# Gravity Waves in Shear and

# Implications for Organized Convection

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# INTRODUCTION

# Abstract

- Gravity waves can trigger/favor the formation of new convection
- If new convection forms repeatedly on a preferred side of preexisting convection, then a *convectively coupled wave* is formed
- What creates a preferred side? Hypothesis: wind shear
- Design a simple model for interactions of gravity waves and wind shear
- Results:
  - Wind shear can create a preferred side
  - Jet shears create the greatest difference in favorability between two sides
  - Predictions of preferred propagation direction of convectively coupled waves in a given background wind shear
- Other application:
  - Formation of new cells within an individual mesoscale convective system
  - Resonance renders upstream more favorable than downstream

## Gravity waves and organized convection



Buoyancy anomalies excited by top-heavy heating



Convection can excite gravity waves

Gravity waves can excite new convection

Theory: the role of (1) deep convection and (2) stratiform heating



from Tulich and Mapes (2008) and Mapes (1993)

# Convectively coupled waves: Envelopes of mesoscale convective systems



- Embedded cloud systems propagate in opposite direction of wave envelope
- New cloud systems tend to form on a preferred side of preexisting cloud systems
- What causes wave trains to form preferentially (rather than scattered convection)?
- What determines the preferred propagation direction of the convectively coupled wave?
- Hypothesis: interactions of gravity waves with wind shear

# SIMPLIFIED MODEL

### Starting point: Hydrostatic Boussinesq equations

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + W \frac{\partial U}{\partial z} + \frac{\partial P}{\partial x} = 0$$
$$\frac{\partial P}{\partial z} = g \frac{\Theta}{\theta_{ref}}$$
$$\frac{\partial \Theta}{\partial t} + U \frac{\partial \Theta}{\partial x} + W \frac{\partial \Theta}{\partial z} + W \frac{d\theta_{bg}}{dz} = 0$$
$$\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} = 0$$

U = horizontal velocity P = pressure W = vertical velocity  $\Theta =$  temperature

### Gravity waves in the tropical atmosphere

Linear waves:

- Independent vertical modes:  $U(x, z, t) = \sum_{j} u_j(x, t) \cos jz$ , etc.
- Shallow water system for each vertical mode j:



#### Nonlinear waves:

• Project nonlinear equations

$$\partial_t U + U \partial_x U + W \partial_z U + \partial_x P = 0$$

onto vertical modes

$$U(x, z, t) = u_1(x, t) \cos z + u_2(x, t) \cos 2z$$

• The result is ...

#### 2-Mode Shallow Water Equations

$$\begin{cases} \frac{\partial u_1}{\partial t} - \frac{\partial \theta_1}{\partial x} = -\frac{3}{\sqrt{2}} \left[ u_2 \frac{\partial u_1}{\partial x} + \frac{1}{2} u_1 \frac{\partial u_2}{\partial x} \right] \\ \frac{\partial \theta_1}{\partial t} - \frac{\partial u_1}{\partial x} = -\frac{1}{\sqrt{2}} \left[ 2u_1 \frac{\partial \theta_2}{\partial x} + 4\theta_2 \frac{\partial u_1}{\partial x} - u_2 \frac{\partial \theta_1}{\partial x} - \frac{1}{2} \theta_1 \frac{\partial u_2}{\partial x} \right] \\ \left( \frac{\partial u_2}{\partial t} - \frac{\partial \theta_2}{\partial x} \right] = 0 \end{cases}$$

$$\begin{cases} \partial t & \partial x \\ \frac{\partial \theta_2}{\partial t} - \frac{1}{4} \frac{\partial u_2}{\partial x} &= -\frac{1}{2\sqrt{2}} \left[ u_1 \frac{\partial \theta_1}{\partial x} - \theta_1 \frac{\partial u_1}{\partial x} \right] \end{cases}$$

• Nonlinear, hydrostatic internal gravity waves with effect of background shear

# CONVECTIVELY COUPLED WAVES

# Numerical experiment WITHOUT wind shear



Results symmetric to east and west of forcing

## Numerical experiment WITH wind shear



- West of forcing is more favorable for new convection than east
- Agrees with observations for this wind shear (Wu and LeMone, 1999)



- Consistent with features of CCW envelope and embedded cloud systems
  - Individual cloud systems propagate  $\mathit{eastward}$
  - Convectively coupled wave propagates westward

#### Optimal shears for east–west asymmetry

A measure of the east–west asymmetry due to wind shear:

• the jump in  $\theta$  across the source,  $[\theta] = \theta^+ - \theta^-$ 

Which shear profiles  $\overline{U}(z)$  maximize  $[\theta_1]$ ? Which shear profiles  $\overline{U}(z)$  lead to  $[\theta_1] = 0$ 

Use linear theory with singular source term:

$$\partial_t \mathbf{u} + A(\bar{\mathbf{u}})\partial_x \mathbf{u} = \mathbf{S}^* \delta(x)$$

#### Results:

Jet shears maximize  $[\theta_1]$ 



Profiles with zero shear at upper levels lead to  $[\theta_1] = 0$ 



# MESOSCALE CONVECTIVE SYSTEMS

# Gravity waves can excite new convective cells



- New cells initiated *ahead of* existing squall line due to gravity waves
- New cells merge with existing squall line
- What are the physical mechanisms involved?

### Numerical experiment with headwind



• Repeat earlier jet-shear experiment with headwind added



- Headwind is equivalent to propagating source (i.e., propagating squall line)
- Headwind confines upstream wave to vicinity of source
- East is more favorable for new convection than west *at low levels*

# Numerical experiment with stronger headwind



Source propagation speed = 15 m/s2nd baroclinic wave speed = 25 m/s $\Rightarrow$  near resonant forcing



- Faster propagation leads to more favorable environment *at low levels*
- If squall line propagation speed  $\approx$  gravity wave speed, then wave amplitude is large due to near-resonance

# Conclusions

- 2-mode shallow water equations:
  - simplified nonlinear model for waves interacting with wind shear
- Predictions of preferred propagation direction of convectively coupled waves in a background wind shear
  - wind shear can lead to east—west asymmetries in favorability for new convection
  - jet shears lead to largest east-west asymmetries
  - linear theory is accurate to within 10 % (usually)
- Initiation of new convective cells ahead of individual convective system
  - Propagation of source leads to *near-resonant forcing and amplification* of upstream waves

Stechmann and Majda (2009), in J. Atmos. Sci. Stechmann et al. (2008), in Theoretical and Computational Fluid Dynamics

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