Supplementary material for "Influence of projected Arctic sea ice loss on polar stratospheric ozone and circulation in spring"

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$\bar{\Psi}^*$ from the downward control principle

In order to establish the connection between changes in stratospheric wave drag and the Brewer-Dobson circulation more quantitatively, we use the downward control principle [*Haynes et al.*, 1991] to diagnose the residual streamfunction associated with the stratospheric wave drag change, similar to *Randel et al.* [2002]. The residual streamfunction for the resolved and parameterized wave drag from downward control is used to compare with the streamfunction derived directly from the momentum equation.

From *Edmon et al.* [1980] and *Andrews et al.* [1987], the Transformed Eulerian Momentum (TEM) equation can be obtained,

$$\frac{\partial \bar{u}}{\partial t} - \hat{f}\bar{v}^* + \bar{\omega}^*\frac{\partial \bar{u}}{\partial p} = DF \tag{1}$$

where DF is the Eliassen-Palm divergence (wave drag) and $\hat{f}$ is the zonal mean absolute vorticity. $\bar{v}^*$ and $\bar{\omega}^*$ are the residual velocity, as follows,

$$\bar{v}^* = \bar{v} - \frac{\partial(\overline{v'\theta'}/\bar{\theta}_p)}{\partial p} \tag{2}$$

$$\bar{\omega}^* = \bar{\omega} + \frac{1}{a\cos\phi}\frac{\partial(\overline{v'\theta'}\cos\phi/\bar{\theta}_p)}{\partial \phi} \tag{3}$$

From zonal-mean continuity equation,

$$\frac{1}{a\cos\phi}\frac{\partial(\bar{v}^*\cos\phi)}{\partial\phi} + \frac{\partial\bar{\omega}^*}{\partial p} = 0 \tag{4}$$

the residual streamfunction can be calculated directly from residual meridonal/vertical velocity, so that

$$(\bar{v}^*, \bar{\omega}^*) = \frac{g}{2\pi a\cos\phi}(\bar{\Psi}^*_p, \; -\frac{1}{a}\bar{\Psi}^*_\phi) \tag{5}$$

where g is acceleration of gravity, a is radius of the Earth. From equation (1) and (5), by neglecting zonal wind tendency, and integrating following angular momentum contours, it is straightforward to derive the residual streamfunction under downward control,

$$\bar{\Psi}^*(\phi, p)|_{D\,control} = -\frac{2\pi a}{g}\frac{\cos\phi}{\hat{f}}\int_0^p DF(\phi, p')dp' \tag{6}$$

The comparison between the direct residual streamfunction at 70 hPa (equation (5)) and the one associated with wave drag diagnosed from downward control principle (equation (6)) is shown in Fig. S1. For the climatology (Fig. S1 a), consistent with *McLandress and Shepherd* [2009]; *Butchart et al.* [2010], the resolved wave and parameterized gravity wave drag (orographic, frontal and convection) can well explain the direct residual streamfunction, with resolved waves playing a more important role in the extratropics. For the response to Arctic sea ice loss (Fig. S1 b), almost all of the BDC reduction within the polar cap is caused by the resolved planetary waves in the stratosphere, with gravity wave drag making only a minor contribution. This provides further evidence for the connection between stratospheric wave change and BDC reduction.

References

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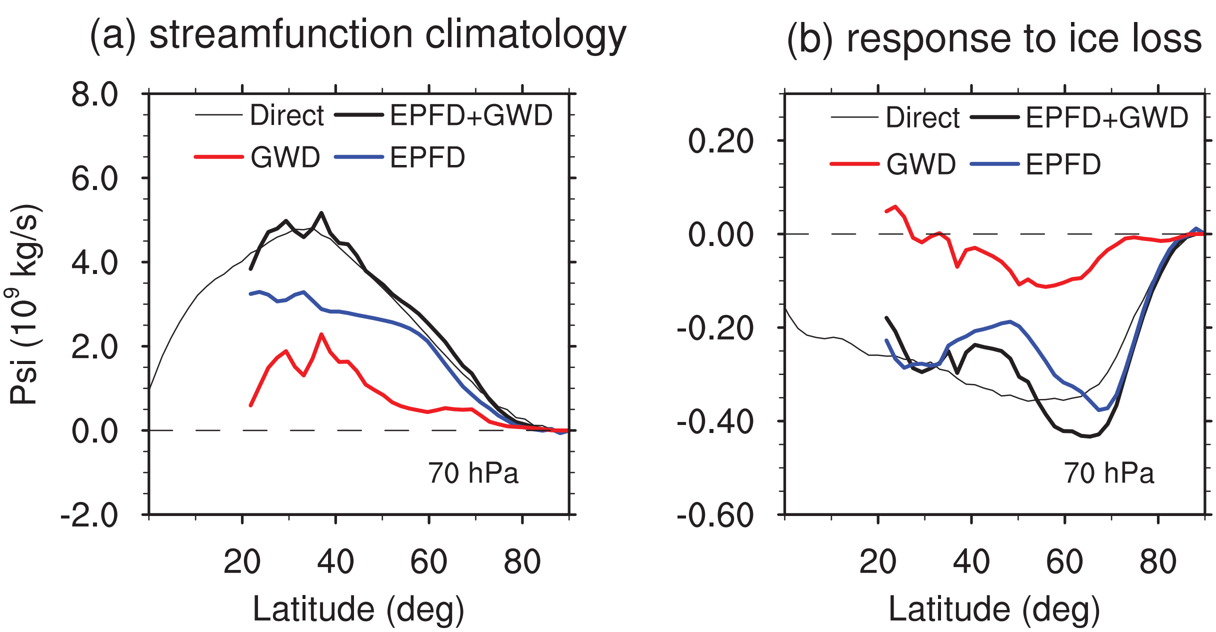

Figure S1: Downward control residual streamfunction in March at 70 hPa for (a) control climatology and (b) response to Arctic sea ice loss from: resolved wave drag (blue curve); parameterized gravity wave drag (red curve), and the sum of the resolved and parameterized wave drag (thick black curve). The thin black curve denotes the streamfunction calculated directly from equation (5).

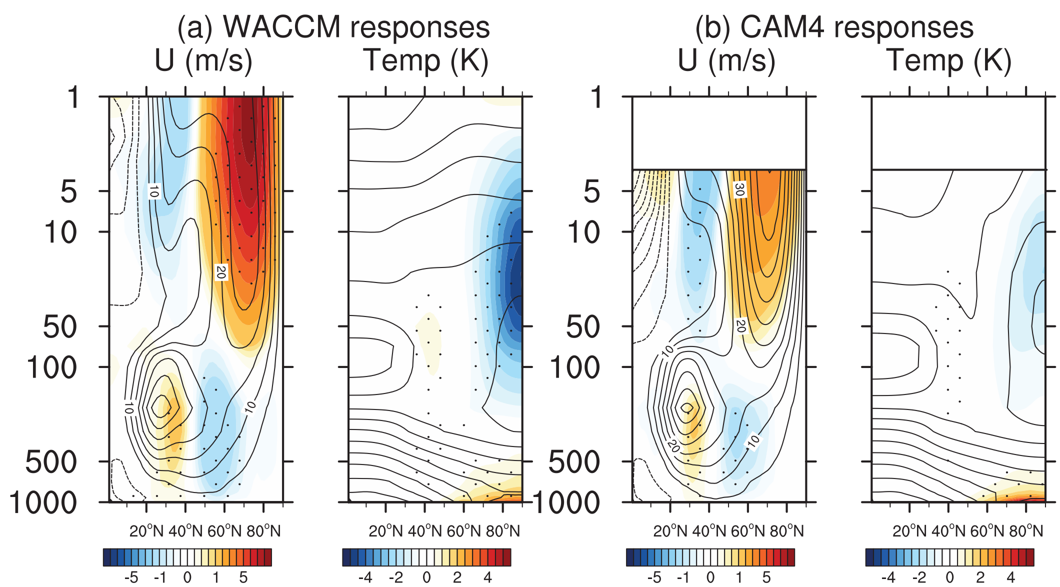

Figure S2: (a) Zonally-averaged zonal wind and temperature for the control run climatology (contours, interval of 5 m s^{-1} and 10 K) and the responses to Arctic sea ice loss (shading, stippled region indicates >95% statistical significance) in March in WACCM simulation. (b) Similar to (a), but for CAM4 simulation.