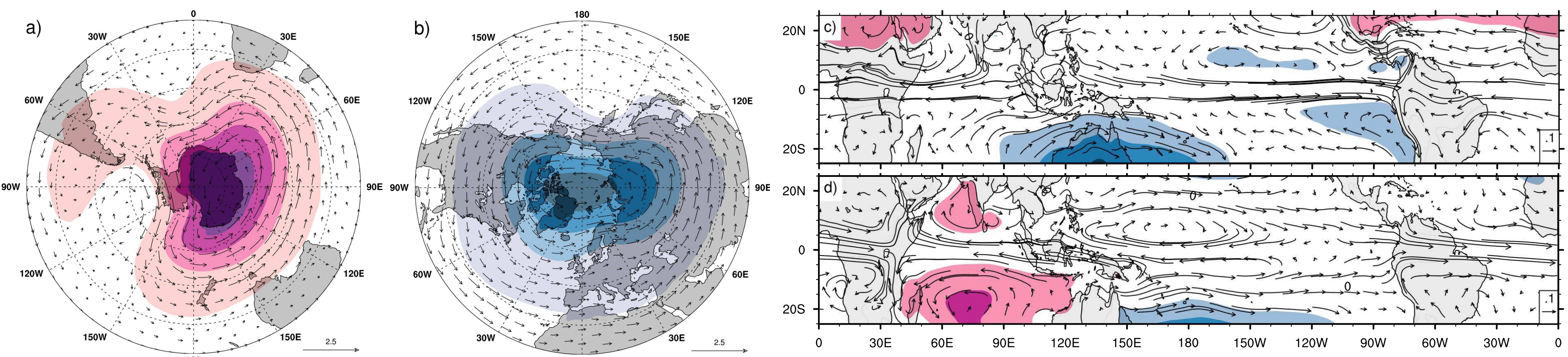
$$\mathcal{R}_n^k(m,\tau) = \frac{1}{N-1}\frac{\sum_{t=1}^{N}\left[\left(\chi_n^k(m,t)-\overline{\chi_n^k(m)}\right)\left(\mathcal{M}_i(t,\tau)-\overline{\mathcal{M}_i}\right)\right]}{Var\left(\overline{\mathcal{M}_i}\right)}$$



Modal representation of MJO

Quantification of the role of various modes in MJO and its teleconnections





Summary

- Representation of the global circulation and mass fields in global NWP and climate models in terms of the normal modes of the Navier-Stokes equations offers an alternative and physically attractive approach to the diagnostic of some properties of the models

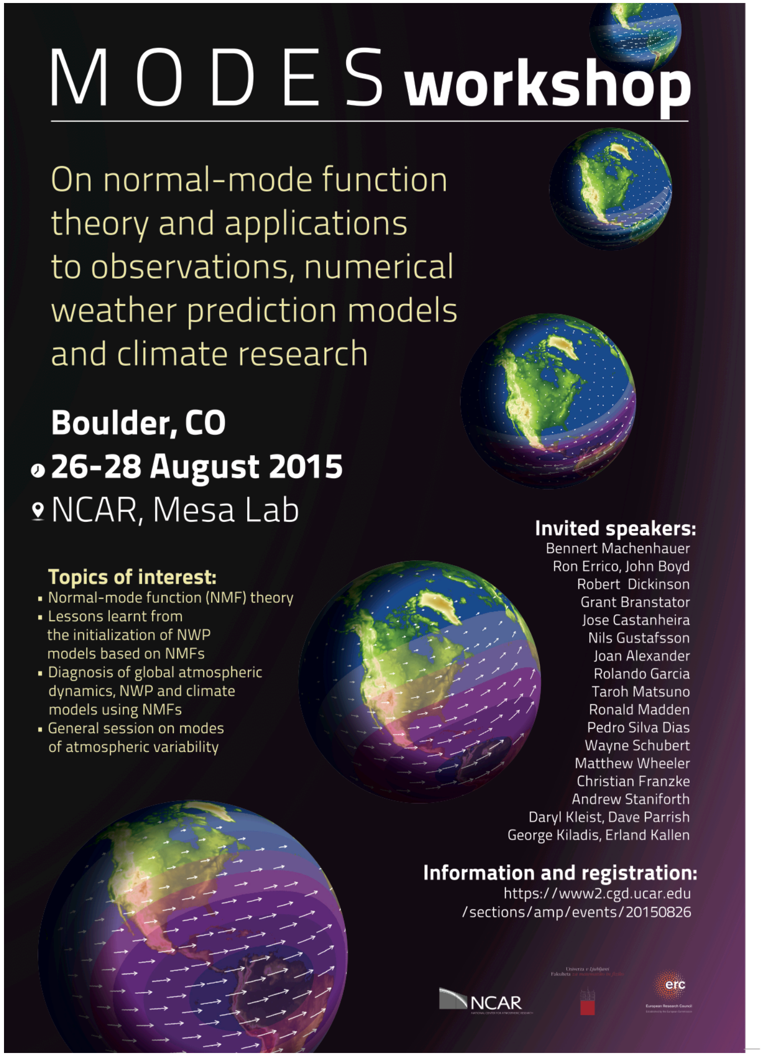
- A new software for the representation of global atmospheric energy, MODES, has been developed under the ERC funding and it is available to the atmospheric research community

- MODES provides possibility to study circulation changes in the coupled climate models in relation to the (un)balanced dynamic. Unbalanced circulation is climatologically small and difficult to diagnose, but it is critical for understanding atmospheric variability

Thank you very

much for your

attention!



http://meteo.fmf.uni-lj.si/MODES